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# ACI 445.1R-12

# Report on Torsion in Structural Concrete

## Reported by Joint ACI-ASCE Committee 445


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The committee would like to thank the following individuals for their contribution to this report: Mohammad Ali, Neal S. Anderson, Shri Bhide, Michael D. Collins, Maria Cristina de Lima, Leonard Elfgren, Christos Karayannis, Liang-Jenq Leu, Mohammad Mansour, Basile Rabbat, Khaldoun Rahal, and Paul Zia.



*A clear understanding of the effects of torsion on concrete members is essential to the safe, economical design of reinforced and prestressed concrete members. This report begins with a brief and systematic summary of the 180-year history of torsion of structural concrete members, new and updated theories and their applications, and a historical overview outlining the development of research on torsion of structural concrete members. Historical theories and truss models include classical theories of Navier, Saint-Venant, and Bredt; the three-dimensional (3-D) space truss of Rausch; the equilibrium (plasticity) truss model of Nielson as well as Lampert and Thürlimann; the compression field theory (CFT) by Collins and Mitchell; and the softened truss model (STM) by Hsu and Mo.*

*This report emphasizes that it is essential to the analysis of torsion in reinforced concrete that members should: 1) satisfy the equilibrium condition (Mohr's stress circle); 2) obey the compatibility condition (Mohr's strain circle); and 3) establish the constitutive relationships of materials such as the "softened" stress-strain relationship of concrete and "smeared" stress-strain relationship of steel bars.*

*The behavior of members subjected to torsion combined with bending moment, axial load, and shear is discussed. This report deals with design issues, including compatibility torsion, spandrel beams, torsional limit design, open sections, and size effects. The final two chapters are devoted to the detailing requirements of transverse and longitudinal reinforcement in torsional members with detailed, step-by-step design examples for two beams under torsion using ACI (ACI 318-11), European (EC2-04), and Canadian Standards Association (CSA-A23.3-04) standards. Two design examples are given to illustrate the steps involved in torsion design. Design Example 1 is a rectangular reinforced concrete beam under pure torsion, and Design Example 2 is a prestressed concrete girder under combined torsion, shear, and flexure.*



**Keywords:** combined action (loading); compatibility torsion; compression field theory; equilibrium torsion; interaction diagrams; prestressed concrete; reinforced concrete; shear flow zone; skew bending; softened truss model; spandrel beams; struts; torsion detailing; torsion redistribution; warping.




## CONTENTS




ACI 445.1R-12 was adopted and published March 2013.

## CHAPTER 1—INTRODUCTION AND SCOPE

### 1.1—Introduction

Accounting for the effects of torsion is essential to the safe design of structural concrete members, requiring a full knowledge of the effects of torsion and a sound understanding of the analytical models that can easily be used for design. For over three decades, considerable research has been conducted on the behavior of reinforced concrete members under pure torsion and torsion combined with other loadings. Likewise, analytical models have been developed based on the truss model concept. Several of these models were developed to predict the full load history of a member, whereas others are simplified and used only to calculate torsional strength. Many models developed since the 1980s account for softening of diagonally cracked concrete.

This report reviews and summarizes the evolution of torsion design provisions in ACI 318, followed with a summary of the present state of knowledge on torsion for design and analysis of structural concrete beam-type members. Despite a vast amount of research in torsion, provisions of torsion design did not appear in ACI 318 until 1971 (ACI 318-71), although ACI 318-63 included a simple clause regarding detailing for torsion. Code provisions in 1971 were based on Portland Cement Association (PCA) tests (Hsu 1968b).

These provisions were applicable only to rectangular nonprestressed concrete members. In 1995, ACI 318-95) adopted an approach based on a thin-tube, space truss model previously used in the Canadian Standards Association (CSA-A23.3-77) code and the Comité Euro-International du Béton (CEB)-FIP code (1978). This model permitted treatment of sections with arbitrary shape and prestressed concrete (Ghoneim and MacGregor 1993; MacGregor and Ghoneim 1995). The ACI 318-02 code extended the application of the (ACI 318) 1995 torsion provisions to include prestressed hollow sections. ACI 318 allows the use of alternative design methods for torsional members with a cross section aspect ratio of 3 or greater, like the procedures of pre-1995 editions of ACI 318 or the Prestressed Concrete Institute (PCI) method (Zia and Hsu 1978).

This report reviews and summarizes the present state of knowledge on torsion and reviews their use as a framework for design and analysis of structural concrete beam-type members. Chapter 3 presents a historical background outlining the development of research on torsion of structural concrete members. The general behavior of reinforced and prestressed concrete members under pure torsion is discussed in Chapter 4. In Chapter 5, the compression field theory (CFT) and softened truss model (STM) are presented in detail. Chapter 5 also includes a description of two graphical methods (Rahal 2000a,b; Leu and Lee 2000). The behavior of members subjected to torsion combined with shear, flexure, and axial load is discussed in Chapter 6. Chapter 7 introduces additional design issues related to torsion, such as precast spandrel beams, torsion limit design, size effect, open sections, and torsional moment distribution. Detailing of torsional members is described in Chapter 8. Chapter 9 covers detailed design examples of several beams subjected to torsion using ACI 318, EC2-04, and CSA-A23.3-04 design equations, and additional graphical design methods reported by researchers.

### 1.2—Scope

Theories presented in this report were developed and verified for building members of typical size. For application to large-scale members, size effects should be considered. They could present a serious safety issue when using the shear strength equations provided in the design standard, which cannot take into account the shear strength reduction in large-scale members caused by loss of aggregate interlock behavior. Experimental information on large-scale torsional members is lacking.

## CHAPTER 2—NOTATION AND DEFINITIONS

The material presented is a summary of research carried out worldwide and spanning more than four decades, making unification of the symbols and notations used by the various researchers and design codes a challenge. In some cases, mostly for graphs and figures, the notation is kept as originally published.

### 2.1—Notation

$a$ = moment arm for bending, mm (in.)
$a_c$ = geometric property index
$a_o$ = depth of equivalent rectangular stress block in concrete strut of torsional member, mm (in.)
$A$ = area of yield surface, mm$^2$ (in.$^2$)
$A_{cp}$ = area enclosed by outside perimeter of concrete cross section, mm$^2$ (in.$^2$)
$A_\ell$ = total area of longitudinal reinforcement to resist torsion, mm$^2$ (in.$^2$)
$A_o$ = gross area enclosed by shear flow path, mm$^2$ (in.$^2$) (noted as $A_{tb}$ in Eq. (7.2.6))
$A_{oh}$ = area enclosed by centerline of outermost closed transverse torsional reinforcement, mm$^2$ (in.$^2$)
$A_{ps}$ = area of prestressing reinforcement in flexural tension zone, mm$^2$ (in.$^2$)
$A_s$ = area of nonprestressed longitudinal tension reinforcement, mm$^2$ (in.$^2$)
$A_s'$ = area of longitudinal compression reinforcement, mm$^2$ (in.$^2$)
$A_t$ = area of one leg of a closed stirrup resisting torsion within spacing $s$, mm$^2$ (in.$^2$) (noted as $A_{tb}$ in Eq. (7.2.6))
$b$ = width of compression face of member, mm (in.)
$b_c$ = width of stirrups, mm (in.)
$B$ = integral of $T_w$
$C$ = cross-sectional constant to define torsional properties of a beam
$d_v$ = distance between top and bottom longitudinal reinforcement, mm (in.)
$D$ = cross-sectional depth used in fracture mechanics calculations, mm (in.)
$D_0$ = size effect constant for computing $\sigma_N$ for plain concrete section
$D_1$ = normalized constant to represent characteristic structural dimensions used in fracture mechanics calculations
$D_b$ = size effect constant for computing $\sigma_N$ for reinforced concrete section
$D_c$ = total energy dissipated on discontinuous concrete yield surface
$D_s$ = total energy dissipated by reinforcement
$e$ = moment arm for torsion, mm (in.)
$E_c$ = modulus of elasticity of concrete, MPa (psi)
$E_{ps}$ = modulus of elasticity of prestressed reinforcement in flexural tension zone, MPa (psi)
$E_{ps}'$ = tangential modulus of Ramberg-Osgood curve at zero load MPa (psi)
$E_s$ = modulus of elasticity of reinforcement and structural steel, MPa (psi)
$EJ_w$ = rigidity of beam under warping torque, N·m$^2$ (lb-in.$^2$)
$f_c'$ = characteristic concrete cylinder compressive strength, MPa (psi)
$f_c^*$ = concrete effective (plastic) compressive stress, MPa (psi)
$f_{ck}$ = characteristic compressive strength of concrete, MPa (psi); $f_{ck} = f_{cm} - 8$ MPa ($f_{ck} = f_{cm} - 1200$ psi)
$f_{cm}$ = mean compressive strength of concrete, MPa (psi)
$f_d$ = diagonal concrete stress, MPa (psi)
$f_{ds}$ = diagonal concrete stress corresponding to strain $\varepsilon_{ds}$, MPa (psi)
$f_\ell$ = reinforcement stress in $\ell$ direction, MPa (psi)
$f_{\ell p}$ = prestressing reinforcement stress in the $\ell$ direction, MPa (psi)
$f_{\ell y}$ = specified yield strength of longitudinal reinforcement, MPa (psi)
$f_p$ = stress in prestressing reinforcement; $f_p$ becomes $f_{\ell p}$ or $f_{tp}$ when applied to longitudinal and transverse reinforcement, respectively, MPa (psi)
$f_{p0.1}$ = characteristic yield strength of prestressing reinforcing strands, MPa (psi); $f_{p0.1} = 0.9f_u$
$f_{pc}$ = compressive stress in concrete due to prestress, MPa (psi)
$f_{pk}$ = characteristic tensile strength of prestressing reinforcing strands, MPa (psi); $f_{pk} = f_{pu}$
$f_{po}$ = effective prestress after losses in prestressing reinforcement, MPa (psi)
$f_{pu}$ = specified tensile strength of prestressing reinforcement, MPa (psi)
$f_{p,ud}$ = design ultimate strength of prestressing reinforcing strands, MPa (psi); $f_{p,ud} = f_{pk}/\gamma_s$ ($\gamma_s = 1.15$)
$f_r$ = modulus of rupture of concrete, MPa (psi)
$f_t$ = reinforcement stress in $t$ direction, MPa (psi)
$f_t'$ = uniaxial tensile strength of concrete, MPa (psi)
$f_t^*$ = concrete effective (plastic) tensile stress, MPa (psi)
$f_{tp}$ = prestressing reinforcement stress in $t$ direction, MPa (psi)
$f_{ty}$ = specified yield strength of transverse reinforcement, MPa (psi)
$f_y$ = specified yield strength of reinforcement, MPa (psi)
$f_{yd}$ = design yield strength reinforcing steel, MPa (psi); $f_{yd} = f_y/\gamma_s$ ($\gamma_s = 1.15$)

$f_{y\ell}$ = yield strength of the torsional longitudinal reinforcement, MPa (psi)
$f_{yv}$ = torsional hoop yield strength reinforcement, MPa (psi)
$G$ = shear modulus, MPa (psi)
$h$ = overall thickness or height of a member, mm (in.)
$H_o$ = horizontal force in radial direction, N (lb) (Chapter 7)
$I_p$ = polar moment of inertia, $mm^4$ ($in.^4$)
$k_1$ = ratio of average stress to peak stress
$K$ = value from Mohr-Coulomb yield criterion
$K_f$ = flexural stiffness of floor beams, N·$m^2$ (lb-$in.^2$)
$K_{ts}$ = torsional stiffness of spandrel beam, N·m/rad (in.-lb/rad)
$\ell$ = span length of beam, mm (in.)
$\ell_f$ = length of flexural beam, mm (in.)
$\ell_q$ = width of shear flow $q$ along top wall (Fig. 4.2(a) and (b)), mm (in.)
$m$ = ratio of effective (plastic) compressive stress to effective (plastic) tensile stress of concrete
$M$ = applied flexural moment at section, N·m (in.-lb)
$M_o$ = pure flexural strength of section, N·m (in.-lb)
$n$ = integer value
$n_R$ = number of redundants
$n_V$ = coefficient describing an under-reinforced, partially under-reinforced, or completely over-reinforced section
$N$ = applied axial load at section, N (lb)
$N_o$ = pure axial strength of section, N (lb)
$p_h$ = perimeter of centerline of outermost closed transverse torsional reinforcement, mm (in.)
$p_o$ = perimeter of outer concrete cross section, mm (in.) (sometimes noted as $p_{cp}$)
$P$ = applied concentrated load, N (lb)
$q$ = shear flow, N/m (lb/in.)
$r$ = ratio of top-to-bottom yield forces of the longitudinal reinforcement
$r$ = size effect constant for computing $\sigma_N$
$R$ = shape parameter used in Ramberg-Osgood
$s$ = center-to-center spacing of longitudinal and transverse reinforcements, mm (in.)
$s_l$ = center-to-center spacing of longitudinal reinforcement, mm (in.)
$s_t$ = center-to-center spacing of transverse reinforcement, mm (in.)
$t$ = wall thickness of hollow section, mm (in.)
$t_d$ = thickness of shear flow zone, mm (in.)
$T$ = applied torsional moment at section, N·m (in.-lb)
$T_c$ = nominal torsional strength provided by concrete, N·m (in.-lb)
$T_{cr}$ = torsional cracking resistance of cross section, N·m (in.-lb)
$T_f$ = applied torsional moment, N·m (in.-lb) (Chapter 9)
$T_{max}$ = maximum torsional moment, N·m (in.-lb) (Chapter 7)
$T_n$ = nominal torsional moment strength, N·m (in.-lb)
$T_o$ = pure torsional strength of section, N·m (in.-lb)
$T_s$ = nominal torsional strength provided by reinforcement, N·m (in.-lb)
$T_u$ = factored torsional moment at section, N·m (in.-lb)
$T_w$ = warping torsional moment, N·m (in.-lb)
$T_{xu}$ = factored balanced torsional strength, N·m (in.-lb)
$T_{xub}$ = balanced torsional strength, N·m (in.-lb)
$\overline{T}_{xub}$ = nondimensional balanced torsional strength, N·m (in.-lb)
$v$ = shearing stress due to shear, MPa (psi)
$v^*$ = plastic flow rate (Chapter 7)
$v_u$ = ultimate shear stress, MPa (psi)
$V$ = applied shear force at section, N (lb)
$V_c$ = nominal shear strength provided by concrete, N (lb)
$V_o$ = pure shear strength of section, N (lb)
$V_u$ = factored shear force at section, N (lb)
$w$ = ultimate distributed load on helical stair, N/m (lb/ft) (Chapter 7)
$W$ = external work, N/m (lb/ft)
$x$ = shorter overall dimension of rectangular part of cross section, mm (in.)
$x_1$ = distance section centroid and an infinitesimally small area of yield surface, mm (in.)
$y$ = longer overall dimension of rectangular part of cross section, mm (in.)
$z$ = distance along axis of beam, mm (in.)
$\alpha$, $\beta$ = Saint-Venant's coefficients for homogeneous torsional section
$\alpha^*$, $\beta^*$ = rotational angles in beam subjected to torsion (Chapter 7)
$\alpha_1$ = stress block factor given as ratio of $f_d$ to $f_c'$ (Chapter 5)
$\beta$ = factor relating effect of longitudinal strain on shear strength of concrete (American Association of State Highway and Transportation Officials (AASHTO) LRFD (general message)
$\beta_1$ = factor relating depth of equivalent rectangular compressive stress block to neutral axis depth; also, block factor given as ratio of $a_o$ to $t_d$ (Fig. 4.5)
$\gamma_1$ = angle along helical stair (in plan) at which maximum torsional moment is assumed to occur
$\gamma_2$ = angle along helical stair (in plan) at which vertical moment is assumed to be zero
$\gamma_{\ell t}$ = shear strain
$\varepsilon_d$ = strain in $d$ direction
$\varepsilon_{dec}$ = strain in prestressing reinforcement at decompression of concrete
$\varepsilon_{ds}$ = maximum strain at concrete strut surface (Fig. 4.3)
$\varepsilon_h$ = strain in hoop direction $\varepsilon_\ell$
$\varepsilon_{\ell y}$ = yield strain in $\ell$ direction
$\varepsilon_o$ = strain at peak compressive stress $f_c'$ in concrete
$\varepsilon_p$ = peak strain in concrete
$\varepsilon_r$ = strain in $r$ direction
$\varepsilon_s$ = strain in nonprestressed reinforcement; $\varepsilon_s$ becomes $\varepsilon_\ell$ or $\varepsilon_t$ when applied to longitudinal or transverse reinforcement, respectively
$\varepsilon_t$ = strain in $t$ direction
$\varepsilon_{ty}$ = yield strain in $t$ direction
$\varepsilon_{uk}$ = characteristic total elongation of reinforcing steel at ultimate load
$\varepsilon_x$ = longitudinal strain at midheight of concrete section
$\zeta$ = softening coefficient of concrete strut

$\eta_\ell$ = normalized reinforcement ratio of longitudinal reinforcement
$\eta_{lb}$ = balanced normalized reinforcement ratio of longitudinal reinforcement
$\eta_t$ = normalized reinforcement ratio of transverse steel reinforcement
$\eta_{tb}$ = balanced normalized reinforcement ratio of transverse steel reinforcement
$\theta$ = angle between axis of strut, compression diagonal, or compression field and tension chord of the member; also, the angle between $\ell$-$t$ direction/axis and $d$-$r$ direction/axis, radians
$\xi$ = coefficient equal to 1 for rectangular sections and to $\pi/4$ for circular cross sections; $\xi$ can be taken as unity for all shapes of cross sections with only negligible loss of accuracy for $A_o$ and $p_o$
$\rho_\ell$ = reinforcement ratio in $\ell$ direction
$\rho_{\ell p}$ = prestressing reinforcement ratio in $\ell$ direction
$\rho_t$ = reinforcement ratio in $t$ direction
$\rho_{tp}$ = prestressing reinforcement ratio in $t$ direction
$\sigma$ = compressive stress acting in combination with torsional moment, psi (MPa)
$\sigma_0$ = nominal torsional strength according to the current code specifications based on plastic limit analysis, MPa (psi)
$\sigma_d$ = principal stress in $d$ direction for concrete struts, MPa (psi)
$\sigma_\ell$ = normal stress in longitudinal direction for reinforced concrete, MPa (psi)
$\sigma_{max}$ = maximum principal tensile stress, MPa (psi)
$\sigma_N$ = nominal strength of structure, MPa (psi)
$\sigma_r$ = principal stress in $r$ direction for the concrete struts, MPa (psi)
$\sigma_t$ = normal stress in the transverse direction for reinforced concrete, MPa (psi)
$\sigma_\infty$ = strength of plain beams according to elastic analysis with maximum stress limited by material strength, MPa (psi)
$\tau$ = shearing stress due to torsion and shear, MPa (psi)
$\tau_{max}$ = maximum shear stress, MPa (psi)
$\tau_{\ell t}$ = applied shear stress in $\ell$-$t$ coordinate for reinforced concrete, MPa (psi)
$\nu$ = uniform plastic effectiveness factor (Chapter 7)
$\nu_c$ = plastic effectiveness factor for compression (Chapter 7)
$\nu_t$ = plastic effectiveness factor for tension (Chapter 7)
$\varphi$ = friction angle
$\phi$ = strength reduction factor
$\phi_c$ = strength reduction factor for concrete (0.65 for cast-in-place, 0.70 for precast concrete)
$\phi_p$ = strength reduction factor for prestressing tendons (0.90)
$\phi_s$ = strength reduction factor for nonprestressed reinforcing bars (0.85)
$\Phi$ = angle of twist in torsional beam, radians/m (radians/in.)
$\Phi'$ = second derivative of rotation with respect to beam's axis $z$
$\Phi''$ = third derivative of rotation with respect to beam's axis $z$
$\Psi$ = bending curvature of concrete strut
$\omega_\ell$ = reinforcement index in $\ell$ direction
$\omega_s$ = functional indicator of an index of reinforcement
$\omega_{s\ell}$ = reinforcement ratio index
$\omega_t$ = reinforcement index in $t$ direction

## 2.2—Definitions

ACI provides a comprehensive list of definitions through an online resource, "ACI Concrete Terminology," http://terminology.concrete.org.

# CHAPTER 8—DETAILING FOR TORSIONAL MEMBERS

## 8.1—General

Torsional moment in a reinforced concrete member is resisted by a circulatory shear flow in a tube along the cross section periphery. The tube can be idealized as a space truss made up of reinforcement ties and concrete struts, as shown in Fig. 3.3.5a. The shear flow induces tensile forces in both the hoop reinforcement and longitudinal reinforcement. Good reinforcement detailing is required to ensure that the hoop and longitudinal reinforcement can develop their yield strength to resist circulatory shear flow.

Good detailing demands consideration of the interaction between the member longitudinal and transverse reinforcement. Although each member type brings about different detailing conditions, the designer should be mindful of this overall force interaction in the member. Transverse reinforcement, oriented either horizontally or vertically, should contain a longitudinal bar at the corners. Enclosure of the longitudinal reinforcement by the transverse reinforcement provides the necessary equilibrium at the joint in the three principal directions, where the three-dimensional force flow is equilibrated.

## 8.2—Transverse reinforcement

**8.2.1** *General*—Once proportioned for torsion and shear, the transverse reinforcement is laid out at a specific longitudinal spacing along the member span. The objective of transverse reinforcement for torsion and shear is to provide the reinforcement around the perimeter to enclose the member core. Typically, this reinforcement has a smaller diameter than the longitudinal reinforcement due to spacing, placement, bending, and proportioning needs. The transverse reinforcement should enclose the perimeter as closely as possible while maintaining clear cover requirements. A closed stirrup is imperative for torsional detailing. In the simplest case—a basic rectangular member cross section—stirrups are provided in a closed rectangular shape to encase the rectangular member core. The hooks of the closed stirrup are developed into the core with 135-degree bends. These bends ensure the hooks are well-anchored to the member core and prevent hook pullout under high torsional loads. Figure 8.2.1a provides an example of a simple rectangular closed stirrup.

Other common examples of cast-in-place member types subject to torsional loads are shown in Fig. 8.2.1b. The key to providing transverse reinforcement in a member subject to torsion is to start with the largest rectangular cross section and provide a rectangular closed stirrup in that section. Alternately, multiple-leg configurations can also be used for this purpose with single or multi-leg pieces or bar layouts to reinforce the cross section. Any protrusions, apertures, ledges, corbels, or other geometric outcroppings are provided with supplemental ties or stirrups developed back into the rectangular core of the individual member.

