E702.2

Designing Concrete Structures:

Interaction Diagrams for Concrete Columns

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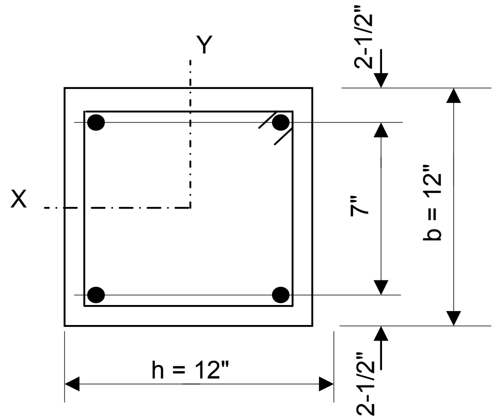
Problem Statement: Draw a interaction diagram for a 12" x 12" non- slender tied(non-spiral) column reinforced w/ 4 - # 8 bars bending around it's x-axis.



f'_c = 4000 psi

f_y = 60000 psi

Notes		ACI-318 05 Reference
Calculate ΦP_{nmax}	Due to the fact concrete structures placed monolithically are continuous, a minimum eccentricity or minimum moment is assummed in this calculation, the code reduces the maximum axial load by 20% to account for this minimum moment.	**Section R10.3.6 & R10.3.7**
	$\phi P_n = .8\phi\left[.85 f'_c \left(A_g - A_{st}\right) + A_{st} f_y\right]$ **(Eq. 1-1)**	**Section 10.3.6.2 Eq (10-2)**
	$A_g = b \times h = 12'' \times 12''$	
	A_g = 144 in^2	
	(Area of 1 - #8 bar = .79 in^2)	
	A_{st} = (4 * .79) = 3.16 in^2	
Find Φ	Φ = 0.65	**Section 9.3.2.2**
	$\phi P_n = .8(.65)\left[.85(4ksi)\left(144in^2 - 3.16in^2\right) + 3.16in^2(60ksi)\right]$	
	$\boxed{\Phi P_{nmax} = 347.60 \text{ kips}}$	
Calculate points on curve	It is possible to derive a group of equations to evaluate the strength of columns subjected to combined bending and axial loads. These equations are tedious to use, therefore interaction diagrams for columns are generally computed by assuming a series of strain distributions. These strain distributions correspond fo a particular point on the interaction diagram, P and M. Each steel strain is selected by multiplying an arbitrary "Z" factor and the yield strain of your steel. $\varepsilon_{s1} = Z \times \varepsilon_y$ **(Eq. 1-2)** The "Z" factors can range from 1 to -1000 and increments between "Z" depend on the required detail of diagram. The smaller the increment the more detailed the diagram will be.	

Notes		ACI-318 05 Reference

With the wide range of possible "Z" factors, every designer must understand there are four mandatory points that must be calculated for each interaction diagram.These four points "Z" factors are 0, -0.5,-1.0,,-2.5, the importance of each point will now be discussed.

Z = 0 (ε_{s1} = 0) - Strain ε_t = 0 in extreme layer in tension. This point marks the change from compression lap splice being allowed on all longitudinal bars to a tension lap splice.

Z = -0.5 ($f_{s1} = -0.5f_y$, $\varepsilon_{s1} = -0.5\varepsilon_y$) This strain distribution affects the length of tension lap splice in a column & is customarily plotted on an interaction diagram.

Z = -1.0 ($f_{s1} = -f_y$, $\varepsilon_{s1} = -\varepsilon_y$) This is the point of balanced failure. This strain distribution marks the change from compression failures originating by crushing of the compression surface of the section to tension failures initiated by yield of the longitudinal reinforcement.

- Also marks beginning of transition zone for Φ for columns in which Φ increases from 0.65 or 