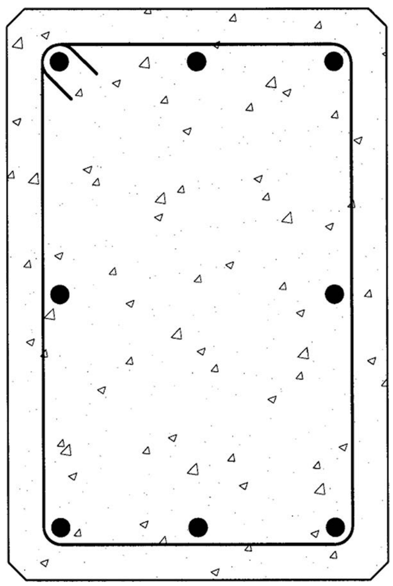


*Fig. 8.2.1a—A typical closed stirrup used in a simple rectangular cross section.*

As shown in Fig. 8.2.1c, good detailing usually dictates that additional ties or stirrups reinforce any protrusions. These ties also have a semi-closed detail with 135-degree or greater hooks developed into the core, which is the region enclosed by the closed stirrup shape. In addition to the closed stirrups and longitudinal bars shown in Fig. 8.2.1b and 8.2.1c, local reinforcement in the disturbed regions or D-regions should accommodate specific load concentrations. To be effective in any size member subjected to torsion, spacing between the closed ties should not exceed about one-half of the smallest dimension of the member, except for slender precast spandrel beams, such as those used in parking structures. In these members, torsional forces cause out-of-plane bending in the web. As described in 7.3, limited testing of such members has not shown signs of spalling or stirrup debonding for which closed stirrups are required. In load tests, slender precast spandrel beams have performed exceptionally well without closed ties. The current state of practice on spandrel beam behavior is contained in a recent study at North Carolina State University (Lucier et al. 2010).

**8.2.2** *Hooks and development considerations*—Stirrups or ties are best terminated with 135- or 180-degree bends. Hooks should be developed into the main core of the member, where greater confinement is present. This detail is important in isolated members, where hook confinement is only provided by the member core, and no other external geometric conditions provide confinement. Practical considerations might dictate the use of simpler stirrup geometry, usually employing 90-degree hooks. When 90-degree hooks are used, confinement should be provided at locations where a slab frames into the beam side or elsewhere as needed. In Fig. 8.2.2a, examples from the *ACI Detailing Manual* (ACI Committee 315 2004) suggest using 90-degrees hooks under

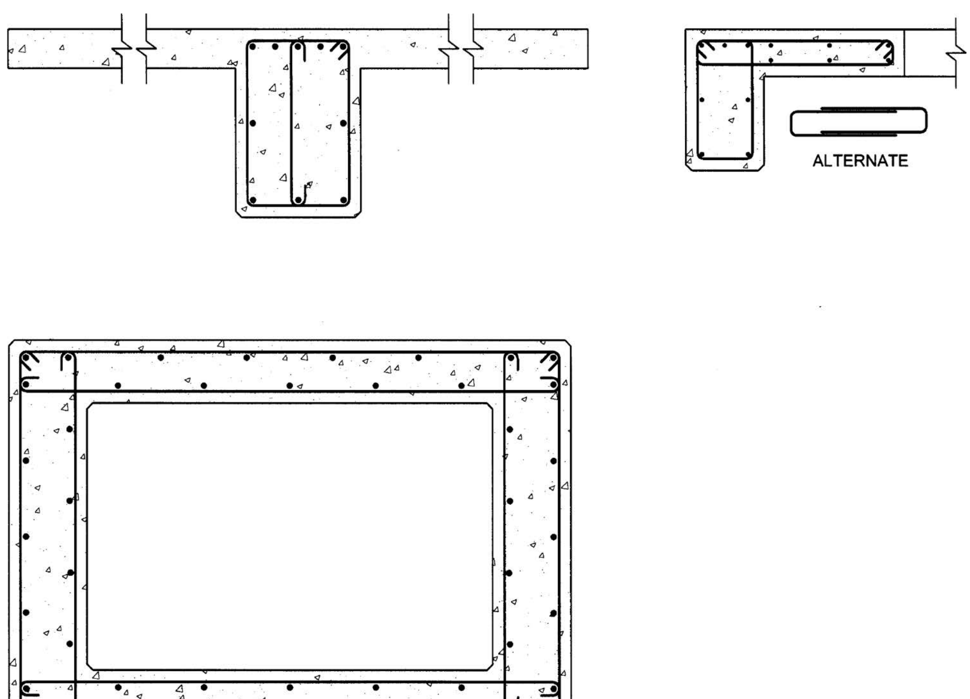



*Fig. 8.2.1b—Examples of transverse torsional detailing in cast-in-place concrete members.*

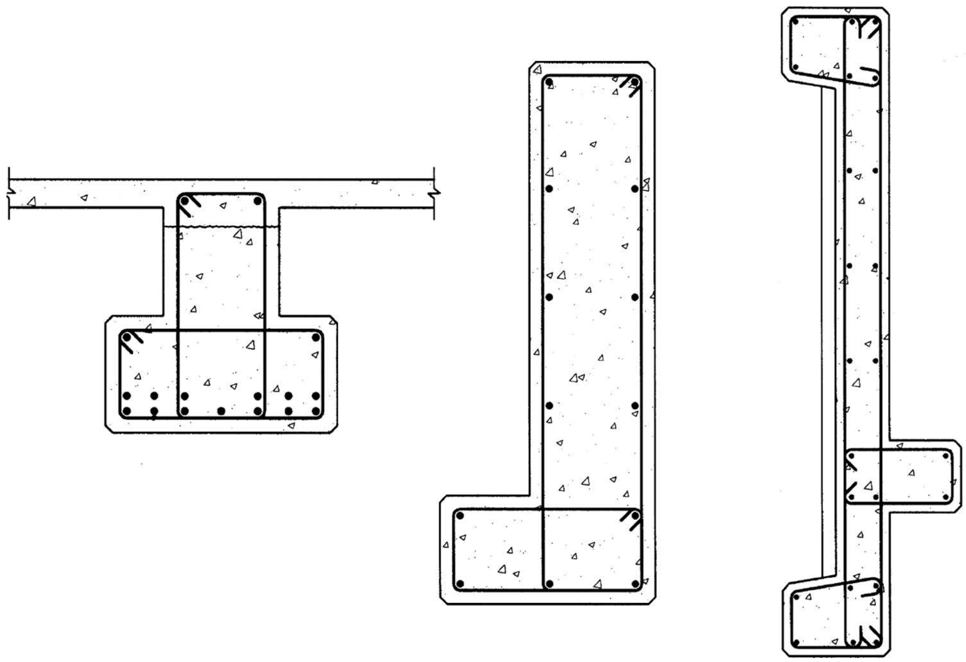


*Fig. 8.2.1c—Examples of transverse torsional detailing in precast concrete members.*

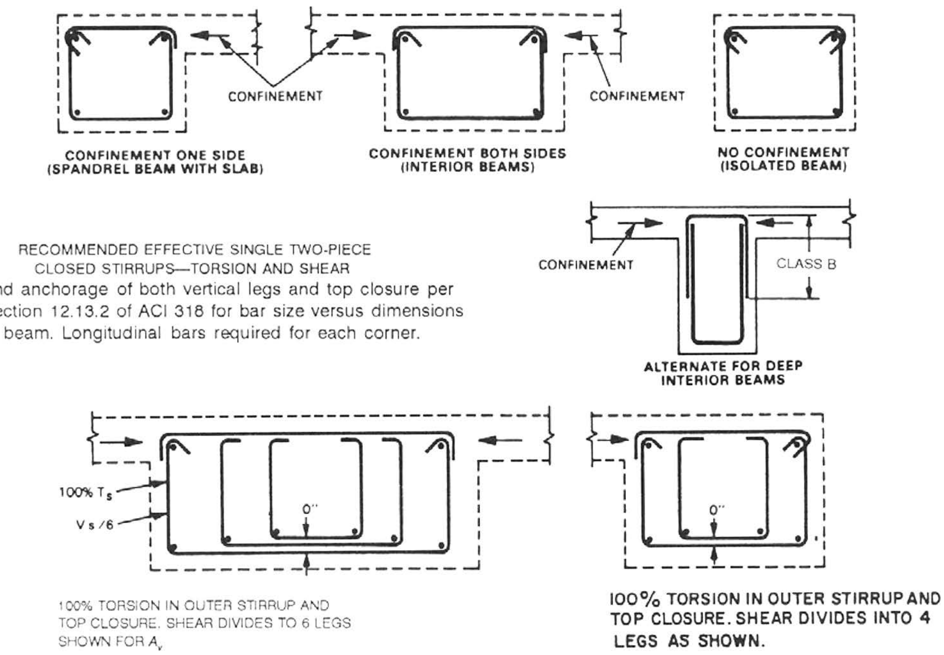



*Fig. 8.2.2a—Recommended two-piece closed single and multiple U-stirrups for members subjected to torsion (ACI Committee 315 2004).*

various side confinement conditions. Examples of poor detailing are reproduced in Fig. 8.2.2b. These recommendations are adopted from the research of Mitchell and Collins (1976).

## 8.3—Longitudinal reinforcement

Longitudinal reinforcement is also proportioned according to torsional requirements and provided around the member cross section perimeter. In beam regions or B-regions, special details need not be provided aside from equal spacing or proportioning of the reinforcement around the perimeter. Splices are proportioned in accordance with ACI requirements. At the end of a cast-in-place member, the perimeter longitudinal reinforcement may have to be developed into a column or other type of rigid vertical member providing torsional restraint.

CSA-A23.3-04 also includes the requirement that "A longitudinal reinforcing bar or bonded prestressing tendon shall be placed in each corner of closed transverse steel reinforcement required for torsion. The nominal diameter of the

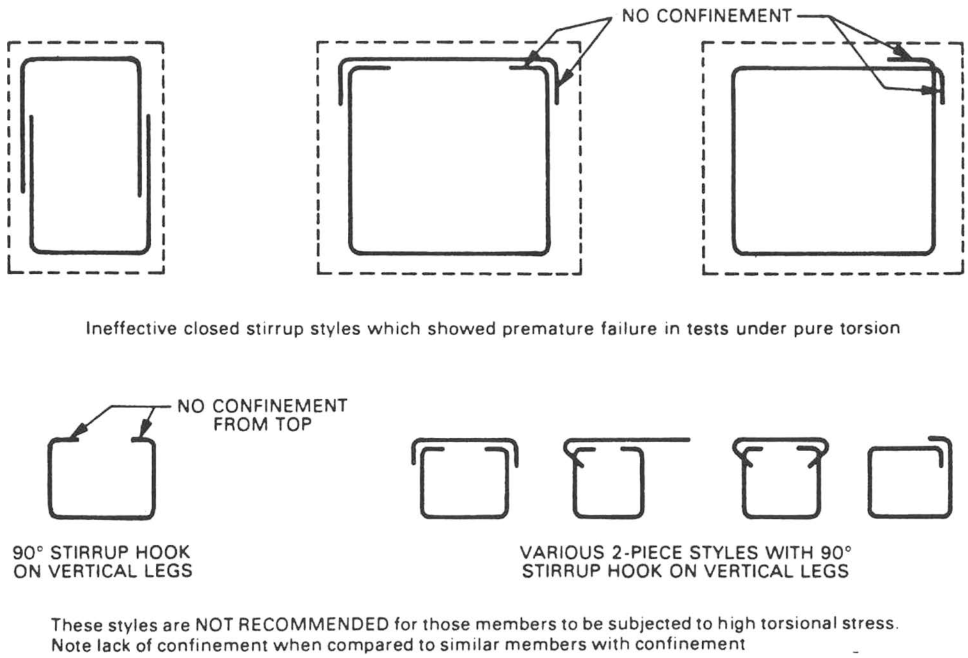



*Fig. 8.2.2b—Ineffective closed stirrup types for members subjected to torsion (ACI Committee 315 2004).*

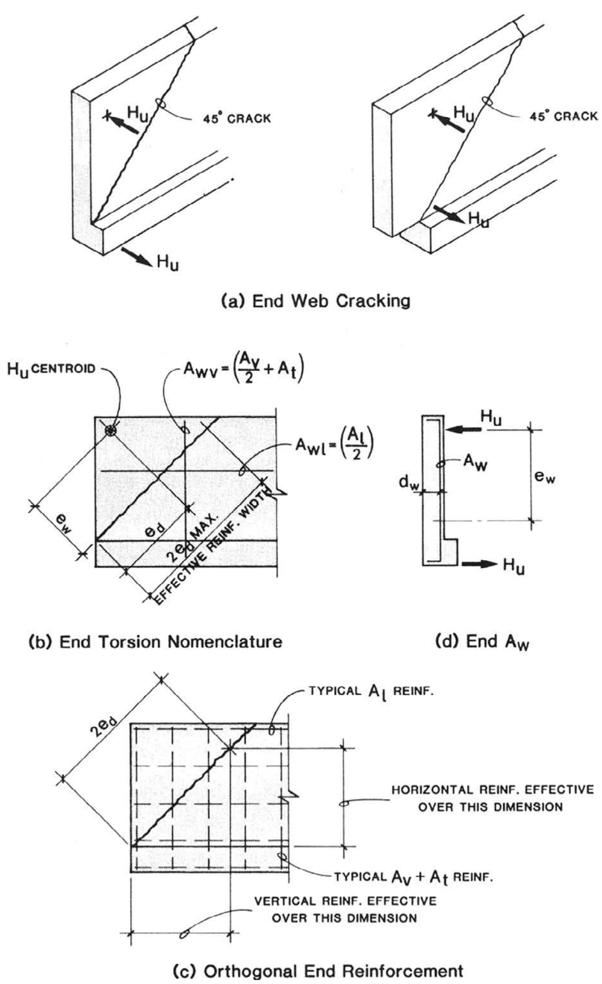



*Fig. 8.4—Support detailing requirements in a precast spandrel member, dependent on the support connection locations (Raths 1984).*

bar or tendon shall not be less than *s*/16." The corner bars help to support the outward thrusts in the zones between the hoops (Mitchell and Collins 1976). Precast concrete members usually have semi-rigid connections consisting of field-welded angles and plates. Although this is not a full torsionally restrained connection, the end region should be examined along with subsequent development of the longitudinal, perimeter reinforcement at the member ends. One common way of developing reinforcement is to use conventional hooks at the member end. In some cases, the hooks might not fit in typically thin precast concrete members. Another common detail is the use of U-bars placed horizontally at the member end and lapped with the longitudinal reinforcement. The U-bars provide additional end confinement to a given precast member. Likewise, such reinforcement can provide sufficient confinement around the connection plate studs or tail bar reinforcement.

## 8.4—Detailing at supports

Precast concrete members require other special detailing considerations due to their horizontal support conditions. Torsional forces in precast members are often equilibrated by out-of-plane, horizontal, or sometimes vertical, reactions at discrete locations along the member depth. The member end conditions and subsequent details are highly dependent on the support configuration.

Figure 8.4 illustrates a common precast spandrel beam and the horizontal force couple that is typically developed at the end. Additional reinforcement is thereby required at the member end near the top to accommodate a potential 45-degree crack that typically develops at the end location due to the couple resisting torsion, as shown in Fig. 8.4(a) and 8.4(b). A possible reinforcement scheme to address this condition is shown in Fig. 8.4(c) and 8.4(d). Similar conditions often exist in other precast members due to their specific support and horizontal restraint conditions. Although these are D-region locations, they are complicated by the three-dimensional or out-of-plane nature of the problem.

# CHAPTER 9—DESIGN EXAMPLES

## 9.1—Torsion design philosophy

Design philosophy for torsion in the ACI 318-11 building code is based on a thin-walled tube, space truss analogy in which compression diagonals wrap around the tube and the tensile contribution of concrete is neglected. Both solid and hollow members are considered tubes in accordance with Saint-Venant's circulatory shear flow pattern both before and after cracking. The outer part of the cross section centered along the stirrups is assumed to provide torsional resistance. The contribution of core concrete cross section is neglected. Once a reinforced concrete beam has cracked in torsion, the torsional resistance is provided primarily by closed stirrups and longitudinal bars located near the member's surface and diagonal compression struts. The inclined angle of the diagonal compression struts is permitted to be taken as 45 degrees for nonprestressed and lightly prestressed members, and 37.5 degrees for most prestressed members. Accordingly, ACI 318-11 makes the specific assumptions in torsion design that:

a) Concrete tensile strength in torsion is neglected

b) Torsion has no effect on the shear strength of concrete

c) Torsion stress determination is based on the closed thin-walled tube with uniform stress distribution and specific thickness, called shear flow

d) The torsional, flexural, and shear strength are accounted for by adding longitudinal reinforcement calculated for torsion and flexure

e) The longitudinal reinforcement are calculated for torsion and shear

The design of torsional resistance in Section 6.3 of EC2-04 is also based on a truss model using the thin-walled closed section theory with an effective wall thickness. The angle between the concrete compression strut and the member's longitudinal axis, θ, may be taken between 22 and 45 degrees. Both the solid and hollow cross section can be modeled by an equivalent hollow section neglecting the core concrete contribution to calculate the torsional resistance, which is limited by the strength of the concrete struts. The longitudinal and transverse reinforcement contributions to torsional resistance are accounted for after the thin-wall cracks. Effects of combined torsion and shear may be superimposed assuming the same value for the strut inclination angle. The required longitudinal and transverse reinforcement for torsion should be added to the existing longitudinal reinforcement for bending and transverse reinforcement for shear, respectively.

The Canadian code (CSA-A23.3-04) provides a General Design Method for torsion derived from the modified compression field theory (CFT) and represents solid cross sections by an equivalent thin-walled tube with a wall thickness determined by cross section dimensions. The space truss analogy provides the basic concept for torsion design assuming that the tension contribution in concrete is neglected and the diagonal compression struts spiraling around the member with variable inclined angle that depend on the loading condition and reinforcement ratio. The bending moment and longitudinal forces due to torsion and shear are considered resistant to four chords, one in each corner of the space truss and the shears by the shear flows in the walls. Dimensions of a cross section are limited to prevent crushing of the diagonal compression struts. In addition to this AASHTO LRFD method (general method), CSA-23.3-04 provides a simplified method for a restricted group of structural members, which states that the inclined angle of diagonal concrete compression strut is fixed at 35 degrees.

## 9.2—Torsion design procedures

**9.2.1** *Torsion design in ACI 318-11*—According to Saint-Venant's circulatory shear flow pattern, the most efficient cross section to resist torsion is tube-shaped. Therefore, torsion of a reinforced concrete member is a three-dimensional (3-D) problem because it involves the shear in a reinforced concrete two-dimensional (2-D) wall element of a hollow tube and the out-of-wall bending of the concrete struts. In ACI 318-11, two simplifications are made. First, the concrete strut bending is neglected and the amount of hoop steel required in the tube determined from Bredt's (1896) equilibrium equation of a cross section

$$q_y = T_u/2A_o \qquad (9.2.1a)$$

where the symbol $q_y$ is the shear flow at yield (N/mm [lb/in.]); $T_u$ is the torsional moment (N·mm [in.-lb]); and $A_o$ [mm$^2$ (in.$^2$)] is the lever arm area enclosed by the centerline of the shear flow.

Second, the hoop and longitudinal steel are assumed to yield at ultimate strength. To design steel reinforcement in a 2-D shear element, it is possible to use only three equilibrium equations (Hsu 1993). Combining the three equations creates a simple equation for yield shear flow

$$q_y = \sqrt{(A_t f_y / s_t)(A_\ell f_y / s_\ell)} \qquad (9.2.1b)$$

where $f_y$ is yield stress of hoop steel and longitudinal steel (MPa [psi]); $A_t$, $A_l$ are area of hoop steel and longitudinal steel (mm$^2$ [in.$^2$]), respectively; and $s_t$, $s_l$ are spacing of hoop steel and longitudinal steel (mm [in.]), respectively. Substituting the shear flow $q_y$ into Bredt's (1896) equation gives

$$T_u = 2A_0\sqrt{(A_t f_y / s_t)(A_l f_y / s_l)} \qquad (9.2.1c)$$

which is the essence of the ACI code provision.

The lever arm area $A_o$ (mm$^2$ [in.$^2$]) is formed by sweeping the lever arm of shear flow one full circle around the axis of twist. The centerline of shear flow was taken by Rausch (1929) to be the centerline of the hoop steel bar, and the corresponding lever arm area is denoted as $A_{oh}$ (mm$^2$ [in.$^2$]). However, this definition of area $A_{oh}$ was found to overestimate the torsional strength by as much as 30 percent. Therefore, the ACI code provides a simple, approximate formula for calculating the lever arm area as

$$A_o = 0.85A_{oh} \quad (9.2.1d)$$

To provide a more accurate formula for the ultimate torsional strength, consider the softening of concrete struts in the reinforced concrete 2-D wall elements of a tube. Under a biaxial tension-compression stress condition, the compressive stress-strain curve of the 2-D elements should be multiplied by a softening coefficient. This softening coefficient is a function of the principal tensile strain (Zhang and Hsu 1998) and varies from approximately 0.25 to 0.50. Applying this softened stress-strain curve of concrete to the study of reinforced concrete tubes under torsion (Hsu 1990, 1993), the thickness $t_d$ (mm [in.]) of the shear flow zone and lever arm area can be determined as

$$t_d = 4T_u / A_{cp}f_c' \qquad A_o = A_{cp} - (2T_u p_{cp} / A_{cp}f_c') \quad (9.2.1e)$$

where $A_{cp}$ is the area enclosed by the outer boundary of cross section (mm$^2$ [in.$^2$]); and $p_{cp}$ is the periphery of the outer boundary (mm [in.]). These formulas are given in the ACI code commentary, and the background was given in a paper by Hsu (1997).

**9.2.2** *Torsion design in EC2-04*—Section 6.3 of EC2-04 requires a full design procedure for a reinforced concrete member under torsion covering both ultimate and serviceability limit states in cases where the static equilibrium of the structure depends on torsional resistance of the elements. In conventional statically indeterminate reinforced concrete structures, torsion arises from consideration of compatibility and it is normally unnecessary to consider torsion at the ultimate limit state. However, even if torsion arises from consideration of compatibility only, it may lead to excessive cracking in the serviceability limit state. Therefore, a minimum reinforcement of stirrups and longitudinal bars should be provided to prevent excessive cracking, as indicated in EC2-04 for cracking control (Section 7.3) and detailing beams (Section 9.2).

In normal slab-and-beam or framed structures, specific calculations for torsion are usually unnecessary when torsional cracking is being adequately controlled by shear and minimum flexural reinforcement. Where torsion is essential for equilibrium of the structure, EC2-04 should be consulted. One example of this is when structure arrangement is such that loads are imposed mainly on one face of a beam without corresponding rotational restraints provided.

The design of torsional resistance moment is based on a truss model using a thin-walled closed section theory with inclined angle θ between the concrete compression strut and the beam axis. The angle θ should be limited and recommended limits are: $1 \leq \cot\theta \leq 2.5$ (45 degrees ≥ θ ≥ 22 degrees). With a solid section, the section can be modeled by an equivalent hollow section from which the torsional resistance is calculated. Complex shapes, such as T-sections, can be divided into a series of subsections modeled as an equivalent thin-walled section, and the total torsional resistance taken as the sum of the capacities of each individual element. The effects of combined torsion and shear for both hollow and solid members can be superimposed assuming the same value for the strut inclination angle θ.

A common value for angle θ is 45 degrees. Eurocode 8 (EN 1998-1:2004) determines that: "In the critical regions of primary seismic beams, the strut inclination θ in the truss model shall be 45 degrees" (Paragraph 5.5.3.1.2(2) of EC2-08). However, in members not designated to resist seismic actions, a reduced value of angle θ could be considered to decrease the required transverse reinforcement and required longitudinal reinforcement. This way, fewer stirrups and more longitudinal bars could be provided. Required torsional reinforcement is added to the required stirrups and bars calculated from the shear and flexural design, respectively. Strength of materials used in EC2-04 is based on characteristic values and depend on whether the value is used for strength or stiffness. The characteristic value used to calculate strength corresponds to the 95 percent fractile of strength from material tests. The characteristic value for stiffness corresponds to mean strength from material tests. Design values are based on multiplying the characteristic value for resistance by the safety factors α and β.

Design procedure in accordance with EC2-04:

- Step 1: Calculation of the equivalent thin-walled section characteristics such as $t_{ef}$, $A_k$, and $u_k$ (also refer to Fig. 6.11 of EC2-04 for notation)

$$t_{ef} = A/u \geq 2c \text{ and, in the case of a hollow section, } t_{ef} < t_{real}$$

where

$t_{ef}$ = effective wall thickness of the equivalent thin-walled section [mm (in.)]
$A$ = total area of the cross section within the outer circumference, including inner hollow areas (for example, $A = bh$ in a rectangular cross section with width and height equal to $b$ and $h$, respectively) [mm$^2$ (in.$^2$)]
$u$ = outer circumference of the cross section (for example, $u = 2(b + h)$ in a rectangular cross section)
$c$ = distance between edge and center of the longitudinal reinforcement (centroid cover) [mm (in.)]
$t_{real}$ = real thickness of a hollow section [mm (in.)]
$A_k$ = area enclosed by the centerlines of connecting walls, including inner hollow areas (for example, $A_k = (b - t_{ef})(h - t_{ef})$ in a rectangular cross section) [mm$^2$ (in.$^2$)]
$u_k$ = perimeter of the area $A_k$ (for example, $u_k = 2(b + h - 2t_{ef})$ in a rectangular cross section)

- Step 2: Assume the value of angle of compression struts, θ, based on the expression: $1 \leq \cot\theta \leq 2.5$ (45 degrees ≥ θ ≥ 22 degrees). For combined shear and torsion, the same value of θ should be assumed and the common value is 45 degrees.

- Step 3: Check the maximum resistance of the member subjected to torsion and shear. This is limited by the strength of the concrete struts. If the following relationship is not satisfied, the member cross section dimensions, the concrete compressive strength, or both, should be increased

$$\frac{T_{Ed}}{T_{Rd.max}} + \frac{V_{Ed}}{V_{Rd.max}} \leq 1 \quad (9.2.2a)$$

where

$T_{Ed}$ = design torsional moment [N·m (in.-lb)]

$T_{Rd.max}$ = design torsional resistance moment according to the following relationship [N·m (in.-lb)]

$$T_{Rd,max} = 2v\alpha_{cw}f_{cd}A_k t_{ef}\sin\theta\cos\theta \quad (9.2.2b)$$

$V_{Ed}$ = design shear force [N (lb)]

$V_{Rd.max}$ = maximum design shear resistance according to the following relationship [N·m (in.-lb)]

$$V_{Rd,max} = \frac{\alpha_c b_w z v f_{cd}}{(\cot\theta + \tan\theta)} \quad (9.2.2c)$$

$b_w$ = width of the web of the cross section [mm (in.)]

$z$ = inner lever arm, for a member with constant depth, corresponding to the bending moment in the element under consideration. In the shear analysis of reinforced concrete without axial force, the approximate value $z = 0.9d$ may normally be used ($d$ is the effective depth of the cross section) [mm (in.)]

$v$ = strength reduction factor for concrete cracked in shear, recommended values (values for use in a country may be found in its National Annex):

$$\begin{aligned} v &= 0.6(1 - f_{ck}/250)\ [f_{ck} \text{ in MPa}] \\ v &= 0.6(1 - f_{ck}/36.26)\ [f_y \text{ in ksi}] \end{aligned} \quad (9.2.2d)$$

$\alpha_{cw}$ = coefficient taking into account the state of compressive stress

$$\alpha_{cw} = \begin{Bmatrix} 1 & \text{non-prestressed} \\ 1+\sigma_{cp}/f_{cd} & 0 < \sigma_{cp} \leq 0.25 f_{cd} \\ 1.25 & 0.25 f_{cd} < \sigma_{cp} \leq 0.5 f_{cd} \\ 2.5(1-\sigma_{cp}/f_{cd}) & 0.5 f_{cd} < \sigma_{cp} \leq f_{cd} \end{Bmatrix} \quad (9.2.2e)$$

$\sigma_{cp}$ = mean compressive concrete stress due to design axial force (measured positive) [MPa (psi)]

$f_{ck}$ = characteristic compressive concrete strength [MPa (psi)]

$f_{cd}$ = design compressive concrete strength (= $f_{ck}/\gamma_c$, where $\gamma_c$ is the partial factor for concrete equal to 1.5 for ultimate limit state and persistent and transient design situations) [MPa (psi)]

- Step 4: Calculation of the required cross-sectional area of the longitudinal reinforcement for torsion, $\Sigma A_{s\ell}$:

$$\Sigma A_{s\ell} = \frac{T_{Ed} u_k \cot\theta}{2 A_k f_{y\ell d}} \quad (9.2.2f)$$

where $f_{y\ell d}$ is the design yield stress of the longitudinal reinforcement [MPa (psi)].

Notes: The longitudinal reinforcement for torsion should be added to the required longitudinal reinforcement for flexure. The longitudinal reinforcement should generally be distributed over the length of side, $z_i$, ($z_i$ is the side length of wall $i$ defined by the distance between intersection points with the adjacent walls [refer to Fig. 6.11 of EC2-04]), but for smaller sections it may be concentrated at the ends of this length. According to EC2-04 provisions (Section 9.2.3(4)), longitudinal bars for torsion should be arranged such that there is at least one bar at each corner, with the others being distributed uniformly around the inner periphery of the links, with a spacing not greater than 350 mm (14 in.).

- Step 5: Calculation of the required cross-sectional area of the transversal reinforcement for torsion

$$\frac{A_{sw}}{s} = \frac{T_{Ed}}{2 A_k f_{ywd} \cot\theta} \quad (9.2.2g)$$

where

$A_{sw}$ = cross-sectional area of the transversal reinforcement (stirrups) [mm$^2$ (in.$^2$)]

$s$ = spacing of the stirrups [mm (in.)]

$f_{ywd}$ = design yield stress of transversal reinforcement [MPa (psi)]

Notes: The transversal reinforcement for torsion should be added to the existing transverse reinforcement for shear. The torsion links (stirrups) should be closed and anchored by means of laps or hooked ends and form an angle of 90 degrees with the axis of the structural element. Refer to Fig. 9.6 of EC2-04 for recommended shapes. According to provisions of EC2-04 (Section 9.2.3(3)), longitudinal spacing of the torsion stirrups should not exceed $u/8$, or the requirements about the maximum longitudinal spacing between shear assemblies (Section 9.2.2(6) of EC2-04) or the lesser dimension of the beam cross section.

- Step 6: Check the value of the angle of compression struts, θ, based on the calculated and provided longitudinal and transversal reinforcement from Steps 4 and 5

$$\theta_{calc} = \tan^{-1}\sqrt{\frac{A_{sw} u_k f_{ywd}}{s \sum A_{sl} f_{y\ell d}}} \quad (9.2.2h)$$

Note: In case of a significant difference between the calculated and the initially assumed angle of compression struts, reassume the angle θ (Step 2) and recalculate Step 3 through 6.

**9.2.3** *Torsion design in CSA-A23.3-04*—The Canadian code stipulates that the effect of torsion should be considered in design only if the torsion due to factored loads, $T_f$ [N·m (in.-lb)], exceeds $0.25T_{cr}$. The cracking torque $T_{cr}$ [N·m (in.-lb)] is assumed to be reached when the principal tensile stress

$f_l$ [MPa (psi)] (equivalent to the shear stress $v$ in pure torsion) equals the factored tensile strength of the concrete, $f_{cr}$ [MPa (psi)]. For the calculation of $T_{cr}$, the following assumptions are made:

- Solid cross sections are represented by an equivalent thin-walled tube with a wall thickness [mm (in.)]

$$t_c = 0.75A_c/p_c \quad (9.2.3a)$$

- Bredt's classical equation for tubular section applies [MPa (psi)]

$$v = T_f/(2A_o t_c) \quad (9.2.3b)$$

- Area enclosed by shear flow path [mm² (in.²)]

$$A_o = 2/3A_c \quad (9.2.3c)$$

- Factored design tensile strength for normal concrete [MPa (psi)]

$$f_{cr} = 0.38\phi_c\sqrt{f_c'} \ [f_c' \text{ in MPa}]$$
$$f_{cr} = 0.38\phi_c 12\sqrt{f_c'} \ [f_c' \text{ in psi}] \quad (9.2.3d)$$

For non-prestressed concrete members, this results in the following expression

$$T_{cr} = (A_c / p_c)0.38\phi_c\sqrt{f_c'} \ [f_c' \text{ in MPa}]$$
$$T_{cr} = (A_c / p_c)0.38\phi_c 12\sqrt{f_c'} \ [f_c' \text{ in psi}] \quad (9.2.3e)$$

The symbols not defined in the above equations are as follows:

$A_c$ = area enclosed by outside perimeter $p_c$ of concrete section [mm² (in.²)]
$f_c'$ = specified compressive strength of concrete
$\phi_c$ = resistance factor for concrete (= 0.65)

If torsion is not negligible ($T_f > T_{cr}$), torsion reinforcement should be provided. The General Design Method (CSA-A23.3-04) for torsion was derived from the MCFT, which represents a holistic approach for both shear and torsion design. For torsion, the basic concept is the space truss analogy, originally envisioned by Rausch (1929), assuming a 45-degree angle for the compression struts. This AASHTO LRFD (general method), originally developed for shear, requires a longitudinal strain indicator $\varepsilon_x$ and the level of normalized shear stress $v_u/f_c'$ to estimate θ and β. In the case of a member subjected to pure torsion, it is not necessary to consider β. For torsion, the General Design Method (CSA-A23.3-04) assumes that:

a) Concrete in the cracked member carries no tension

b) The angle θ of the diagonal compression struts spiraling around the member is variable and depends on longitudinal strain at mid-depth of section, $\varepsilon_x$

The longitudinal strain $\varepsilon_x$ is affected by the bending moment: shear, torsion, and if present, by axial load and prestressing in the member. In the presence of bending moment, shear, and torsion, the strain at mid-depth of the section, $\varepsilon_x$, is computed from the expression

$$\varepsilon_x = (M_f / d + \sqrt{V_f^2 + [0.9p_h T_f / 2A_o]^2}) / (2E_s A_s) \quad (9.2.3f)$$

where

$M_f$ = moment due to factored loads [N·m (in.-lb)]
$V_f$ = shear force due to factored loads [N (lb)]
$p_h$ = perimeter of the centerline of the closed transverse reinforcement [mm (in.)]
$E_s$ = modulus of elasticity of the reinforcement [MPa (psi)]
$A_s$ = area of flexural reinforcement on the flexural tension side of the member [mm² (in.²)]
θ = 29 + 7000$\varepsilon_x$ (degrees)

For a given angle θ, the transverse reinforcement to resist the factored torque $T_f$ is derived from equilibrium and given by

$$A_t = \frac{T_f s}{2A_o\phi_s f_y \cot\theta} \quad (9.2.3g)$$

where

$s$ = spacing of transverse reinforcement measured parallel to the axis of the member [mm (in.)]
$\phi_s$ = resistance factor for non-prestressed reinforcing bars
$f_y$ = specified yield strength of transverse reinforcement [MPa (psi)]

Additional longitudinal reinforcement is required to resist the longitudinal forces generated by torsion. As usual, the transverse reinforcement due to torsion should be added to the shear reinforcement.

Dimensions of the cross section of the member have to be such that crushing of the diagonal compression struts is prevented. This is achieved if the combined stress due to shear and torsion does not exceed 25 percent of the factored compressive strength of the concrete. This is expressed by Eq. (9.2.3h)

$$\frac{V_f}{b_w d_v} + \frac{T_f p_h}{1.7A_{oh}^2} \le 0.25\phi_c f_c' \quad (9.2.3h)$$

**9.2.4** *A comparison of torsion design procedures for ACI, EC2, and CSA*—The design philosophy and procedures for pure torsion and combined loads including bending, shear, and torsion are discussed previously according to ACI 318-11, EC2-04, and CSA-A23.3-04. All the design procedure and equation citations from these codes are summarized and compared in Table 9.2.4.

## 9.3—Introduction to design examples

Two examples were selected to illustrate the steps involved in torsion design: 1) a solid reinforced concrete rectangular beam under pure torsion; and 2) A prestressed box girder under combined loading including torsion. Although this report focuses on recent torsion developments and theories, the design examples are solved by three major building codes: ACI 318-11, Eurocode 2, and CSA-A23.3-

**Table 9.2.4—Comparison of torsion design procedures for ACI, EC2, and CSA codes**

| Pure torsion design procedure | | ACI 318-11 | EC2-04 | CSA-A23.3-04 |
|---|---|---|---|---|
| 1 | Determine the factored torsional moment and if torsion effects can be disregarded | Section 11.5.1,<br>$T_u < \phi\lambda\left(\frac{\sqrt{f'_c}}{12}\right)\left(\frac{A_{cp}^2}{p_{cp}}\right)$ [$f'_c$ in MPa]<br>$T_u < \phi\lambda\sqrt{f'_c}\left(\frac{A_{cp}^2}{p_{cp}}\right)$ [$f'_c$ in psi] | | Section 11.2.9.1, Eq. 11-2<br>$T_{cr} = \left(\frac{A_c^2}{p_c}\right)0.38\lambda\phi_c\sqrt{f'_c}$ [$f'_c$ in MPa]<br>$T_{cr} = \left(\frac{A_c^2}{p_c}\right)0.38\lambda\phi_c 12\sqrt{f'_c}$ [$f'_c$ in psi] |
| 2 | Calculate properties of the equivalent thin-walled section | | Section 6.3.2(1)<br>Eq. 6-26 and 6-27<br>$t_{ef} = \max\begin{Bmatrix} A/p_c \\ 2c_t \end{Bmatrix}$ $\tau_r t_{ef} = \frac{T_{Ed}}{2A_k}$, $V_{Ed} = \tau_t t_{ef} z$ | |
| 3 | Determine if dimensions of the cross section are adequate | Section 11.5.3.1,<br>Eq. (11-18) and Section 11.2.1.1, Eq. (11-3)<br>$\frac{T_u p_h}{1.7A_{oh}^2} \leq \phi\left(\frac{V_c}{b_w d} + \frac{2\sqrt{f'_c}}{3}\right)$ [$f'_c$ in MPa]<br>$\frac{V_c}{b_w d} = \frac{2\lambda\sqrt{f'_c}}{12}$ [$f'_c$ in MPa]<br>$\frac{T_u p_h}{1.7A_{oh}^2} \leq \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right)$ [$f'_c$ in psi]<br>$\frac{V_c}{b_w d} = 2\lambda\sqrt{f'_c}$ [$f'_c$ in psi] | Section 6.3.2(4)<br>Eq. 6-29 and 6-30<br>$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1 \xrightarrow{\text{pure torsion}} T_{Ed} <= T_{Rd,max}$<br>$T_{Rd,max} = 2\nu\alpha_c f_{cd} A_k t_{ef} \sin\theta\cos\theta$ | Section 11.3.10.4,<br>Eq.11-19<br>$\sqrt{\left(\frac{V_f - V_p}{b_w d_v}\right)^2 + \left(\frac{T_f p_h}{1.7A_{oh}^2}\right)^2} \leq 0.25\phi_c f'_c$ |
| 4 | Calculate the amount of stirrups required for pure torsion<br><br>Select and check the stirrups' details for the cross section | Section 11.5.3.5,<br>Eq. (11-20) and Section 11.5.3.6, Eq. (11-21) transformed into<br>$\frac{A_t}{s} \geq \frac{T_u}{\phi 2 f_{yt} A_o \cot\theta}$<br>and Section 11.5.5.2, Eq. (11-23) can be expressed by<br>$\left(\frac{A_t}{s}\right)_{min} = \frac{0.75}{2}\frac{\sqrt{f'_c}}{12}\frac{b_w}{f_{yv}}$ [$f'_c$ in MPa]<br>$\left(\frac{A_t}{s}\right)_{min} = \frac{0.75}{2}\sqrt{f'_c}\frac{b_w}{f_{yv}}$ [$f'_c$ in psi] | Section 6.3.2 (2) and 9.2.3<br>Eq. 9.5N<br>$\rho_t = \left(0.08\frac{\sqrt{f'_c}}{12}\right)/f_{yk}$ [$f'_c$ in MPa]<br>$\rho_t = \left(0.08\sqrt{f'_c}\right)/f_{yk}$ [$f'_c$ in ]<br>and Eq. 9.6N<br>$s_{max} = \min\begin{Bmatrix} u/8 \\ 0.75d(1+\cot\alpha) \\ \min(b_w, h) \end{Bmatrix}$ | Section 11.3.10.3,<br>11.3.8.1, and 11.3.6.4<br>Eq.11-1, 11-12, 11-13, and 11-17<br>$\left(\frac{A_v}{s}\right)_{min} = 0.06\sqrt{f'_c}\frac{b_w}{f_{yv}}$ [$f'_c$ in MPa]<br>$\left(\frac{A_v}{s}\right)_{min} = 0.72\sqrt{f'_c}\frac{b_w}{f_{yv}}$ [$f'_c$ in psi]<br>$\theta = 29 + 7000\varepsilon_x$<br>$\varepsilon_x = \frac{\frac{M_f}{d_v} + \sqrt{(V_f - V_p)^2 + \left(\frac{0.9 p_h T_f}{2A_o}\right)^2} + 0.5N_f - A_p f_{po}}{2[E_s A_s + E_p A_p]}$<br>$\frac{A_t}{s} \geq \frac{T_f}{2A_o\phi_s f_y \cot\theta}$<br>$s_{max} = \min\begin{Bmatrix} 0.7d_v \\ 600 \text{ mm (23.62 in.)} \end{Bmatrix}$ |

| | | | | |
|---|---|---|---|---|
| 5 | Calculate the longitudinal bars required for pure torsion<br><br>Select and check the longitudinal bar details for the cross section | Section 11.5.3.7, $A_l = \frac{A_t}{s} p_h \frac{f_{yt}}{f_{yl}} \cot^2\theta$<br>Eq. (11-22): $A_{\ell,min} = \frac{5\sqrt{f_c'}A_{cp}}{12 f_{yt}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yt}}{f_{yt}}$ [$f_c'$ in MPa]<br>and Section 11.5.5.3, Eq. (11-24)<br>$A_{\ell,min} = \frac{5\sqrt{f_c'}A_{cp}}{f_{yl}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yt}}{f_{yl}}$ [$f_c'$ in psi] | Section 6.3.2(3) and 9.2.1<br>Eq. 6-28 $\sum A_l \geq \frac{T_u u_k \cot\theta}{2A_k f_{yd}}$<br>and Eq. 9.1N $A_{\ell,min} = 0.26 \frac{f_{ctm}}{f_{yk}} b_t d$ | Section 11.3.10.6 and 11.3.9.2, Eq. 11-14 and 1-21 can be transformed into<br>$F_{lt} = \frac{M_f}{d_v} + \cot\theta\sqrt{(V_f - 0.5V_s - V_p)^2 + \left(\frac{0.45 p_h T_f}{2A_o}\right)^2}$<br>$= \cot\theta \frac{0.45 p_h T_f}{2A_o}$ |
| **Design procedure for prestressed concrete members under combined loadings (Bending, shear, and torsion)** | | **ACI 318-11** | **EC2-04** | **CSA-A23.3-04** |
| 1 | Determine the material and cross-sectional properties | | Concrete compressive strength: Section 3.1.2 and 3.1.6<br>Concrete tensile strength: Section 3.1.2, 3.1.6 and 3.1.7<br>Reinforcing steel properties: Section 2.4.2.4 and 3.2.7<br>Pre-stressed reinforcing strands: Section 2.4.2.4, 3.3.6 and 3.3.3<br>Concrete covers: Section 4.4.1.2 | |
| 2 | Determine the factored loads and calculate factored shear, torque, and bending moment | Section 8.3 and 9.2.1 | Section 2.4.3 and 5.1.3 | Annex C |
| 3 | Determine if torsion effects can be disregarded | Section 11.5.1<br>$T_u < \phi\lambda\left(\frac{\sqrt{f_c'}}{12}\right)\left(\frac{A_{cp}^2}{p_{cp}}\right)$ [$f_c'$ in MPa]<br>$T_u < \phi\lambda\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$ [$f_c'$ in psi] | | Section 11.2.9.1, Eq. 11-2<br>$T_{cr} = \frac{(1.5A_g)^2}{p_c} 0.38\lambda\phi_c\sqrt{f_c'}\sqrt{1 + \frac{\phi_p f_{cp}}{0.38\lambda\phi_c\sqrt{f_c'}}}$ [$f_c'$ in MPa]<br>$T_{cr} = \frac{(1.5A_g)^2}{p_c} 0.38\lambda\phi_c 12\sqrt{f_c'}\sqrt{1 + \frac{\phi_p f_{cp}}{0.38\lambda\phi_c 12\sqrt{f_c'}}}$ [$f_c'$ in psi] |

| | | | | |
|---|---|---|---|---|
| 4 | Determine if dimensions of the cross section are adequate | Section 11.2.1.1,<br>Eq. (11-3), Section 11.3.2, Eq. (11-9), and Section 11.5.3.1, Eq. (11-18)<br>$\frac{V_c}{b_w d} = \frac{2\lambda\sqrt{f'_c}}{12}$ [$f'_c$ in MPa]<br>$\frac{V_c}{b_w d} = 2\lambda\sqrt{f'_c}$ [$f'_c$ in psi]<br>$V_c = \left(0.6\lambda\frac{\sqrt{f'_c}}{12} + 700(0.00689)\frac{V_u d}{M_u}\right)b_w d$ [$f'_c$ in MPa]<br>$V_c = \left(0.6\lambda\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right)b_w d$ [$f'_c$ in psi]<br>$\frac{T_u p_h}{1.7A_{oh}^2} \le \phi\left(\frac{V_c}{b_w d} + \frac{2\sqrt{f'_c}}{3}\right)$ [$f'_c$ in MPa]<br>$\frac{T_u p_h}{1.7A_{oh}^2} \le \phi\left(\frac{V_c}{b_w d} + 8\sqrt{f'_c}\right)$ [$f'_c$ in psi] | Section 6.3.2(4)<br>Eq. 6-14, 6-29, and 6-30<br>$V_{Rd,max} = \frac{\alpha_c b_w z v f_{cd}}{(\cot\theta + \tan\theta)}$ $\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \le 1$<br>$T_{Rd,max} = 2\nu\alpha_c f_{cd} A_k t_{ef} \sin\theta\cos\theta$ | Section 11.3.10.4,<br>Eq. 11-18<br>$\frac{V_f - V_p}{b_w d_v} + \frac{T_f}{1.7A_{oh} t} \le 0.25\phi_c f'_c$ |
| 5 | Shear design to determine transverse reinforcement for shear force | Shear resistance of concrete: Section 11.2.1.1, Eq. (11-3) and Section 11.3.2, Eq. (11-9)<br>$\frac{V_c}{b_w d} = \frac{2\lambda\sqrt{f'_c}}{12}$ [$f'_c$ in MPa] ,<br>$\frac{V_c}{b_w d} = 2\lambda\sqrt{f'_c}$ [$f'_c$ in psi]<br>$V_c = \left(0.6\lambda\frac{\sqrt{f'_c}}{12} + 700(0.00689)\frac{V_u d}{M_u}\right)b_w d$ [$f'_c$ in MPa]<br>$V_c = \left(0.6\lambda\sqrt{f'_c} + 700\frac{V_u d}{M_u}\right)b_w d$ [$f'_c$ in psi]<br>Required shear reinforcement: Section 11.4.7<br>$\frac{A_v}{s} = \frac{V_u - \phi V_c}{\phi d f_{yv}}$<br>Minimum shear reinforcement: Section 11.4.6.3, Eq. 11-13<br>$A_{v,min} = 0.75\frac{\sqrt{f'_c}}{12}\frac{b_w}{f_{yt}}$ [$f'_c$ in MPa]<br>$A_{v,min} = 0.75\sqrt{f'_c}\frac{b_w}{f_{yt}}$ [$f'_c$ in psi] | Design shear resistance of a member without shear reinforcement: Section 6.2.2(1). If not adequate,<br>Required shear reinforcement: Section 6.2.3(4); Eq. 6-13 and 6-14<br>$V_{Rd,s} = \frac{A_{sw}}{s} z f_{ywd} \cot\theta \ge V_{Ed} \Rightarrow \frac{A_{sw}}{s} \ge \frac{V_{Ed}}{z f_{ywd} \cot\theta}$<br>$V_{Rd,max} = \frac{\alpha_c b_w z v f_{cd}}{(\cot\theta + \tan\theta)}$<br>Minimum shear reinforcement: Section 9.2.2(5);<br>Maximum effective cross-sectional area of the shear reinforcement: Section 6.2.3(3) | The angle of diagonal compression strut: Section 11.3.6.4, Eq. 11-12 and 11-13<br>$\varepsilon_x = \frac{\frac{M_f}{d_v} + \sqrt{(V_f - V_p)^2 + \left(\frac{0.9 p_h T_f}{2A_o}\right)^2} + 0.5N_f - A_p f_{po}}{2[E_s A_s + E_p A_p]}$<br>$\frac{A_t}{s} \ge \frac{T_f}{2A_o \phi_s f_y \cot\theta}$<br>Shear resistance of concrete: Section 11.3.4, Eq. 11-6 and 11-11<br>$V_c = \phi_c \lambda\beta\sqrt{f'_c} b_w d_v$ [$f'_c$ in MPa]<br>$V_c = \phi_c \lambda\beta 12\sqrt{f'_c} b_w d_v$ [$f'_c$ in psi]<br>$\beta = \frac{0.40}{1 + 1500\varepsilon_x} \times \frac{1300}{1000 + s_{ze}}$<br>Required shear reinforcement: Section 11.3.5.1, Eq. 11-7 can be transformed into<br>$\frac{A_v}{s} = \frac{V_s}{\phi_s f_y d_v \cot\theta}$ |

| | | | | |
|---|---|---|---|---|
| 6 | Torsion design to determine transverse reinforcement for torsion | Assume the angle of diagonal compression strut. Required transverse reinforcement for torsion: Section 11.5.3.6 and 11.5.6, Eq. (11-21) transformed into<br>$\frac{A_t}{s} = \frac{T_u}{\phi 2A_o f_{yv} \cot\theta}$ | Calculate the terms of the equivalent thin-walled section for torsion design: Section 6.3.2(1)<br>Required transverse reinforcement for torsion: Section 6.3.2(2)<br>$\frac{A_{sw}}{s} \geq \frac{T_{Ed}}{2A_k f_{ywd} \cot\theta}$ | Required transverse reinforcement for torsion: Section 11.3.10.3, Eq. 11-17 can be transformed into<br>$\frac{A_t}{s} \geq \frac{T_f}{2A_o \phi_s f_y \cot\theta}$ |
| 7 | Determine the total transverse reinforcement for shear and torsion<br>Select and check transverse reinforcement details for cross section | Add the shear and torsion transverse reinforcement in one web: Section 11.5.3.8<br>Minimum transverse reinforcement: Section 11.5.5, Eq. (11-23)<br>$\left(\frac{A_t}{s}+\frac{1}{2}\frac{A_v}{s}\right)_{min} = 0.375\frac{\sqrt{f'_c}}{12}\frac{b_w}{f_{yt}}$ [$f'_c$ in MPa]<br>$\left(\frac{A_t}{s}+\frac{1}{2}\frac{A_v}{s}\right)_{min} = 0.375\sqrt{f'_c}\frac{b_w}{f_{yt}}$ [$f'_c$ in psi]<br>Maximum longitudinal spacing of transverse reinforcement for shear: ACI 318-11, Section 11.4.5<br>Maximum longitudinal spacing of transverse reinforcement for torsion: ACI 318-11, Section 11.5.6 | Add the shear and torsion transverse reinforcement in one web: Section 6.3.2(2)<br>Maximum longitudinal spacing of the stirrups for shear: Eq. 9.6N in EC2-04<br>$s_{max} = 0.75d(1+\cot\alpha) \xrightarrow{\alpha=90^\circ} s_{max} = 0.75d$<br>Maximum longitudinal spacing of the stirrups for torsion: Section 9.2.3(3)<br>$s_{max} = \min\begin{Bmatrix} u/8 \\ 0.75d(1+\cot\theta) \\ \min(b,h) \end{Bmatrix}$ | Minimum transverse reinforcement: Section 11.2.8.2, Eq. 11-1<br>$\frac{A_{v,\min}}{s} = 0.06\sqrt{f'_c}\frac{b_w}{f_y}$ [$f'_c$ in MPa]<br>$\frac{A_{v,min}}{s} = 0.72\sqrt{f'_c}\frac{b_w}{f_y}$ [$f'_c$ in psi]<br>Maximum longitudinal spacing of transverse reinforcement: Section 11.3.8.1<br>$s_{max} = \min\begin{Bmatrix} 0.7d_v \\ 600\text{ mm } (23.62\text{ in.}) \end{Bmatrix}$<br>Add the shear and torsion transverse reinforcement in one web |
| 8 | Determine longitudinal reinforcement for torsion<br>Select and check longitudinal reinforcement details for cross section | Required torsional longitudinal reinforcement: Section 11.5.3.7, Eq. (11-22)<br>$A_l = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yl}}\right)\cot^2\theta$<br>Minimum longitudinal reinforcement for torsion: Section 11.5.5.3, Eq. (11-24)<br>$A_{\ell,min} = \frac{5\sqrt{f'_c}A_g}{12 f_{yl}} - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yv}}{f_{yl}}\right)$ [$f'_c$ in MPa]<br>$A_{\ell,min} = \frac{5\sqrt{f'_c}A_g}{f_{yl}} - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yv}}{f_{yl}}\right)$ [$f'_c$ in psi]<br>Spacing of longitudinal reinforcement for torsion: Section 11.5.6.2 and 11.5.6.3 | Required torsional longitudinal reinforcement: Section 6.3.2(3), Eq. 6-28<br>$\Sigma A_{st} \geq \frac{T_{Ed} u_k \cot\theta}{2A_k f_{ywd}}$<br>Spacing of longitudinal reinforcement for torsion: Section 9.2.3 (4) | Required torsional longitudinal reinforcement: Section 11.3.10.6 11.3.9.2, Eq. 11-14 and 11-21<br>can be transformed into<br>$F_{lt} = \frac{M_f}{d_v} + \cot\theta\sqrt{(V_f - 0.5V_s - V_p)^2 + \left(\frac{0.45 p_h T_f}{2A_o}\right)^2}$<br>$A_p = \frac{F_{lt}}{f_p f_{pr}}$ |

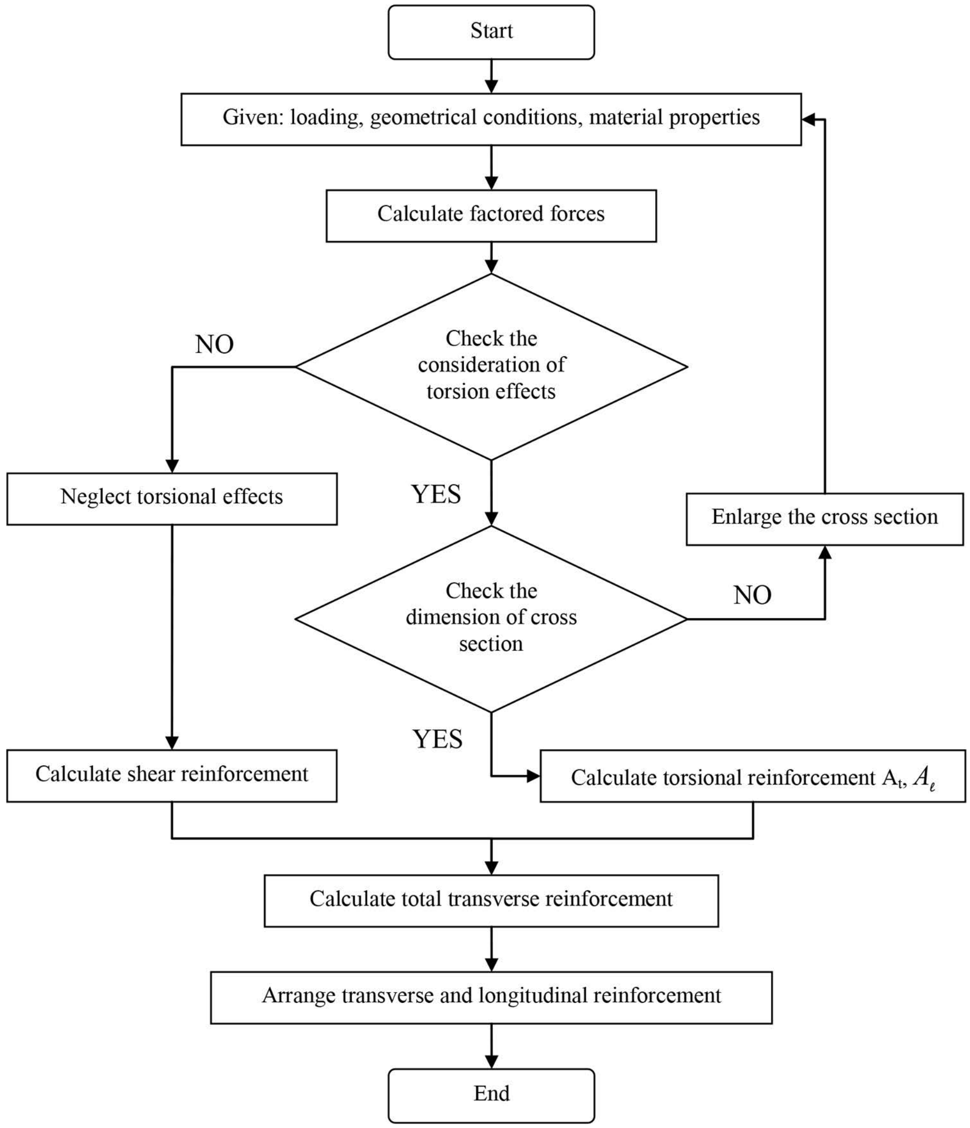



*Fig. 9.3—Design process flow chart for combined shear and torsion effects.*

04. Example 1 was also solved by the two graphical methods proposed by Rahal (2000b) and Leu and Lee (2000). These graphical methods are not suitable to treat prestressed concrete sections under combined loads and, therefore, were not used for Example 2. Following each design example, a comparison table summarizes results of the various codes and methods. Design equations and expressions may vary slightly from one code to another, but the general design procedure is the same. In the case of torsion combined with shear, design procedures for all three codes follow the flow chart given in Fig. 9.3. Hsu (1997) provides background on ACI 318-95 that is still applicable to the updated ACI 318-11. His paper includes a detailed design example of a prestressed hollow box girder subjected to torsion, shear, and flexure that constitutes Example 2.

## 9.4—*Design Example 1: solid rectangular reinforced concrete beam under pure torsion*

**9.4.1** *Design problem statement*—As shown in Fig. 9.4.1, the cross-sectional dimensions of beam are $b_w$ = 300 mm (12 in.) and $h$ = 500 mm (20 in.). The characteristic concrete cylinder compressive strength is $f_c'$ = 20 MPa (2900 psi) [Class C20/25], and the characteristic steel yield strength is $f_y$ = 420 MPa (60,000 psi). The applied torsional moment is $T_u$ = 30 kN·m (266 in.-kip). The mean cylinder strength of the concrete: $f_{cm} = f_c' + 8$ MPa $\Rightarrow f_{cm}$ = 28 MPa (4000 psi). Assume 40 mm (1.5 in.) from exterior face to stirrup centerline typical.

**9.4.2** *Solution according to ACI 318-11*

1. Determine if torsion effects may be disregarded (ACI 318-11, Section 11.5.1).

The torsion effects can be disregarded if the following expression is valid

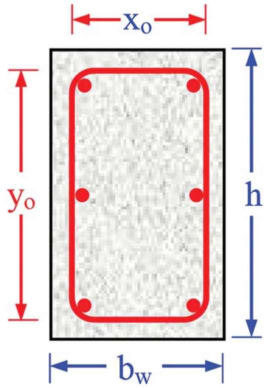



*Fig. 9.4.1—Rectangular reinforced concrete cross section subjected to pure torsion.*

$$T_u < \phi\lambda\left(\frac{\sqrt{f_c'}}{12}\right)\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

where

$\phi$ = strength reduction factor for torsion and shear = 0.75

$\lambda$ = modification factor of lightweight concrete (normalweight concrete, $\lambda$ = 1.0)

$p_{cp}$ = perimeter of outer concrete cross section, mm (in.) = 2 × ($b_w$ + $h$) = 1600 mm (64.0 in.)

$A_{cp}$ = total area enclosed by the outside perimeter of concrete cross section, mm (in.) = $b_w \times h$ = 150,000 mm$^2$ (240 in.$^2$)

Therefore,

$$\phi\lambda\left(\frac{\sqrt{f_c'}}{12}\right)\left(\frac{A_{cp}^2}{p_{cp}}\right) = 0.75(1.0)\left(\frac{\sqrt{20}}{12}\right)\left(\frac{150{,}000^2}{1600}\right)\ \text{N·mm}$$

= 3.93 kN·m (36.3 in.-kip)

$T_u$ = 30.0 kN·m (266 in.-kip).

Torsion effects must be considered.

2. Determine if dimensions of the cross section are adequate (ACI 318-11, Section 11.5.3.1).

Dimensions of the cross section are adequate if

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 8\frac{\sqrt{f_c'}}{12}\right)$$

In case of pure torsion, the expression becomes

$$\frac{T_u p_h}{1.7A_{oh}^2} \le \phi\frac{5\sqrt{f_c'}}{6},\ \text{taking}\ \frac{V_c}{b_w d} = \frac{2\lambda\sqrt{f_c'}}{12}$$

where

$p_h$ = perimeter of centerline of outermost closed transverse torsional reinforcement, mm (in.)

$p_h$ = 2($x_o$ + $y_o$) = 1280 mm (50.4 in.)

$x_o$, $y_o$ = horizontal and vertical dimension of the centerline of outermost closed transverse torsional reinforcement, respectively

$x_o$ = (300 – 2 × 40) = 220 mm (9 in.)

$y_o$ = (500 – 2 × 40) = 420 mm (17 in.)

$A_{oh}$ = area enclosed by centerline of outermost closed transverse torsional reinforcement, mm (in.)

$A_{oh}$ = $x_o y_o$ = 92,400 mm$^2$ (153 in.$^2$)

Therefore

$$\frac{T_u p_h}{1.7A_{oh}^2} = \frac{30\times10^6\times1280}{1.7\times92{,}400^2} = 2.65\ \text{MPa (347.6 psi)}$$

whereas

$$\phi\frac{5\sqrt{f_c'}}{6} = 0.75\frac{5\sqrt{20}}{6} = 2.80\ \text{MPa (403 psi)}$$

Because the expression in Eq. (11-18) (ACI 318-11) is valid, the cross section dimensions are adequate.

3. Calculate the amount of stirrups required for pure torsion (ACI 318-11, Section 11.5.3.3).

To calculate transverse reinforcement for torsion, Eq. (11-20) and (11-21) in ACI 318 can be transformed into

$$\frac{A_t}{s} \ge \frac{T_u}{\phi 2 f_{yt} A_o \cot\theta}$$

where

$A_t$ = area of one leg of a closed stirrup resisting torsion

$s$ = spacing of the stirrups

$f_{yt}$ = design strength of torsion transverse reinforcement = 420 MPa (60,000 psi)

$A_o$ = gross area enclosed by shear flow path = 0.85$A_{oh}$

$\theta$ = angle of compression diagonals in truss analogy for torsion = 45 degrees

Therefore,

$$\frac{A_t}{s} \ge \frac{T_u}{\phi 2 f_{yt} A_o \cot\theta} = \frac{30\times10^6}{0.75\times2\times420\times0.85\times92{,}400\times1}$$

= 0.61 mm$^2$/mm (0.0227 in.$^2$/in.)

The minimum transverse reinforcement is computed by Eq. (11-23) in ACI 318.

In the case of pure torsion, this minimum is expressed by

$$\left(\frac{A_t}{s}\right)_{min} = \frac{0.75}{2}\frac{\sqrt{f_c'}}{12}\frac{b_w}{f_{yv}} = \frac{0.75\times\sqrt{20}\times300}{24\times420}$$

= 0.1 mm$^2$/mm (0.0040 in.$^2$/in.)

but shall not be less than

$$\left(\frac{A_t}{s}\right)_{min} = \frac{0.35}{2}\frac{b_w}{f_{yv}} = \frac{0.175 \times 300}{420}$$

$$= 0.125 \text{ mm}^2/\text{mm } (0.005 \text{ in.}^2/\text{in.})$$

The maximum spacing of the stirrups is given by (ACI 318-11, Section 11.5.6.1)

$$s_{max} = \min\begin{Bmatrix} p_h/8 \\ 300 \text{ mm}\end{Bmatrix} = 160 \text{ mm } (6.50 \text{ in.})$$

For stirrups ∅8 ($d_s$ = 8.0 mm [0.315 in.]), $A_t$ = 50.3 mm$^2$ (0.078 in.$^2$) and $s \leq$ 81 mm (3.20 in.). Select: $s$ = 80 mm (3.15 in.).

For stirrups No. 3 ($d_s$ = 9.5 mm (0.375 in.)), $A_t$ = 71.0 mm$^2$ (0.11 in.$^2$) and $s \leq$ 115 mm (4.55 in.). Select: $s$ = 110 mm (4.50 in.).

4. Calculate the longitudinal bars required for pure torsion (ACI 318-11, Section 11.5.3.7).

Total longitudinal reinforcement for torsion ($A_\ell$) (Eq. (11-22) in ACI 318) is calculated by

$$A_\ell = \frac{A_t}{s} p_h \frac{f_{yt}}{f_{y\ell}} \cot^2\theta = 0.610 \times 1280 \times 1 \times 1$$

$$= 780.8 \text{ mm}^2 \ (1.18 \text{ in.}^2)$$

Minimum longitudinal reinforcement (Eq. (11-24) in ACI 318) is calculated by

$$A_{\ell,min} = \frac{5\sqrt{f_c'}A_{cp}}{12 f_{y\ell}} - \left(\frac{A_t}{s}\right)p_h \frac{f_{yt}}{f_{y\ell}} = \frac{5\sqrt{20} \times 150{,}000}{12 \cdot 420} - 0.610 \times 1280 \times 1$$

$$< 0 \text{ mm}^2$$

Therefore, $A_\ell$ = 780.8 mm$^2$ (1.18 in.$^2$)

At least one longitudinal bar is required in each stirrup's corner and bar spacing distributed around the perimeter of the closed stirrups is 305 mm (12 in.). Therefore, the number of longitudinal bars is at least six and each bar requires780.8/6 = 130 mm$^2$ (0.20 in.$^2$).

For six longitudinal bars ∅14 ($d_s$ = 14.0 mm [0.551 in.]), $A_l$ = 6 × 153.9 = 923 mm$^2$ (1.43 in.$^2$).

For six longitudinal bars No. 5 ($d_s$ = 15.9 mm [0.625 in.]), $A_l$ = 6 × 200 = 1200 mm$^2$ (1.86 in.$^2$).

**9.4.3** *Solution according to EC2-04 code*

1. Calculate the terms of the equivalent thin-walled section (EC2-04, Section 6.3.2(1)).

Effective wall thickness is calculated by

$$t_{ef} = \max\begin{Bmatrix} A/p_c \\ 2c_\ell \end{Bmatrix} = \max\begin{Bmatrix} 150{,}000/1600 = 94 \text{ mm} \\ 2 \times 50 = 100 \text{ mm}\end{Bmatrix}$$

$$\Rightarrow t_{ef} = 100 \text{ mm } (4.0 \text{ in.})$$

where

$A$ = total area of the outside perimeter of the concrete cross section
= $b_w h$ = 300 × 500 = 150,000 mm$^2$ (240 in.$^2$)

$p_c$ = outside perimeter of the concrete cross section
= 2($b_w$ + $h$) = 2(300 + 500) = 1600 mm (64.0 in.)

$c_\ell$ = distance between edge of member and center of the longitudinal reinforcement
= 50 mm (2.0 in.)

The area enclosed by the centerlines of the connecting thin-walls is calculated by

$$A_k = (b_w - t_{ef})(h - t_{ef}) = 200 \times 400 = 80{,}000 \text{ mm}^2 \ (124.0 \text{ in.}^2)$$

The perimeter of the area enclosed by the centerlines of the connecting thin-walls is determined by

$$u_k = 2\left[\left(b_w - t_{ef}\right) + \left(h - t_{ef}\right)\right] = 2(200 + 400)$$

$$= 1200 \text{ mm } (48.0 \text{ in.})$$

2. Determine if dimensions of the cross section are adequate (evaluate the strength of concrete struts) (*EC2-04, Section 6.3.2(4)).*

Dimensions of the cross section are adequate if (Eq. (6-29) in EC2-04)

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1 \xrightarrow{\text{pure torsion}} T_{Ed} <= T_{Rd,max}$$

where

$T_{Ed}$ = design torsional moment = $T_u$

$V_{Ed}$ = design shear force = $V_u$

$T_{Rd,\,max}$ = design torsional resistance moment

$V_{Rd,\,max}$ = design shear resistance force

The design torsional resistance moment is given as (Eq. (6-30) in EC2-04)

$$T_{Rd,max} = 2v\alpha_c f_{cd} A_k t_{ef} \sin\theta\cos\theta$$

where

$v$ = strength reduction factor for concrete cracked in shear
= $0.6(1 - f_c'/250) = 0.6(1 - 20/250) = 0.552$ ($f_c'$ in MPa)

$\alpha_c$ = 1 (for non-prestressed elements)

$\theta$ = angle of compression diagonals in truss analogy for torsion

$1 \leq \cot\theta \leq 2.5$; assume $\theta$ = 35 degrees

$f_{cd}$ = design compressive strength of concrete
= $f_{ck}/\gamma_c$ = 20/1.5 = 13.3 MPa (1933 psi)

Therefore,

$$T_{Rd,max} = 2v\alpha_c f_{cd} A_k t_{ef} \sin\theta\cos\theta$$
$$= 2 \times 0.552 \times 1 \times 13.3 \times 80{,}000 \times 100 \times \sin 35° \cos 35° (10^{-6})$$
$$= 55.2 \text{ kN•m } (513.4 \text{ in.-kip})$$
$$\Rightarrow T_{Rd,max} = 55.2 \text{ kN•m } (513.4 \text{ in.-kip}) > T_u = 30 \text{ kN•m } (266 \text{ in.-kip})$$

Because the above expression is valid, dimensions of the cross section are adequate.

3. Calculate the amount of stirrups required for pure torsion (EC2-04, Section 6.3.2 (2)).

The amount of stirrups required is calculated using the equation

$$\frac{A_t}{s} \geq \frac{T_u}{2A_k f_{ywd} \cot\theta}$$

where

$A_t$ = area of one leg of a closed stirrup resisting torsion
$s$ = spacing of the stirrups
$f_{ywd}$ = design strength of transverse reinforcement
= $f_{ty}/\gamma_s$ = 420/1.15 = 365.2 MPa (52,174 psi)

Therefore,

$$\frac{T_u}{2A_k f_{ywd} \cot\theta} = \frac{30\times10^6}{2\times80{,}000\times365.2\cot35°}$$
$$= 0.359 \text{ mm}^2\text{/mm } (0.014 \text{ in.}^2\text{/in.})$$

$$\Rightarrow \frac{A_t}{s} \geq 0.359 \text{ mm}^2\text{/mm } (0.014 \text{ in.}^2\text{/in.})$$

The ratio of the required transverse reinforcement is given by

$$\rho_t = \frac{A_t}{sb_w \sin\alpha} \xrightarrow{\alpha\,=\,90} \rho_t = \frac{A_t}{sb_w} = \frac{0.359}{300} = 0.0012$$

where α is angle between the stirrups and longitudinal axis.

The ratio of the minimum stirrups is (Eq. (9.5N) in EC2-04)

$$\rho_t = \left(0.08\sqrt{f_c'}\right)/f_{yk} = 0.08\sqrt{20}/420 = 0.00085$$

The maximum longitudinal spacing of the stirrups is given as (Eq. (9.6N) and Section 9.2.3(3) in EC2-04).

$$s_{max} = \min\begin{Bmatrix} u/8 \\ 0.75d(1+\cot\alpha) \\ \min(b_w,h) \end{Bmatrix} \Rightarrow$$

$$s_{max} = \min\begin{Bmatrix} 1{,}600/8 = 200 \\ 0.75\cdot450 = 338 \\ 300 \end{Bmatrix} = 200 \text{ mm (8.0 in.)}$$

For stirrups ∅8 ($d_s$ = 8.0 mm [0.315 in.]), $A_t$ = 50.3 mm² (0.078 in.²) and $s \leq$ 137 mm (5.40 in.). Select: $s$ = 125 mm (5.0 in.).

For stirrups No. 3 ($d_s$ = 9.5 mm [0.375 in.]), $A_t$ = 71 mm² (0.110 in.²) and $s \leq$ 194 mm (7.66 in.). Select: $s$ = 180 mm (7.1 in.).

4. Calculate the longitudinal bars required for pure torsion (EC2-04, Section 6.3.2(3)).

The total longitudinal reinforcement needed for torsion ($\Sigma A_\ell$) is given as (Eq. (6-28) in EC2-04)

$$\sum A_\ell \geq \frac{T_u u_k \cot\theta}{2A_k f_{yd}} = \frac{30\times10^6\times1200\times\cot35°}{2\times80{,}000\times365.2} \Rightarrow \sum A_\ell$$
$$\geq 880 \text{ mm}^2 \text{ (1.37 in.}^2\text{)}$$

Minimum longitudinal reinforcement is determined by (Eq. (9.1N) in EC2-04)

$$A_{\ell,min} = 0.26\frac{f_{ctm}}{f_{yk}}b_t d \geq 0.0013 b_t d$$

where

$f_{ctm}$ = mean tension strength of concrete
= $0.30 f_{ck}^{2/3} = 0.3(20^{2/3})$ = 2.21 MPa (319 psi)
$b_w$ = mean width of tension zone = 300 mm (12 in.)

Therefore,

$$A_\ell = \max\begin{Bmatrix} 0.26(2.21/420)300\times450 = 184.7 \text{ mm}^2 \\ 0.0013\times300\times450 = 175 \text{ mm}^2 \end{Bmatrix}$$
$$= 185 \text{ mm}^2 \text{ (0.287 in.}^2\text{)}$$

Longitudinal bars are arranged with at least one bar at each corner of the stirrups and the others distributed uniformly around the inner periphery of the torsion links (closed stirrups) with a maximum spacing of 350 mm (13.8 in.). Therefore, the number of longitudinal bars is at least six and each bar requires 880/6 = 147 mm² (0.23 in.²).

For six longitudinal bars ∅14 ($d_s$ = 14 mm [0.55 in.]), $A_l$ = 6 × 154 = 924 mm² (1.43 in.²).

For six longitudinal bars No. 5 ($d_s$ = 15.8 mm [0.625 in.]), $A_l$ = 6 × 198 = 762 mm² (1.88 in.²).

**9.4.4** *Solution according to CSA-A23.3-04 code*

1. Determine if torsion effects may be disregarded (CSA-A23.3-04, Section 11.2.9.1).

If the magnitude of the torsion, $T_f$, satisfies the following expressions (Eq. (11-2) in CSA-A23.3-04), torsional effects need not be considered

$$T_f < 0.25T_{cr}$$

$$T_{cr} = \left(\frac{A_c^2}{p_c}\right)0.38\lambda\phi_c\sqrt{f_c'}\sqrt{1+\frac{f_p f_{cp}}{0.38\lambda\phi_c\sqrt{f_c'}}} = \left(\frac{A_c^2}{p_c}\right)0.38\lambda\phi_c\sqrt{f_c'}$$

$$= \left(\frac{150{,}000^2}{1600}\right)0.38\times1\times0.65\sqrt{20} \text{ N·mm}$$
$$= 15.5 \text{ kN·m (144 in.-kip)}$$

where

$A_c$ = $A_{cp}$ = 150,000 mm² (240 in.²)
$p_c$ = $p_{cp}$ = outside perimeter of concrete cross section = 1600 mm (64.0 in.)

$\lambda$ = factor to account for low-density concrete
= $1.0\phi_c$ = resistance factor for concrete = 0.65
$f_c'$ = specified compressive strength of concrete, which is the same as the characteristic concrete cylinder compressive strength in previous definition = 20 MPa (2.9 ksi)

Therefore,

$$T_f\,(= 30.0\text{ kN·m}) > 0.25T_{cr}\,(= 0.25 \times 15.5 = 3.9\text{ kN·m})$$

Torsion effect, therefore, must be considered.

2. Determine if dimensions of the cross section are adequate (CSA-A23.3-04, Section 11.3.10.4).

Dimensions of the cross section are adequate if the equation below (Eq. (11-19) in CSA-A23.3-04) is satisfied

$$\sqrt{\left(\frac{V_f - V_p}{b_w d_v}\right)^2 + \left(\frac{T_f p_h}{1.7A_{oh}^2}\right)^2} \leq 0.25\phi_c f_c'$$

In case of pure torsion, the expression is simplified to

$$\frac{T_f p_h}{1.7A_{oh}^2} \leq 0.25\phi_c f_c'$$

With a concrete cover of 40 mm (1-1/2 in.) and 10M (No. 3) stirrups (diameter 12 mm [1/2 in.]), the following calculations/values apply

$$A_{oh} = (300 - 2 \times 46)(500 - 2 \times 46) = 84{,}864\text{ mm}^2\ (141.0\text{ in.}^2)$$

$$p_h = 2[(300 - 2 \times 46) + (500 - 2 \times 46)] = 1232\text{ mm}\ (50.0\text{ in.}^2)$$

Therefore,

$$\frac{T_u p_h}{1.7A_{oh}^2} = \frac{30 \times 10^6 \times 1232}{1.7 \times 84{,}860^2} = 3.02\text{ MPa (394 psi)}$$

$$0.25 f_c f_c' = 0.25 \times 0.65 \times 20 = 3.25\text{ MPa (471 psi)}$$

Because 3.02 MPa < 3.25 MPa, dimensions of the cross section are adequate.

3. Calculate the stirrups required for pure torsion (CSA-A23.3-04, Section 11.3.10.3);

The equation of nominal torsional strength $T_n$ is same in all codes except for differences in strength reduction factors. For design, the equation for $T_n$ is rearranged to express the required area of transverse reinforcement per unit length (Eq. (11-7) in CSA-A23.3-04)

$$\frac{A_t}{s} \geq \frac{T_f}{2A_o\phi_s f_y \cot\theta}$$

where

$A_t$ = area of one leg of a closed stirrup resisting torsion
$s$ = spacing of the stirrups
$A_o$ = gross area enclosed by shear flow path = $0.85A_{oh}$
$\theta$ = angle of inclination of compression stresses to the longitudinal member axis

4. Calculate angle of inclination of compression strut (CSA-A23.3-04, Section 11.3.6.4).

The angle of inclination of the diagonal compression strut is given by the expression (Eq. (11-12) in CSA-A23.3-04)

$$\theta = 29 + 7000\varepsilon_x$$

The longitudinal strain indicator $\varepsilon_x$ is defined by (Eq. (11-13) in CSA-A23.3-04)

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + \sqrt{(V_f - V_p)^2 + \left(\dfrac{0.9p_h T_f}{2A_o}\right)^2} + 0.5N_f - A_p f_{po}}{2[E_s A_s + E_p A_p]}$$

For non-prestressed sections subjected to pure torsion, the expression for $\varepsilon_x$ is simplified to

$$\varepsilon_x = \frac{\dfrac{0.9p_h T_f}{2A_o}}{2E_s A_s} = \frac{0.9p_h T_f}{4E_s A_s A_o}$$

with $A_o = 0.85A_{oh} = 0.85 \times 84{,}860 = 72{,}130\text{ mm}^2$ (120 in.$^2$) and the longitudinal reinforcement $A_s = 413\text{ mm}^2$ (0.64 in.$^2$) (established in Section 6 of this example), the following is obtained

$$\varepsilon_x = \frac{0.9 \times 1232 \times 30 \times 10^6}{4 \times 200{,}000 \times 413 \times 72{,}130} = 0.00140$$

Therefore

$$\theta = 29 + 7000 \times 0.00140 = 38.8\text{ degrees}$$

5. Calculate stirrups required for pure torsion.

The required stirrup area per unit length is calculated using the equation (Eq. (11-17) in CSA-A23.3-04)

$$\frac{A_t}{s} \geq \frac{T_f}{2A_o\phi_s f_y \cot\theta} = \frac{30 \times 10^6}{2 \times 72{,}130 \times 0.85 \times 420 \times \cot 38.8°}$$
$$= 0.468\text{ mm}^2\text{/mm (0.0175 in.}^2\text{/in.)}$$

The minimum transverse reinforcement is (Eq. (11-1) in CSA-A23.3-04)

$$\left(\frac{A_v}{s}\right)_{min} = 0.06\sqrt{f_c'}\frac{b_w}{f_{yv}} = 0.06 \times \sqrt{20} \times \frac{300}{420}$$
$$= 0.192\text{ mm}^2\text{/mm (0.00775 in.}^2\text{/in.)}$$

The maximum spacing of the stirrups is (CSA-A23.3-04, Section 11.3.8.1)

$$s_{max} = \min\begin{Bmatrix} 0.7d_v \\ 600 \text{ mm} \end{Bmatrix} = 277 \text{ mm (11.2 in.)}$$

where

$d_v$ = effective shear depth = max(0.9$d$, 0.72$h$) = max(396 mm, 360 mm) [(16.0 in., 14.4 in.)]

For stirrups 10M ($d_s$ = 11.3 mm [0.444 in.]), $A_t$ = 100 mm$^2$ (0.155 in.$^2$) → $s \leq$ 210 mm (8.32 in.). Select $s$ = 200 mm (8.0 in.).

For stirrups No. 3 ($d_s$ = 9.5 mm [0.375 in.]), 71.0 mm$^2$ ($A_t$ = 0.110 in.$^2$) → $s \leq$ 150 mm (5.89 in.). Select: $s$ = 150 mm (5. 9 in.).

6. Calculate the amount of longitudinal bars required for pure torsion (CSA-A23.3-04, Section 11.3.9.2 and 11.3.10.6).

Total longitudinal reinforcement is calculated using the following equation (Eq. (11-21) and (11-14) in CSA-A23.3-04)

$$F_{\ell t} = \frac{M_f}{d_v} + \cot\theta\sqrt{(V_f - 0.5V_s - V_p)^2 + \left(\frac{0.45 p_h T_f}{2A_o}\right)^2} = \cot\theta\frac{0.45 p_h T_f}{2A_o}$$

where

$F_{\ell t}$ = required tension force in longitudinal reinforcement = $\phi A_\ell f_y$

Therefore

$$A_\ell \geq \frac{0.45 p_h T_f}{2\phi_s A_o f_y}\cot\theta = \frac{0.45\times 1232\times 30\times 10^6}{2\times 0.85\times 72{,}130\times 420}\cot 38.8°$$

$$= 402 \text{ mm}^2 \text{ (0.61 in.}^2)$$

One longitudinal reinforcing bar should be placed in each corner of the closed transverse reinforcement required for torsion. The nominal diameter of the corner bars should be no less than $s$/16. If 10M stirrups are used, the bar nominal diameter can be no less than 200/16 = 12.5 mm (8/16 = 0.5 in.), and for No. 3: 150/16 = 9.38 mm (6/16 = 0.375 in.).

Select: four longitudinal bars 15M ($d_s$ = 16.0 mm [0.628 in.]) at the corners and two 10M bars at mid-depth: $A_\ell$ = 4 × 200 + 2 × 100 = 1000 mm$^2$ (1.55 in.$^2$).

Or, six longitudinal bars No. 3 ($d_s$ = 9.5 mm [0.375 in.]), $A_\ell$ = 6 × 71 = 426 mm$^2$ (0.66 in.$^2$)

Or, six longitudinal bars No. 4 ($d_s$ = 12.7 mm [0.50 in.]), $A_\ell$ = 6 × 127 = 762 mm$^2$ (1.18 in.$^2$).

**9.4.5** *Rahal's graphical method*—Rahal's graphical method uses the ACI general requirements, such as an upper limit on spacing of transverse and longitudinal reinforcement, an upper limit on yield strength of reinforcing rebars, and a minimum of four corner longitudinal bars. Based on ACI requirements, it also disregards the torque effect if it is smaller than 25 percent of the cracking torque. The cross section outer perimeter and area enclosed within this perimeter were calculated in the ACI design example 9.4.1 as: $A_c$ = 150,000 mm$^2$ (240 in.$^2$) and $p_c$ = 1600 mm (64.0 in.).

1. Determine if torsion effects may be disregarded.

The torsion effects can be disregarded if this expression is valid: $T_u \leq 0.25\phi T_{cr}$

$$T_{cr} = 0.328\sqrt{f_c'}\frac{A_c^2}{p_c} = 0.328\sqrt{20}\,\frac{150{,}000^2}{1600}\ \text{N•mm}$$

$$= 20.\ 6 \text{ kN·m (193.87 in.-kip)}$$

The torque is disregarded if $T_u$ = 30 kN·m (266 in.-kip) is smaller than $0.25\phi T_{cr}$ = 0.25 × 0.75 × 20.6 = 3.9 kN·m (35.8 in.-kip). Torsion effects, therefore, must be considered.

2. Calculate normalized shear stress and determine if size of cross section is adequate.

Shear stress in the walls of the cross section is calculated by

$$\frac{v}{f_c'} = \frac{\frac{T_u}{\phi}p_c}{0.67A_c^2 f_c'} = \frac{\frac{30\times 10^6}{0.75}\times 1600}{0.67(150{,}000)^2 20} = 0.212$$

The normalized shear stress fits well within Region I in Fig. 5.7.1. Therefore, the section can be designed as under-reinforced, and the section dimensions are adequate.

3. Calculate the required reinforcement.

The most straightforward design of an under-reinforced section uses equal amounts of longitudinal and transverse reinforcement indexes ($\omega_l = \omega_t = v/f_c'$). Therefore, $\omega_l$ = 0.212 and $\omega_t$ = 0.212. From Eq. (5.7.1b) and (5.7.1c), the longitudinal and transverse reinforcement are

$$\frac{A_t}{s} = \frac{0.42A_c f_c'}{f_{ty}p_c}\omega_t = \frac{(0.42)(150{,}000)(20)}{(420)(1600)}0.212$$

$$= 0.40 \text{ mm}^2\text{/mm (0.016 in.}^2\text{/in.)}$$

$$A_\ell = \frac{0.375A_c f_c'}{f_{\ell y}}\omega_l = \frac{(0.375)(150{,}000)(20)}{420}0.212$$

$$= 568 \text{ mm}^2 \text{ (0.92 in.}^2)$$

The stirrups' maximum spacing is taken as 160.0 mm (6.30 in.), which is the smaller of $p_h$/8 = 2(220 + 420)/8 = 160.0 mm (6.50 in.) and 300.0 mm (12.0 in.), as per ACI code.

For stirrups ∅8 ($d_s$ = 8.0 mm [0.315 in.]), $A_t$ = 50.3 mm$^2$ (0.08 in.$^2$), the maximum spacing is calculated as $s \leq$ 124.5 mm (4.9 in.), and it can be used: $s$ = 120 mm (4.50 in.).

For stirrups No. 3 ($d_s$ = 9.5 mm [0.375 in.]), $A_t$ = 71.0 mm$^2$ (0.11 in.$^2$), the maximum spacing is calculated as $s \leq$ 175.7 mm (6.9 in.), and it can be used: $s$ = 160 mm (6.50 in.).

To provide four longitudinal corner bars and limit the spacing to 300 mm (12 in.), six bars are needed (ACI design in 8.2.2). The minimum bar area is of 578/6 = 96 mm$^2$ (0.15 in.$^2$).

For six longitudinal bars ∅12 ($d_s$ = 12.0 mm [0.472 in.]), $A_\ell$ = 6 × 113 = 678 mm$^2$ (1.05 in.$^2$).

For six longitudinal bars No. 4 ($d_s$ = 12.7 mm [0.50 in.]), $A_\ell$ = 6 × 129 = 774 mm$^2$ (1.20 in.$^2$).

**9.4.6** *Leu and Lee's graphical method*

1. Calculate the required torsional strength.

Nondimensional balanced torsional strength is calculated using Eq. (5.7.2g)

$$\overline{T}_{xub} = \frac{140}{300+f_y}\ (f_y \text{ in MPa}) = \frac{140}{300+420} = 0.196$$

Balanced torsional strength $T_{xub}$ is determined by Eq. (5.7.2h)

$$T_{xub} = \frac{\overline{T}_{xub} f_c' A_c^2}{p_c} = \frac{0.196 \times 20 \times 150{,}000^2}{1600}\ \text{N·mm}$$
$$= 55.1 \text{ kN·m (512 in.-kip)}$$

where

$A_c$ = total area of the outside perimeter of the concrete cross section
= $b_w h = 300 \times 500 = 150{,}000$ mm$^2$ (240 in.$^2$)

$p_c$ = outside perimeter of the concrete cross section
= $2(b_w + h) = 2(300 + 500) = 1600$ mm (64.0 in.)

The torsional strength indicated by Eq. (5.7.2h) is greater than required

$$\omega_s T_{xub} \geq \frac{T_u}{f} \rightarrow \omega_s \times 55.1 \geq \frac{30}{0.75} \rightarrow f_s \geq 0.73\omega$$

where

$T_u$ = applied torsional moment = 30.0 kN·m (266 in.-kip)

$\phi$ = strength reduction factor for shear and torsion; which is assumed to be the same as that in ACI 318
= $0.75\omega_s$ = strength contour value (Fig. 5.7.2)

2. Calculate the longitudinal bars and stirrups required for pure torsion.

Required reinforcement indexes (Fig. 5.7.2): $\omega_l$ and $\omega_t$

For convenience, assume $\omega_\ell = \omega_t$.

Referring to (Fig. 5.7.2), $\omega_s \geq 0.73 \rightarrow \omega_\ell = \omega_t \geq 0.70$.

Balanced normalized reinforcement ratios $\eta_{\ell b}$ and $\eta_{tb}$ are calculated by Eq. (5.7.2c) and (5.7.2d)

$$\eta_{\ell b} = \frac{76}{200+f_y}\ (f_y \text{ in MPa}) = \frac{76}{200+420} = 0.125$$

$$\eta_{tb} = \frac{76}{100+f_y}\ (f_y \text{ in MPa}) = \frac{76}{100+420} = 0.135$$

Required normalized reinforcement ratios $\eta_\ell$ and $\eta_t$ are determined using Eq. (5.7.2e) and (5.7.2f)

$$\eta_\ell = \omega_\ell \eta_{\ell b} \geq 0.70 \times 0.125 \rightarrow \eta_\ell \geq 0.0861$$

$$\eta_t = \omega_t \eta_{tb} \geq 0.70 \times 0.135 \rightarrow \eta_t \geq 0.105$$

Required longitudinal bars are calculated with Eq. (5.7.2a)

$$\eta_\ell \geq 0.0868 \rightarrow \frac{f_y}{f_c'}\frac{A_\ell}{A_{cp}} \geq 0.0868 \rightarrow \frac{420}{20}\frac{A_\ell}{150{,}000} \geq 0.0861$$
$$\rightarrow A_\ell \geq 615 \text{ mm}^2 \text{ (1.0 in.}^2)$$

At least one longitudinal bar is placed at each corner of the stirrups with the others distributed uniformly around the inner periphery of the closed stirrups with a maximum spacing of 300 mm (12 in.), as indicated in ACI 318. Therefore, the number of longitudinal bars is at least six and each bar requires 615/6 = 102.5 mm$^2$ (0.17 in.$^2$).

For six longitudinal bars ∅12 ($d_s$ = 12.0 mm [0.472 in.]), $A_\ell = 6 \times 113 = 678$ mm$^2$ (1.05 in.$^2$).

For six longitudinal bars No. 4 ($d_s$ = 12.7 mm [0.500 in.]), $A_\ell = 6 \times 127 = 762$ mm$^2$ (1.18 in.$^2$).

Required stirrups are calculated using Eq. (5.7.2b)

$$\eta_t \geq 0.1036 \rightarrow \frac{f_y}{f_c'}\frac{A_t p_{cp}}{A_{cp} s} \geq 0.1036$$
$$\rightarrow \frac{420}{20}\frac{1600 A_t}{150{,}000 s} \geq 0.1036$$
$$\rightarrow \frac{A_t}{s} \geq 0.470 \text{ mm}^2/\text{mm (0.019 in.}^2/\text{in.)}$$

Using ACI 318, the maximum stirrups' spacing is calculated to be 160 mm (6.3 in.).

For stirrups ∅8 ($d_s$ = 8.0 mm [0.315 in.]) $\rightarrow A_t = 50.3$ mm$^2$ (0.08 in.$^2$) and $s \leq 107$ mm (4.2 in.) and it can be used: $s$ = 100 mm (4.0 in.).

For stirrups No. 3 ($d_s$ = 9.5 mm [0.375 in.]) $\rightarrow A_t = 71$ mm$^2$ (0.11 in.$^2$) and $s \leq 151$ mm (5.9 in.) and it can be used: $s$ = 150 mm (5.5 in.).

## 9.5—*Design Example 2: Prestressed concrete box girder under combined torsion, shear, and flexure*

**9.5.1** *Design problem statement*

**9.5.1.1** *Design problem description*—Design the shear and torsional reinforcement of a box girder. A 3658 mm (12 ft) wide and 1270 mm (4 ft 2 in.) deep box girder with overhanging flanges (Fig. 9.5.1.1(a)) was designed as an alternative to the double-tee girder in Dade County, FL (Hsu 1997). The standard prestressed box girder is simply supported, 24.00 m (79.00 ft) long, and prestressed with 64 strands at 1860 MPa (270 ksi), 13.0 mm (1/2 in.), seven-wire strands as shown in Fig. 9.5.1.1(b). Total prestress force is 6076 kN (1366 kips) after prestress loss. The design of flexural reinforcement is omitted for simplicity. The concrete cover is 40 mm (1.5 in.), and material strengths are normalweight concrete: $f_c'$ = 48.0 MPa (7000 psi) and $f_y$ = 420 MPa (60,000 psi).

**9.5.1.2** *Sectional properties*

$L$ = 24.00 m (79.00 ft)

$h$ = 1270 mm (50.00 in.)

$d$ = 1016 mm (40.00 in.) at $0.3L$ from support

$t$ = 251 mm (9.88 in.) (average of stem width)

$b_w$ = 502 mm (20 in.)

$A$ = $1.523 \times 10^6$ mm$^2$ (2361.4 in.$^2$)

**Table 9.4.6—Summary of design solution of Example 1 using all five solution methods**

| Code | | Transverse reinforcement | Longitudinal reinforcement |
|---|---|---|---|
| ACI 318 | required: | 0.61 mm²/mm (0.0227 in.²/in.) | 780.8 mm² (1.18 in.²) |
| | *minimum:* | *0.125 mm²/mm (0.0042 in.²/in.)* | *0 mm² (0 in.²)* |
| | provided: | ∅8/80 mm (No. 3 at 4.50 in.)<br>0.625 mm²/mm (0.0244 in.²/in.) | 6∅14 (6 No. 5)<br>923 mm² (1.86 in.²) |
| EC2-04 | required: | 0.359 mm²/mm (0.014 in.²/in.) | 880 mm² (1.37 in.²) |
| | *minimum:* | *0.261 mm²/mm (0.0103 in.²/in.)* | *185 mm² (0.287 in.²)* |
| | provided: | ∅8/125 mm (No. 3 at 7.10 in.)<br>0.402 mm²/mm (0.0155 in.²/in.) | 6∅14 (6 No. 5)<br>924 mm² (1.88 in.²) |
| CSA A23.3-04 | required: | 0.468 mm²/mm (0.0175 in.²/in.) | 402 mm² (0.61 in.²) |
| | *minimum:* | *0.192 mm²/mm (0.00775 in.²/in.)* | — |
| | provided: | 10M/200.0 mm (No. 3 at 5.90 in.)<br>0.500 mm²/mm (0.0186 in.²/in.) | Four 15M + two 10M<br>(6 No. 3 or 6 No. 4)<br>1000 mm² (0.66 or 1.18 in.²) |
| Rahal (2000b) | required: | 0.40 mm²/mm (0.0160 in.²/in.) | 568.1 mm² (0.92 in.²) |
| | *minimum:* | — | — |
| | provided: | ∅8/120.0 mm (No. 3 at 6.50 in.)<br>0.417 mm²/mm (0.0183 in.²/in.) | 6∅12 (6 No.4)<br>678 mm² (1.18 in.²) |
| Leu and Lee (2000) | required: | 0.470 mm²/mm (0.019 in.²/in.) | 615 mm² (1.0 in.²) |
| | *minimum:* | — | — |
| | provided: | ∅8/100.0 mm (No. 3 at 5.50 in.)<br>0.503 mm²/mm (0.0200 in.²/in.) | 6∅12 (6 No. 4)<br>678 mm² (1.18 in.²) |

$I$ = $319.8 \times 10^9$ mm² (768,336 in.⁴)
$y_t$ = 516 mm (20.34 in.)
$y_b$ = 753 mm (29.66 in.)
$\lambda$ = modification factor of lightweight concrete ($\lambda = 1.0$)

**9.5.1.3** *Loading criteria*—The standard girders are designed to carry a train of cars, each 22.86 m (75 ft 0 in.) long. Each car has two trucks with a center-to-center distance of 16.46 m (54 ft 0 in.). Each truck consists of two axles 1981 mm (6 ft 6 in.) apart. The crush live load of each car is 513.8 kN (115.5 kip). The maximum web reinforcement amount was obtained at section $0.3L$ from the support under a derailment load, which consists of two truckloads located symmetrically at a distance 3200 mm (10 ft 6 in.) from midspan. Each axle load is taken as 24 percent of the crush live load (513.8 kN/4) with 100 percent impact and a maximum side shift of 914 mm (36.0 in.). The self-weight of the girder is 34.4 kN/m (2.36 kip/ft). The girder is also subjected to a superimposed dead load caused by the track rails' weight, rail plinth pads, power rail, guard rail, cableway, acoustic barrier, and other permanent loads. At derailment, this superimposed dead load is assumed to produce a uniform vertical load of 12.8 kN/m (0.88 kip/ft) and a uniformly distributed torque of 3.16 kN-m/m (0.71 ft-kip/ft). This torque is neglected in the calculation because the magnitude of the distributed torque is small, and the torque is acting in a direction opposite to the derailment torque.

**9.5.2** *Solution according to ACI 318-11*

1. Determine the factored forces for (ACI 318-11, Section 9.2.1).

*Factored dead and live loads*—

The load factor for live loads is taken as 1.6.

The derailment load per axle is calculated by

$$P_{u,L} = 16 \times \frac{513.8}{4} \times 2 = 411 \text{ kN/axle (92.4 kip/axle)}$$

The derailment torque per axle is calculated by

$$T_{u,L} = 16 \times \frac{513.8}{4} \times 2 \times 0.914$$
$$= 375.7 \text{ kN·m/axle (277 ft-kip/axle)}$$

The load factor for dead loads is taken as 1.2.

The girder weight is calculated by

$$w_{u,g} = 1.2(34.4) = 41.3 \text{ kN/m (2.83 kip/ft)}$$

The superimposed dead weight is calculated by

$$w_{u,s} = 1.2(12.8) = 15.4 \text{ kN/m (1.05 kip/ft)}$$

*Factored shear, torque, and bending moment*—

The $V_u$, $T_u$, and $M_u$ at $0.3L$ from the support are

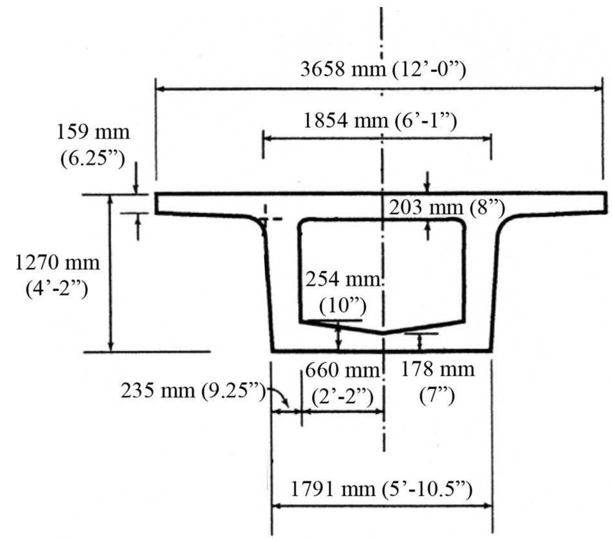
$V_u$ = $(w_{u,g} + w_{u,s})(0.2L) + 2P_{u,L} = (41.3 + 15.3) \times 0.2 \times 24.00 + 2 \times 411 = \underline{1094 \text{ kN}}$ (246 kips)
$T_u$ = $2T_{u,L} = 2 \times 375.7 = \underline{752 \text{ kN·m}}$ (554 ft-kip)
$M_u$ = $0.5(w_{u,g} + w_{u,s})(L - 0.3L)(0.3L) + 2P_{u,L}(0.3L)$
= $0.5(41.3 + 15.4)(24.00 - 7.2) \times 7.2 + 2 \times 411 \times 7.2$
= 9347 kN·m (6922 ft-kip)

2. Determine if torsion effects may be disregarded.

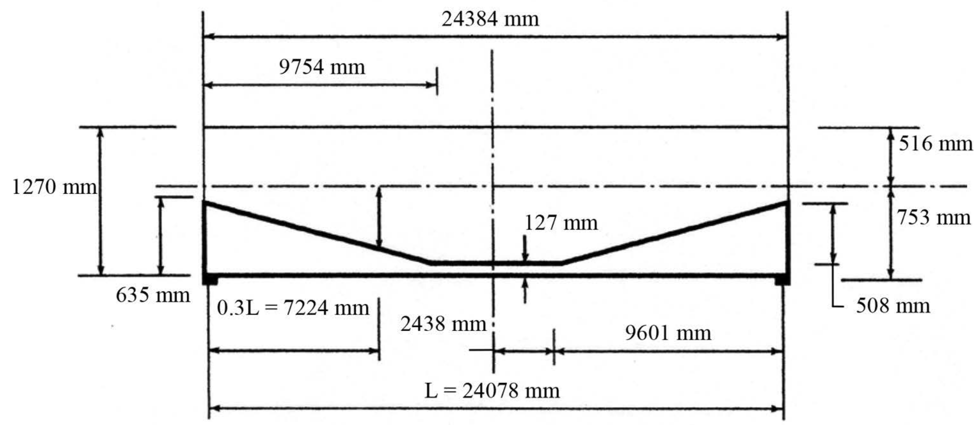
*Check outstanding flanges*—

As indicated by Fig. 9.5.1.1(a), the parameter $A_{cp}^2/p_{cp}$ is determined by (disregard overhanging flanges)

$A_{cp}$ = $1854 \times 203 + 0.5(1854 + 1791) \times 1067 = 2.32 \times 10^6$ mm² (3597 in.²)
$p_{cp}$ = $1854 + 1791 + 2 \times 1270 = 6185$ mm (243.5 in.)
$A_{cp}^2/p_{cp} = (2.32 \times 10^6)^2/6185 = 8.70 \times 10^8$ mm³ (53,090 in.³)




**(a) Cross section**




**(b) Elevation and prestress profile**

*Fig. 9.5.1.1—Cross section and elevation of box girder.*

*Check threshold torque (ACI 318-11, Section 11.5.1)—*

$A_g$ = $2.32 \times 10^6 - (1270 - 419)(1320) = 1.20 \times 10^6$ mm$^2$ (1855 in.$^2$)($fT_{cr}/4$)

=

$$\phi(0.083)\lambda\sqrt{f'_c}\frac{A_{g^2}}{p_{cp}}\left(1+\frac{f_{pc}}{0.33\sqrt{f'_c}}\right)=\phi(0.083)\lambda\sqrt{f'_c}\frac{A_{g^2}}{p_{cp}}\left(1+\frac{6076/A}{0.33\sqrt{f'_c}}\right)$$

$$=0.75(0.083)(1.0)\sqrt{48.0}\,\frac{\left(1.20\times10^6\right)^2}{6185}\left(1+\frac{6076/1523}{0.33\sqrt{48.0}}\right)10^{-6}$$

= 276 kN·m (202 ft-kip) < 752 kN·m (554 ft-kip)

Factored torsional moment should be considered in design.

3. Determine if dimensions of the cross section are adequate (ACI 318-11, Section 11.5.3.1).

*Check cross section—*

Assume a clear concrete cover of 40 mm (1.5 in.) and 13 mm No. 4 bars for web reinforcement

$A_{oh}$ = $0.5[(1854 - 93) + (1791 - 93)](1270 - 93)$

= $2.04 \times 10^6$ mm$^2$ (3177 in.$^2$)

$p_h$ = $(1854 - 93) + (1791 - 93) + 2(1270 - 93)$

= 5813 mm (229.5 in.)

$$e = (753 - 127) - \frac{9754 - 7224}{9754}508$$

= 496 mm (19.47 in.) at 0.3$L$ from support

$d$ = $y_t + e = 516 + 496 = 1012$ mm (39.81 in.) at 0.3$L$ from support

$d$ = $0.8h = 0.8 \times 1270 = 1016$ mm (40.00 in.) governs

$b_w$ = $2t = 2 \times 251 = 502$ mm (20.0 in.)

$b_wd$ = $502 \times 1016 = 510 \times 10^3$ mm$^2$ (790 in.$^2$)

The interaction equation for hollow box sections is (Eq. (11-18) in ACI 318-11)

$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u}{1.7A_{oh}t}\right)\leq \phi\left(\frac{V_c}{b_w d}+8\times 0.083\sqrt{f_c'}\right) \text{ when } t<\frac{A_{oh}}{p_h}$$

$$\frac{A_{oh}}{p_h} = \frac{2.04\times 10^6}{5829} = 351 \text{ mm (13.85 in.)} > t = 251 \text{ in.}$$

$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u}{1.7A_{oh}t}\right)=\frac{1094\times 1000}{510\times 10^3}+\frac{751.4\times 10^6}{1.7\left(2.04\times 10^6\right)251}$$

$$= 2.15 + 0.86 = 3.01 \text{ MPa (437 psi)}$$

$$V_c = \left(0.6\times 0.083\lambda\sqrt{f_c'}+700\times 0.0689\frac{V_u d}{M_u}\right)b_w d,$$

where $\frac{V_u d}{M_u} \leq 1$ (Eq. (11-9) in ACI 318-11)

$$\frac{V_u d}{M_u}=\frac{1094\times 1016}{9347\times 1000} = 0.119 < 1 \quad \text{OK}$$

$$V_c = (0.6 \times 0.083(1.0)\sqrt{48.0} + 700 \times 0.00689\times 0.119)\left(\frac{510\times 10^3}{1000}\right) = 469 \text{ kN (105.4 kip)}$$

$$V_{c,min} = 2 \times 0.083\lambda\sqrt{f_c'}\, b_w d$$

$$= 2 \times 0.083(1.0)\left(\sqrt{48.0}\,\frac{510\times 10^3}{1000}\right)$$

(Eq. (11-3) in ACI 318-11)

$= 587$ kN (132.13 kip) governs

$$\phi\left(\frac{V_c}{b_w d}+8\times 0.083\sqrt{f_c'}\right)=0.75\left(\frac{588\times 1000}{510\times 10^3}+8\times 0.083\sqrt{48.0}\right)$$

$= 0.75(1.15 + 4.61) = 4.31$ MPa (627 psi) $> 3.01$ MPa (437 psi) OK

4. Calculate number of transverse bars required

*Design of shear reinforcement (ACI 318-11, Section 11.4.7)—*

$V_c = V_{c,min} = 587$ kN (132.2 kip)

$$\frac{A_v}{s}=\frac{V_u-\phi V_c}{\phi d f_{yv}}=\frac{(1094-0.75\times 587)1000}{0.75\times 1016\times 420}$$

$= 2.043$ mm²/mm (0.0818 in.²/in.)

$$> \frac{A_{v,min}}{s} = 0.75\frac{\sqrt{f_c'}}{12}\frac{b_w}{f_{yt}} = 0.51 \text{ mm}^2\text{/mm (0.021 in.}^2\text{/in.)}$$

$s_{max} =$ 305.0 mm (12.00 in.) for torsion governs

*Design of torsional hoop reinforcement (ACI 318-11, Section 11.5.3.6)—*

$$A_o = A_{cp} - \frac{2T_u p_{cp}}{\phi f_c' A_{cp}} = 2.32 \times 10^6 - \frac{2\times 752\times 6185(1000)^2}{0.75\times 48\times 2.32\times 10^6}$$

$= 2.21 \times 10^6$ mm² (3582.7 in.²)

Assume $\theta = 37.5$ degrees, as recommended by the code provision for prestressed members:

$$\frac{A_{ts}}{s}=\frac{T_u}{\phi 2A_o f_{yv}\cot\theta}=\frac{752(1000)^2}{0.75\times 2\left(2.21\times 10^6\right)\times 420\times 1.303}$$

$= 0.414$ mm²/mm (0.0166 in.²/in.)

$s_{max} =$ $p_h/8 = 5813/8 = 727$ mm (28.7 in.) $> 305$ mm (12 in.) (ACI 318-11, Section 11.5.6);

These calculations indicate that 305 mm (12 in.) spacing governs.

*Transverse reinforcement for vertical walls (ACI 318-11, Section 11.5.3.8)—*

Transverse reinforcement in the vertical walls is contributed by both torsion and shear:

$$\frac{A_t}{s}+\frac{1}{2}\frac{A_v}{s}=0.414+0.5\times 2.043$$

$= 1.44$ mm²/mm (0.0575 in.²/in.)

$$\left(\frac{A_t}{s}+\frac{1}{2}\frac{A_v}{s}\right)_{min}=0.375\frac{\sqrt{f_c'}}{12}\frac{b_w}{f_{yt}}$$

$= 0.259$ mm²/mm (0.01 in.²/in.)
$< 1.44$ mm²/mm (0.0575 in.²/in.) OK

(Eq. (11-23) in ACI 318-11)

Select two layers of ∅18 mm bars (No. 6 bars) in each vertical wall at 305 mm (12 in.) spacing

$$\frac{2(254)}{(305)} = 1.666 \text{ mm}^2\text{/mm (0.0656 in.}^2\text{/in.)}$$

$> 1.440$ mm²/mm (0.0575 in.²/in.) OK

*Transverse reinforcement for horizontal walls—*

Transverse reinforcement in horizontal walls is contributed by torsion only:

$$\frac{A_t}{s} = 0.414 \text{ mm}^2\text{/mm (0.0166 in.}^2\text{/in.)}$$

Select two layers of ∅10 mm bars (No. 3 bars) in each horizontal wall at 305 mm (12 in.) spacing:

$$\frac{2\times 78.5}{305} = 0.515 \text{ mm}^2\text{/mm (0.0203 in.}^2\text{/in.)}$$

$$> 0.259 \text{ mm}^2\text{/mm (0.01 in.}^2\text{/in.)}$$

The transverse reinforcement in the top wall should be added to the flexural reinforcement required in the top flange acting as a transverse continuous slab.

5. Calculate number of longitudinal bars required.

*Design of torsional longitudinal reinforcement (Eq. (11-22) in ACI 318-11)—*

$$A_\ell = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{y\ell}}\right)\cot^2\theta = 0.414 \times 5813 \times 1 \times 1.303^2$$

$$= 4085.9 \text{ mm}^2 \text{ (6.46 in.}^2\text{)}$$

Check minimum limitation for $A_t/s$ and $A_{l,min}$ (Eq. (11-23) in ACI 318-11)

$$\frac{A_t}{s} = 0.375\sqrt{f_c'}\frac{b_w}{f_{yt}} = 0.259 \text{ mm}^2\text{/mm (0.01 in.}^2\text{/in.)}$$

$$< 0.414 \text{ mm}^2\text{/mm (0.0166 in.}^2\text{/in.)}$$

$$A_{\ell,\min} = \frac{5\sqrt{f_c'}A_g}{f_{yl}} - \left(\frac{A_t}{s}\right)p_h\left(\frac{f_{yv}}{f_{yl}}\right)$$

(Eq. (11-24) in ACI 318-11)

$$= \frac{5\times 0.083\sqrt{48.0}\left(1.2\times 10^6\right)}{420} - 0.414 \times 5813 \times 1$$

$$= 8215 - 2407 = 5808 \text{ mm}^2 \text{ (9.17 in.}^2\text{) governs}$$

Select 36 ∅16 bars (No. 5 bars) longitudinal bars

$$A_\ell = 36 \times 201 = 7236 \text{ mm}^2 \text{ (11.22 in.}^2\text{)} > 5808 \text{ mm}^2 \text{ (9.17 in.}^2\text{)}$$

*Arrangement of reinforcing bars—*

The arrangement of the reinforcing bars for torsion and shear is summarized in Table 9.4.6. This reinforcement arrangement could be conservatively used throughout the girder length.

**9.5.3** *EC2-04 code*

**9.5.3.1** *Material properties*

**9.5.3.1.1** *Concrete*

*Concrete compressive cylinder strength (EC2-04, Section 3.1.2 and 3.1.6)—*

- Specified compressive strength (The characteristic concrete cylinder compressive strength)

$$f_c' = 48.0 \text{ MPa (7000 psi)}$$

- Required average compressive strength (psi)

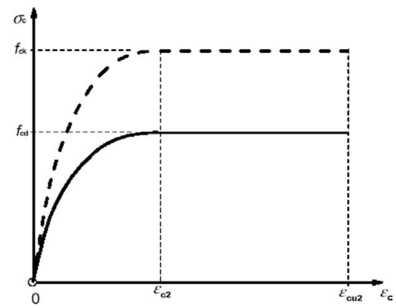



*Fig. 9.5.3.1.1—Parabola-rectangle diagram for concrete under compression.*

$$f_{cr}' = f_c' + 1400 \text{ (for } f_c' > 5000 \text{ psi)}$$

$$f_{cr}' = 7000 + 1400 = 8400 \text{ psi (57.9 MPa)}$$

- Mean compressive strength (based on $f_{cr}'$)

$$f_{cm} = 57.9 \text{ MPa (8400 psi)}$$

- Characteristic compressive strength (MPa) (EC2-04, Section 3.1.2(5))

$$f_c' = f_{cm} - 8 \Rightarrow f_{ck} = 49.9 \text{ MPa (7240 psi)}$$

- Strength class for concrete according to Section 3.1.2 (Table 3.1) in EC2-04 corresponding to mean compressive strength

Strength Class C50, therefore: $f_c' = 50$ MPa (7250 psi)

- Design compressive strength (EC2-04, Section 3.1.6(1))

$$f_{cd} = \alpha_{cc}\frac{f_c'}{\gamma_c} = 1\left(\frac{50}{1.5}\right) = 33.3 \text{ MPa (4833 psi)}$$

- Mean compressive strength according to EC2-04 (Section 3.1.6(5)) and the strength class of concrete

$$f_c' = f_{cm} + 8 \Rightarrow f_{cm} = 58.0 \text{ MPa (8410 psi)}$$

*Concrete tensile strength* (*EC2-04, Section 3.1.6 (2)—*

- Mean tensile strength

$$f_t' = 0.3 f_c'^{2/3} = 4.07 \text{ MPa (590 psi)}$$

- Design tensile strength

$$f_{ctd} = \alpha_{ct}\frac{f_{ctk,0.05}}{\gamma_c} = 1\times\frac{0.7 f_t'}{1.5} = 1.89 \text{ MPa (275 psi)}$$

- Modulus of elasticity

$E_m = 22\left(\frac{f_{cm}}{10}\right)^{0.3}$ = 37.28 GPa (5405 ksi) (EC2-04, Section 3.1.3, Table 3.1)

Parabola-rectangle diagram for concrete under compression, as shown in Fig. 9.5.3.1.1 (*EC2-04, Section 3.1.7)*

$$\sigma_c = f_{cd}\left[1-\left(1-\frac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right] \text{ for } \varepsilon_c < \varepsilon_{c2}$$

$$\sigma_c = f_{cd} \text{ for } \varepsilon_c \geq \varepsilon_{c2}$$

where

$$n = 1.4 + 23.4\left(\frac{90-f_c'}{100}\right)^4 = 2.00$$

$$\varepsilon_{c2} = (2 + 0.085(f_c' - 50)^{0.53})/100 = 0.0020$$

$$\varepsilon_{cu2} = \left(2.6 + 35\left(\frac{90-f_c'}{100}\right)^4\right)/100 = 0.0035$$

**9.5.3.1.2** *Reinforcing steel*
Type: Ribbed reinforcing bars (high bond)—
Diameter for longitudinal bars = 12 mm [∅12] (0.472 in.)
Diameter for closed stirrups = 10 mm [∅10] (0.394 in.)
Characteristic yield strength $f_y$ = 420 MPa (60 ksi)
*Design yield strength (EC2-04, Section 2.4.2.4)—*

$$f_{yd} = \frac{f_y}{\gamma_s} = \frac{420}{1.15} = 365 \text{ MPa (52 ksi)}$$

Modulus of elasticity (EC2-04, Section 3.2.7(4))

$$E_s = 200 \text{ GPa (29,000 ksi)}$$

Strain at design yield strength

$$\varepsilon_y = \frac{f_{yd}}{E_s} = 0.0018$$

Ultimate design strain (EC2-04, Section 3.2.7(2))

$$\varepsilon_{ud} = 0.9\varepsilon_{uk} = 0.0200$$

**9.5.3.1.3** *Prestressing reinforcing strands*
Type: Low relaxation, 270K, seven-wire strands—
Diameter = 13.0 mm (0.50 in.)
Area $A_s$ = 99 mm$^2$ (0.153 in.$^2$)
Characteristic tensile strength $f_{pk} = f_{pu}$ = 1860 MPa (270 ksi)
Design ultimate strength (EC2-04, Section 2.4.2.4)

$$f_{p,ud} = \frac{f_{pk}}{\gamma_s} = \frac{1860}{1.15} = 1617 \text{ MPa (234.78 ksi)}$$

Characteristic yield strength (EC2-04, Section 3.3.3(1))

$$f_{p0.1k} = 0.9f_{pu} = 1674 \text{ MPa (243 ksi)}$$

Design yield strength (EC2-04, Section 3.3.6(6))

$$f_{pd} = \frac{f_{p0.1k}}{\gamma_s} = \frac{1674}{1.15} = 1456 \text{ MPa (211 ksi)}$$

Modulus of elasticity (EC2-04, Section 3.3.6(3))

$$E_p = 196 \text{ GPa (28,420 ksi)}$$

Strain at design yield strength

$$\varepsilon_{p,yd} = \frac{f_{pd}}{E_p} = 0.0074$$

Ultimate design strain (EC2-04, Section 3.3.6(7))

$$\varepsilon_{p,ud} = 0.0200$$

**9.5.3.2** *Sectional areas and concrete cover*
**9.5.3.2.1** *Gross concrete area*
- Area of the gross cross section including overhanging flanges

$$= 2.65 \times 10^6 \text{ mm}^2 \text{ (4107 in.}^2\text{)}$$

- Area of the gross cross section (disregarding overhanging flanges)

$$= 2.32 \times 10^6 \text{ mm}^2 \text{ (3596 in.}^2\text{)}$$

- Area of the concrete cross section (disregarding overhanging flanges)

$$A_c = 1.20 \times 10^6 \text{ mm}^2 \text{ (1860 in.}^2\text{)}$$

**9.5.3.2.2** *Concrete covers* (*EC-04, Section 4.4.1.2);*
- Minimum cover with regard to bond

$c_{min,b}$ = diameter of bar = 12 mm (0.47 in.) (reinforcing steel)
= 13 mm (0.51 in.) (prestressing steel)

- Exposure class related to environmental conditions

Exposure Class XD3 (Cyclic wet and dry. Parts of bridges exposed to spray containing chlorides.)

- Structural class (XD3) = 4 + 2 (service life of 100 years) – 1 (concrete class ≥ C45) = 5

- Minimum cover with regard to durability

$c_{min,dur}$ = 50 mm (1.97 in.) (reinforcing steel)
= 60 mm (2.36 in.) (prestressing steel)

- Minimum cover

$$c_{min} = \max(c_{min,b};\ c_{min,dur};\ 10\text{ mm [0.39 in.]})$$
$$= 50\text{ mm (1.97 in.) for reinforcing steel}$$
$$= 60\text{ mm (2.36 in.) for prestressing steel}$$

- Nominal cover

$$c_{nom} = c_{min} + 10\text{ mm (0.39 in.)}$$
$$= 60\text{ mm (2.36 in.) for reinforcing steel}$$
$$= 70\text{ mm (2.76 in.) for prestressing steel}$$

- Distance from center of longitudinal reinforcing bars to extreme concrete fiber (cover from bars centroid)

$$= 66\text{ mm (2.56 in.)} > c_{min} + 12\text{ mm/2 (0.47 in./2)}$$

- Distance from center of prestressed tendons to extreme bottom fiber (cover from tendons centroid)

$$= 102\text{ mm (3.94 in.)} > c_{min} + 13\text{ mm/2 (0.51 in./2)}$$

**9.5.3.3** *Factored shear, torque, bending moment, and prestress*

Factored shear force

$$V_u = 1.4 \times (34.4 + 12.8) \times 0.2 \times 24.0 + 1.5 \times (513.8/2) \times 2 = 1089\text{ kN (245 kip)}$$

Factored torsional moment

$$T_u = 1.5 \times (513.8/2)2 \times 0.914 = 705\text{ kN·m (520 ft-kip)}$$

Factored bending moment

$$M_u = 0.5 \times 1.4 \times (34.4 + 12.8) \times 0.7 \times 0.3 \times (24.00)^2 + 1.5(513.8/2)(2 \times 7.2) = 9561\text{ kN·m (7074 ft-kip)}$$

Prestress force at time $t = \infty$

$$P_t = 6076\text{ kN (1366 kip)}$$

Total prestress losses

$$20\text{ percent or } \omega = P_t/P_o = 0.80$$

Therefore, prestress force at time $t = 0$

$$P_o = P_t/\omega = 7595\text{ kN (1708 kip)}$$

**9.5.3.4** *Prestressed tendons requirements*

Minimum longitudinal reinforcement (EC2-04, Section 9.2.1.1)

$$A_\ell = 0.26\frac{f_t'}{f_{yk}}b_t d \geq 0.0013 b_t d$$

where $b_t$ = 470 mm (18.5 in.) mean width of tension zone, and $d$ = 1172 mm (46.1 in.) distance from extreme top fiber to the centroid of the reinforcement.

Therefore,

$$A_{\ell,min} = 716\text{ mm}^2\text{ (1.1 in.}^2)$$

The area of the prestressed tendons is

$$A_P = 64 \times 99 = 6336\text{ mm}^2\text{ (9.82 in.}^2) \geq A_{s\ell,min} \quad \text{OK}$$

**9.5.3.5** *Shear design*

**9.5.3.5.1** *Design shear resistance of a member without shear reinforcement* (*EC2-04, Section 6.2.2(1)*)

$$V_{Rd,c} = \left[C_{Rd,c}k\left(100\rho_\ell f_{ck}\right)^{1/3} + k_1\sigma_{cp}\right]b_w d \geq \left(v_{min} + k_1\sigma_{cp}\right)b_w d$$

where

$d$ = 517 + 494 = 1011 mm (39.8 in.) at $0.3L$ = 7.2 m (23.7 ft) (9.5.1.1(b))

$b_w$ = 470 mm (18.50 in.)

$C_{Rd,c}$ = $0.18/\gamma_c = 0.18/1.5 = 0.12$

$k$ = $1 + \sqrt{200/d} \leq 2 \Rightarrow k = 1.44$

$$\rho_\ell = \frac{A_p}{b_w d} \leq 0.02 \Rightarrow \rho_\ell = 0.0133$$

$$\sigma_{cp} = \frac{N_{Ed}}{A_c} = \frac{P_t}{A_c} = 5.1\text{ MPa (734.4 psi) (compressive)}$$

$k_1$ = 0.15

$v_{min}$ = $0.035k^{3/2}f_{ck}^{1/2} \Rightarrow v_{min} = 0.428$

Therefore

$$V_{Rd,c} = 698\text{ kN (156.2 kip)}$$

Because

$$V_{Rd,c} < V_{Ed} = 1089\text{ kN (245 kip)},$$

shear reinforcement must be provided

**9.5.3.5.2** *Minimum shear reinforcement* (*EC2-04, Section 9.2.2(5)*)

The ratio of the minimum stirrups is

$$\rho_{w,min} = \left(0.08\sqrt{f_{ck}}\right)\Big/f_{yk} = 0.08\sqrt{49.9}/(1.2 \times 420) = 0.00113$$

Therefore

$$\left(\frac{A_{sw}}{sb_w}\right)_{min} \geq 0.00113 \rightarrow \left(\frac{A_{sw}}{s}\right)_{min} \geq 0.5311\text{ mm}^2\text{/mm (0.0209 in.}^2\text{/in.)}$$

**9.5.3.5.3** *Maximum effective cross-sectional area of the shear reinforcement* (*EC2-04, Section 6.2.3(3)*)

$$\frac{A_{sw.max} f_{yd}}{s b_w} \leq \frac{1}{2}\alpha_c v f_{cd} \rightarrow \left(\frac{A_{sw}}{s}\right)_{max} \leq \frac{\alpha_c v b_w f_{cd}}{2 f_{yd}}$$

where
$v = 0.6(1 - f_{ck}/250) = 0.48$
$\alpha_c = 1 + \sigma_{cp}/f_{cd} = 1.154$ (because $\sigma_{cp} \leq 0.25 f_{cd}$)
Therefore

$$\left(\frac{A_{sw}}{s}\right)_{max} \leq 11.86 \text{ mm}^2\text{/mm } (0.47 \text{ in.}^2\text{/in.})$$

**9.5.3.5.4** *Required shear reinforcement* (*EC2-04, Section 6.2.3(4))*

$$V_{Rd} = \min\begin{Bmatrix} V_{Rd,s} \\ V_{Rd,max} \end{Bmatrix} \geq V_{Ed}$$

$$V_{Rd,max} = \frac{\alpha_c b_w z v f_{cd}}{(\cot\theta + \tan\theta)} \qquad \text{(Eq. (6.14) in EC2-04)}$$

With the above data and

$$z = 0.9d \Rightarrow z = 910 \text{ mm (35.8 in.)}$$

$$f_{cd} = 33.3 \text{ MPa (4830 psi)}$$

$$1 \leq \cot\theta \leq 2.5 \Rightarrow 22 \text{ degrees} \leq \theta \leq 45 \text{ degrees}$$
(mean value: $\theta = 35$ degrees)

$$V_{Rd,max} = \frac{1.154 \times 470 \times 910 \times 0.48 \times 33.3}{(\cot\theta + \tan\theta)}$$

For
$\theta = 45$ degrees

$$V_{Rd,max} = 3945 \text{ kN (886 kips)} > V_{Ed},$$

$\theta = 35$ degrees

$$V_{Rd,max} = 3707 \text{ kN (832 kips)} > V_{Ed}, \text{ and}$$

$\theta = 22$ degrees

$$V_{Rd,max} = 2740 \text{ kN (615 kips)} > V_{Ed}.$$

Thus for (Eq. (6.13) in EC2-04)

$$V_{Rd} = V_{Rd,s} \geq V_{Ed}$$

$$V_{Rd,s} = \frac{A_{sw}}{s} z f_{yd} \cot\theta \geq V_{Ed} \Rightarrow \frac{A_{sw}}{s} \geq \frac{V_{Ed}}{z f_{yd} \cot\theta} \Rightarrow \frac{A_{sw}}{s} \geq \frac{1089 \times 10^3}{910 \times 365 \times \cot\theta}$$

$\theta = 45$ degrees

$$\frac{A_{sw}}{s} \geq 3.28 \text{ mm}^2\text{/mm } (0.131 \text{ in.}^2\text{/in.}),$$

$\theta = 35$ degrees

$$\frac{A_{sw}}{s} \geq 2.30 \text{ mm}^2\text{/mm } (0.092 \text{ in.}^2\text{/in.}), \text{ and}$$

$\theta = 22$ degrees

$$\frac{A_{sw}}{s} \geq 1.32 \text{ mm}^2\text{/mm } (0.053 \text{ in.}^2\text{/in.}).$$

Check

$$\left(\frac{A_{sw}}{s}\right)_{min} < \frac{A_{sw}}{s} < \left(\frac{A_{sw}}{s}\right)_{max} \qquad \text{OK}$$

**9.5.3.6** *Torsion design*

**9.5.3.6.1** *Terms of the equivalent thin-walled section* (*EC2-04, Section 6.3.2(1));*

$$t_{ef} = \max\begin{Bmatrix} A/u \\ 2c_\ell \end{Bmatrix} \leq t_{real} = \max\begin{Bmatrix} 375 \text{ mm} \\ 134 \text{ mm} \end{Bmatrix} \leq 178 \text{ mm}$$

$$\Rightarrow t_{ef} = 178 \text{ mm (7 in.)}$$

where
$A$ = $2.32 \times 10^6$ mm² (3597 in.²) total area of the cross section within the outer circumference, including inner hollow areas (disregarding overhanging flanges)
$u$ = 6185 mm (243.5 in.) outer circumference of the cross section (disregarding overhanging flanges)
$c_\ell$ = 66 mm (2.56 in.) distance from edge to center of the longitudinal reinforcement
$t_{real}$ = 178 mm (7 in.) minimum wall thickness of the real concrete thin-walled section.

The continuous area and perimeter enclosed by centerlines of the connecting thin-walls, as shown in Fig. 9.5.3.6.1, are: $A_k = 1.8 \times 10^6$ mm² (2794 in.²) and $u_k = 5486$ mm (216 in.).

**9.5.3.6.2** *Determine if cross-sectional dimensions (for example, strength of concrete struts under torsion) are adequate* (*EC2-04, Section 6.3.2(4))*

Dimensions of the cross section are adequate if (Eq. (6.29) in EC2-04)

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1$$

The design torsional resistance moment is calculated as (Eq. (6.30) in EC2-04)

$$T_{Rd,max} = 2 v \alpha_c f_{cd} A_k t_{ef} \sin\theta \cos\theta$$

where

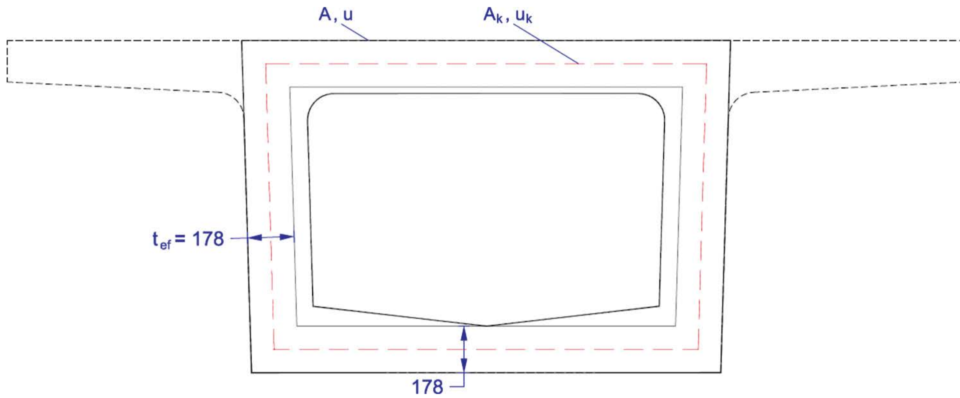



*Fig. 9.5.3.6.1—Assumed effective thin-wall section.*

$v = 0.6(1 - f_{ck}/250) = 0.48$ and $\alpha_c = 1 + \sigma_{cp}/f_{cd} = 1.154$

and for
θ = 45 degrees

$$T_{Rd,max} = 5915 \text{ kN·m (4356 ft-kip)},$$

θ = 35 degrees

$$T_{Rd,max} = 5559 \text{ kN·m (4093 ft-kip)}$$

θ = 22 degrees

$$T_{Rd,max} = 4109 \text{ kN·m (3026 ft-kip)}$$

Design shear resistance force is determined by (Eq. (6.14) in EC2-04)

$$V_{Rd,max} = \frac{\alpha_c b_w z v f_{cd}}{(\cot\theta + \tan\theta)}$$

Thus for
θ = 45 degrees

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} = \frac{705}{5910} + \frac{1089}{3945} = 0.39 \leq 1 \quad \text{OK}$$

θ = 35 degrees

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} = \frac{705}{5554} + \frac{1089}{3707} = 0.42 \leq 1 \quad \text{OK}$$

θ = 22 degrees

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} = \frac{705}{4085} + \frac{1089}{2740} = 0.57 \leq 1 \quad \text{OK}$$

Dimensions of the cross section, therefore, are adequate for every case (every value of θ).

**9.5.3.6.3** *Calculate the required stirrup area for torsion.* (*EC2-04, Section 6.3.2(2)*)

The required stirrup area per unit length is calculated by

$$\frac{A_{sw}}{s} \geq \frac{T_{Ed}}{2A_k f_{yd}\cot\theta} \Rightarrow \frac{A_{sw}}{s} \geq \frac{705\times10^6}{2\times1.8\times10^6\times365\times\cot\theta}$$

For
θ = 45 degrees

$$\frac{A_{sw}}{s} \geq 0.54 \text{ mm}^2\text{/mm (0.021 in.}^2\text{/in.)},$$

θ = 35 degrees

$$\frac{A_{sw}}{s} \geq 0.38 \text{ mm}^2\text{/mm (0.015 in.}^2\text{/in.), and}$$

θ = 22 degrees

$$\frac{A_{sw}}{s} \geq 0.22 \text{ mm}^2\text{/mm (0.009 in.}^2\text{/in.)}$$

where $A_{sw}$ and $s$ are the area of one leg of a closed stirrup resisting torsion and the spacing of the stirrups, respectively.

**9.5.3.6.4** *Add stirrup areas for torsion and shear, and select the stirrups* (*EC2-04, Section 6.3.2(2)*)

$$\frac{A_{sw,S+T}}{s} = \frac{1}{2}\frac{A_{sw,S}}{s} + \frac{A_{sw,T}}{s} \quad \text{(using two single-legged stirrups)}$$

For
θ = 45 degrees

$$\frac{A_{sw,S+T}}{s} \geq 2.18 \text{ mm}^2\text{/mm (0.087 in.}^2\text{/in.)},$$

θ = 35 degrees

$$\frac{A_{sw,S+T}}{s} \geq 1.53 \text{ mm}^2/\text{mm } (0.061 \text{ in.}^2/\text{in.}), \text{ and}$$

θ = 22 degrees

$$\frac{A_{sw,S+T}}{s} \geq 0.88 \text{ mm}^2/\text{mm } (0.035 \text{ in.}^2/\text{in.}).$$

Maximum longitudinal spacing of the stirrups for shear (Eq. (9.6N) in EC2-04)

$$s_{max} = 0.75d\left(1+\cot\alpha\right) \xrightarrow{\alpha=90^\circ} s_{max} = 0.75d$$
$$= 758 \text{ mm (29.8 in.)}$$

where α is the inclination of the stirrups.

Maximum longitudinal spacing of the stirrups for torsion (EC2-04, Section 9.2.3(3))

$$s_{max} = \min\begin{Bmatrix} u/8 \\ 0.75d\left(1+\cot\theta\right) \\ \min(b,h) \end{Bmatrix} = \min\begin{Bmatrix} 773 \text{ mm} \\ 758 \text{ mm} \\ 235 \text{ mm} \end{Bmatrix}$$
$$= 235 \text{ mm (9.25 in.)}$$

For two single-legged ∅10 stirrups, spacing is calculated from shear and torsion

For

θ = 45 degrees

$$s \leq 72 \text{ mm (2.8 in.)} < s_{max} \quad \text{OK}$$

θ = 35 degrees

$$s \leq 102 \text{ mm (4.0 in.)} < s_{max} \quad \text{OK}$$

θ = 22 degrees

$$s \leq 178 \text{ mm (7.0 in.)} < s_{max} \quad \text{OK}$$

Total selection (θ = 35 degrees): two single-legged stirrups of ∅10/100 mm (diameter 0.39 in. at 4.0 in.).

**9.5.3.6.5** *Longitudinal reinforcement required for torsion (EC2-04, Section 6.3.2(3))*

Total longitudinal reinforcement for torsion (Eq. (6-28) in EC2-04)

$$\Sigma A_{s\ell} \geq \frac{T_{Ed} u_k \cot\theta}{2A_k f_{yd}} \Rightarrow \Sigma A_{s\ell} \geq \frac{705\times10^6 \times 5486 \times \cot\theta}{2\times1.8\times10^6 \times 365}$$

For

θ = 45 degrees

$$\Sigma A_{s\ell} \geq 2943 \text{ mm } (4.62 \text{ in.}^2),$$

θ = 35 degrees

$$\Sigma A_{s\ell} \geq 4204 \text{ mm}^2 \text{ } (6.60 \text{ in.}^2), \text{ and}$$

θ = 22 degrees

$$\Sigma A_{s\ell} \geq 7285 \text{ mm}^2 \text{ } (11.50 \text{ in.}^2).$$

Longitudinal bars shall be arranged so that at least one bar is placed at each corner of the stirrups and the others are distributed uniformly around the inner periphery of the torsion links (closed stirrups) with a maximum spacing of 350 mm (13.8 in.) (EC2-04, Section 9.2.3(4)). Therefore, the number of the longitudinal bars is at least 36.

For

θ = 45 degrees

$$38\varnothing10 \rightarrow \Sigma A_{s\ell} = 2985 \text{ mm } (4.63 \text{ in.}^2) > 2943 \text{ mm } (4.63 \text{ in.}^2),$$

θ = 35 degrees

$$38\varnothing12 \rightarrow \Sigma A_{s\ell} = 4298 \text{ mm}^2 \text{ } (6.66 \text{ in.}^2) > 4204 \text{ mm}^2 \text{ } (6.61 \text{ in.}^2), \text{ and}$$

θ = 22 degrees

$$38\varnothing16 \rightarrow \Sigma A_{s\ell} = 7640 \text{ mm}^2 \text{ } (11.84 \text{ in.}^2) > 7285 \text{ mm}^2 \text{ } (11.58 \text{ in.}^2).$$

Final selection of longitudinal bars (θ = 35 degrees)

$$\varnothing12 \text{ } (0.47 \text{ in.}) \rightarrow A_{s\ell} = 113 \text{ mm}^2 \text{ } (0.175 \text{ in.}^2).$$

Due to uniform distribution and symmetry

$$40\varnothing12 \rightarrow \Sigma A_{s\ell} = 4524 \text{ mm}^2 \text{ } (7.0 \text{ in.}^2).$$

The torsional longitudinal reinforcement is in addition to the prestressing tendons. Low values of angle θ (θ = 22 degrees) lead to a design with lower area requirements of transverse reinforcement (∅10/175 mm) and higher area requirements of longitudinal reinforcement (40∅16). High values of angle θ (θ = 45 degrees) lead to higher area requirements of stirrups (∅10/70 mm) and lower area requirements of longitudinal bars (40∅10).

The selection of θ = 45 degrees maximizes the concrete strength components, such as $V_{Rd,max}$ = 3945 kN and $T_{Rd,max}$ = 5910 kN·m. This value of angle θ could be used when checking the adequacy of the cross section dimensions.

The following relationship between the strength of concrete struts under torsion is satisfied

$$\frac{T_{Ed}}{T_{Rd,max}} + \frac{V_{Ed}}{V_{Rd,max}} \leq 1$$

**9.5.3.6.6** *Arrangement of reinforcing bars*—Total reinforcement of the member with a hollow cross section under combined loading of prestressing, torsion, shear, flexure, and axial force is depicted in detail in Fig. 9.5.3.6.6.

**9.5.4** *Design solution using CSA-A23.3-04 code*—Load factors of the CSA-A23.3-04 code are used here to establish

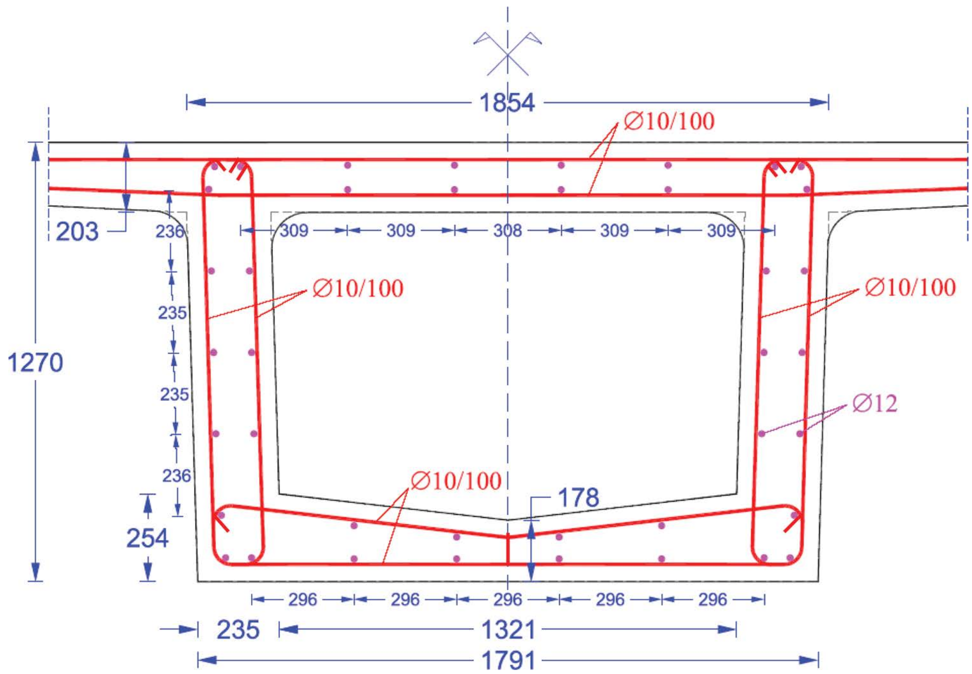



*Fig. 9.5.3.6.6—Reinforcement configuration (EC2-04).*

the forces due to factored loads. These factors are 1.25 for dead loads and 1.50 for live loads. The cross-sectional area of the prestressing strands ($f_{pu}$ = 1860 MPa [270 ksi]) is 6336 mm$^2$ (9.82 in.$^2$), the effective prestressing force is 6076 kN (1366 kip), and the average prestress is 3.99 MPa (589 psi). The concrete strength is 48.0 MPa (7000 psi), and the yield strength of the non-prestressed reinforcement is 420 MPa (60,000 psi).

1. Determine the factored forces (CSA-A23.3-04, Annex C).

*Factored dead and live loads—*

The derailment load per axle is calculated by

$$P_{u,L} = 1.5\left(\frac{513.8}{4}\right)2 = 385 \text{ kN/axle (86.6 ft/axle)}$$

The derailment torque per axle is determined by

$$T_{u,L} = 1.5\left(\frac{513.8}{4}\right)(2 \times 0.914) = 352 \text{ kN·m (259.8 ft-kip)}$$

The girder weight (with a load factor of 1.25) is determined by

$$w_{u,g} = 1.25 \times 34.3 = 43.0 \text{ kN/mm (2.95 kip/ft)}$$

The superimposed dead weight (with a load factor of 1.25) is calculated by

$$w_{u,s} = 1.25 \times 12.8 = 16.0 \text{ kN/mm (1.10 kip/ft)}$$

*Factored shear, torque, and bending moment—*

At distance 0.3$L$ from support, the following values are obtained

$$V_u = (43.0 + 16.0)(0.2 \times 24.00) + 2 \times 385 = 1053 \text{ kN (237 kip)}$$

$$T_u = 2 \times 352.3 = 705 \text{ kN·m (520 ft-kip)}$$

$$M_u = 0.5(43.0 + 16.0)(24.00 - 7.2)7.2 + 2 \times 385 \times 7.2 = 9112 \text{ kN·m (6759 ft-kip)}$$

2. Determine if torsion effects can be disregarded (CSA-A23.3-04, Section 11.2.9.1).

*Threshold torque—*

For a hollow section with a wall thickness of less than $0.75A_c/p_c$ (= 281 mm > 178 mm), torsion must be considered if the torque due to factored loads, $T_f$, exceeds $0.25T_{cr}$

$$T_{cr} = \frac{\left(1.5A_g\right)^2}{p_c}0.38\lambda\phi_c\sqrt{f_c'}\sqrt{1+\frac{\phi_p f_{cp}}{0.38\lambda\phi_c\sqrt{f_c'}}}$$

where

$A_c$ = $A_{cp}$ = 2.32 × 10$^6$ mm$^2$ (3597 in.$^2$)

$p_c$ = $p_{cp}$ = outside perimeter of concrete cross section = 6185 mm (243.5 in.)

$A_g$ = gross area of section (without flanges) = 1.20 × 10$^6$ mm$^2$ (1855 in.$^2$)

$\lambda$ = factor to account for low-density concrete = 1.0

$\phi_c$ = resistance factor for concrete = 0.70 for precast concrete

$\phi_p$ = resistance factor for prestressing reinforcement = 0.90

$f_c'$ = specified compressive strength of concrete = 48.0 MPa (7.0 ksi)

$f_{cp}$ = compression stress in concrete due to effective prestress = 3.99 MPa (0.6 ksi)

With this information, the following calculation can be made

$$T_{cr} = \frac{(1.5\times1.20\times10^6)^2}{6185}0.38\times0.70\sqrt{48.0}\sqrt{1+\frac{0.9\times3.99}{0.38\times0.70\sqrt{48.0}}}$$
$$= 1658 \text{ kN·m (1216 ft-kip)}$$

Because $0.25T_{cr}$ = 415 kN·m (304 ft-kip) < 705 kN·m (520 ft-kip), torsion must be considered in the design.

3. Determine if dimensions of the cross section are adequate.

*Check cross-sectional dimensions* (*CSA-A23.3-04, Section 11.3.10.4)—*

For box sections with a wall thickness of less than $A_{oh}/p_h$, the cross-sectional dimensions must satisfy the following criterion (Eq. (11-19) in CSA-A23.3-04)

$$\frac{V_f - V_p}{b_w d_v} + \frac{T_f}{1.7A_{oh}t} \le 0.25\phi_c f_c'$$

where

$V_f$ = $V_u$ = shear force due to factored loads = 1054 kN (237.2 kip)
$V_p$ = shear force due to prestressing factored by $\phi_p$
= 0.9(6076)508/9754 = 284 kN (64.0 kip)
$A_{oh}$ = [(1854 + 1791)/2 – 2 × 48)](1270 – 2 × 48) = 2.03 × $10^6$ mm$^2$ (3142 in.$^2$)
$p_h$ = (1854 – 96) + (1791 – 96) + 2(1270 – 96) = 5801 mm (228.4 in.)
$A_{oh}/p_h$ = 350 mm (13.8 in) > 178 mm (7.0 in.) for bottom flange
> 235 mm (9.25 in.) for web
> 203 mm (8 in.) for top flange
$d_v$ = larger of 0.9$d$ = 0.9 × 1016 = 914 mm and 0.72$h$ = 0.72 × 1270 = 914 mm (36 in.)
$t$ = minimum wall thickness = 178 mm (7.0 in.)
$T_u$ = torque due to factored loads = 705 kN·m (520 ft-kip)

Therefore,

$$\frac{(1054-284)\times10^3}{235\times2\times914} + \frac{705\times10^6}{1.7\times2.03\times10^6\times235}$$
$$= 2.66 \text{ MPa (386 psi)} < 0.25 \times 0.70 \times 48.0 = 8.40 \text{ MPa (1223 psi)} \quad \text{OK}$$

Because the web governs design, the above equation was the web thickness $t$ = 235 mm, not the bottom flange thickness.

4. Calculate θ and β (CSA-A23.3-04, Section 11.3.6.4).

*Angle of diagonal compression strut and shear resistance of concrete—*

The angle of inclination of the diagonal compression strut is given by the expression (Eq. (11-12) in CSA-A23.3-04)

$$\theta = 29 + 7000\varepsilon_x$$

In the absence of an axial load normal to the cross section, the strain at mid-depth of the section is defined by (Eq. (11-13) in CSA-A23.3-04)

$$\varepsilon_x = \frac{\dfrac{M_f}{d_v} + \sqrt{(V_f - V_p)^2 + \left(\dfrac{0.9p_hT_f}{2A_o}\right)^2} - A_pf_{po}}{2[E_sA_s + E_pA_p]}$$

The terms not defined above are

$M_f$ = $M_u$ = moment due to factored loads = 9166 kN·m (6761 ft-kip)
$A_o$ = 0.85$A_{oh}$ = 0.85 × 2.04 × $10^6$ mm$^2$ = 1.74 × $10^6$ mm$^2$ (2697 in.$^2$)
$A_p$ = area of prestressing reinforcement = 6336 mm$^2$ (9.8 in.$^2$)
$f_{po}$ = stress in prestressing tendons (may be taken as 0.7$f_{pu}$ = 1302 MPa [189.0 ksi])
$E_s$ = 200,000 MPa (29,000 ksi)
$A_s$ = area of non-prestressed reinforcement in tension zone (assume fourteen 15M bars with 2800 mm$^2$ (4.34 in.$^2$)
$E_p$ = 190,000 MPa (28,000 ksi)

With this information, the following calculation can be made

$$\varepsilon_x = \frac{\dfrac{9112\times10^6}{914} + \sqrt{[(1054-284)\times10^3]^2 + \left(\dfrac{0.9\times5801\times705\times10^6}{2\times1.74\times10^6}\right)^2} - 1302\times6336}{2(200,000\times2800 + 190,000\times6336)} = 0.00086$$

Therefore

$$\theta = 29 + 7000 \times 0.00086 = 35.0°$$

The shear force resisted by the concrete is (Eq. (11-6) in CSA-A23.3-04)

$$V_c = \phi_c\lambda\beta\sqrt{f_c'}b_wd_v$$

where (Eq. (11-11) in CSA-A23.3-04)

$$\beta = \frac{0.40}{1+1500\varepsilon_x}\times\frac{1300}{1000+s_{ze}}$$

Because minimum transverse reinforcement is provided, $s_{ze}$ = 300 mm (12 in.)

Therefore

$$\beta = \frac{0.40}{1+1500(0.00086)}\times\frac{1300}{1000+300} = 0.175$$

and

$$V_c = 0.70 \times 1.0 \times 0.175(\sqrt{48.0})235 \times 2 \times 914 \text{ N}$$
$$= 365 \text{ kN (80.7 kip)}$$

5. Calculate transverse reinforcement.

*Design of transverse reinforcement for shear (CSA-A23.3-04, Section 11.3.5.1)*—

Shear with $V_s = V_f - V_c - V_p = 1054 - 365 - 284 = 405$ kN (92.6 kip)

$$\frac{A_v}{s} = \frac{V_s}{\phi_s f_y d_v \cot\theta} = \frac{405\times10^3}{0.85\times420\times914\times\cot 35.0°}$$

$$= 0.87 \text{ mm}^2\text{/mm } (0.0356 \text{ in.}^2\text{/in.})$$

*Design of transverse reinforcement for torsion (CSA-A23.3-04, Section 11.3.10.3)*—

The required transverse reinforcement for torsion is given by (Eq. 11-17 in CSA-A23.3-04)

$$\frac{A_t}{s} = \frac{T_u}{2A_o\phi_s f_y \cot\theta} = \frac{705\times10^6}{2\times1.74\times10^6\times0.85\times420\times\cot 35.0°}$$

$$= 0.40 \text{ mm}^2\text{/mm } (0.0160 \text{ in.}^2\text{/in.})$$

This amount of transverse reinforcement must be provided in the top and bottom slabs.

Total transverse reinforcement in one web is calculated as

$$\frac{A_t}{s} + \frac{A_v}{2s} = 0.40 + 0.87/2 = 0.835 \text{ mm}^2\text{/mm } (0.0338 \text{ in.}^2\text{/in.})$$

The selection of two 10M bars ($A_{bar} = 100$ mm$^2$) per web yields the following bar spacing

$s = 2(100)/0.835 = 240$ mm; select $s = 225$ mm (9.17 in.)

*Minimum transverse reinforcement (CSA-A23.3-04, Section 11.2.8.2)*—

The minimum transverse reinforcement is established with the maximum web thickness at the top of the web, $b_w$ = 534 mm (21.0 in.)

$$\frac{\min A_v}{s} = 0.06\sqrt{f_c'}\frac{b_w}{f_y} = 0.06\sqrt{48.0}\left(\frac{534}{420}\right)$$

$$= 0.528 \text{ mm}^2\text{/mm } (0.0212 \text{ in.}^2\text{/in.})$$

$$< 0.835 \text{ mm}^2\text{/mm } (0.0338 \text{ in.}^2\text{/in.})$$

For $T_f > 0.25T_{cr}$, max $s = 0.35d_v = 0.35(914) = 320$ mm (12.6 in.) > 225 mm. (9.17 in.) OK

6. Calculate the longitudinal reinforcement.

*Design of longitudinal reinforcement for torsion (CSA-A23.3-04, Section 11.3.10.6 and 11.3.9.2)*—

Longitudinal reinforcement on the tension side of the girder shall have dimensions such that its factored resistance shall not be less than that given by the following expression. In the absence of an axial load normal to the cross section, the required longitudinal force due to torsion, shear, and flexure is

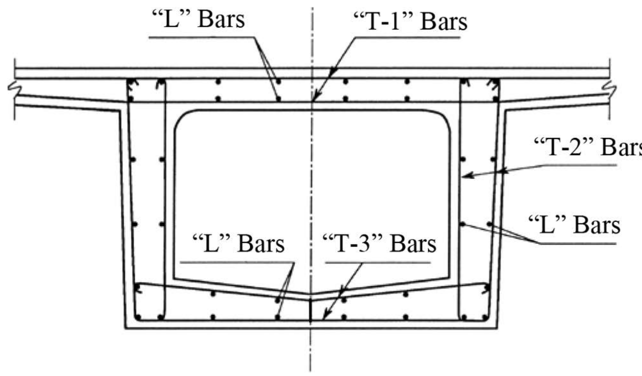



*Fig. 9.5.5—Reinforcement locations for box girder under combined torsion, shear, and flexure.*

$$F_{lt} = \frac{M_f}{d_v} + \cot\theta\sqrt{(V_f - 0.5V_s - V_p)^2 + \left(\frac{0.45p_hT_f}{2A_o}\right)^2}$$

$$= \frac{9112\times10^6}{914} + \cot 35.0°\sqrt{\left[\left(1054 - \frac{405}{2} - 284\right)(10^3)\right]^2 + \left(\frac{0.45\times5801\times705\times10^6}{2\times1.74\times10^6}\right)^2}$$

$$= 9969 + 1108 = 11{,}077 \text{ kN (2503 kip)}$$

Factored tension resistance provided by the prestressing reinforcement with $f_{pr} = 0.96f_{pu} = 1786$ MPa (259 ksi) (CSA-A23.3-04, Section 18.6.2)

$$F_{ltr} = \phi_p A_p f_{pr} = 0.9\times6336\times1786\times10^{-3}$$

$$= 10{,}184 \text{ kN (2290 kips)}$$

The non-prestressed reinforcement required in tension zone is calculated by

$$A_s = \frac{(11{,}077 - 10{,}184)(10^3)}{0.85\times420} = 2501 \text{ mm}^2 \text{ (4.26 in.}^2\text{)}$$

These calculations indicate that fourteen 15M bars are required, providing $A_s = 2800$ mm$^2$. This reinforcement is to be distributed in the tension zone.

Because the compression due to moment is much greater than the tension due to torsion and shear, there is no need to check the tension reinforcement required in the compression zone.

**9.5.5** *Comparison of design solutions according to ACI 318, CSA-A23.3-04, and EC2-04*—The required longitudinal and transverse reinforcement calculated using ACI 318, CSA-A23.3-04, and EC2-04 is shown in Table 9.5.5. The longitudinal reinforcement ("L" Bars) given is the total amount required for the entire cross section. The required transverse reinforcement is given for the top wall ("T-1" Bars), side walls ("T-2" Bars), and bottom walls ("T-3" Bars). The transverse reinforcement in the top wall should be added to the flexural reinforcement required in the top wall acting as a transverse continuous slab.

*Arrangement of reinforcing bars*—

**Table 9.5.5—Summary of design solution of Example 2 using design codes ACI 318-11, EC2-04, and CSA-A23.3-04**

| Reinforcing bars | Design code | | |
|---|---|---|---|
| | ACI 318-11 | EC2-04 (45, 35, and 22 degrees) | CSA-A23.3-04 |
| “L” bars | 5808 mm$^2$ (9.2 in.$^2$) | 4204 mm$^2$ (6.61 in.$^2$) | 2503 mm$^2$ (4.26 in.$^2$) |
| “T-1” and “T-3” bars (torsion) | 0.414 mm$^2$/mm (0.0166 in.$^2$/in.) | 2.18, 1.53, 0.88 mm$^2$/mm (0.087, 0.061, 0.035 in.$^2$/in.) | 0.40 mm$^2$/mm (0.0160 in.$^2$/in.) |
| “T-2” bars (torsion + shear) 1.44 mm$^2$/mm (0.0575 in.$^2$/in.) | | 2.18, 1.53, 0.88 mm$^2$/mm (0.087, 0.061, 0.035 in.$^2$/in.) | 0.835 mm$^2$/mm (0.0338 in.$^2$/in.) |

Arrangement of the reinforcing bars for torsion and shear is shown in Fig. 9.5.5 and summarized in Table 9.5.5.

The general location of longitudinal and transverse reinforcement in the section, which is shown in Fig. 9.5.5, is based on all three design codes.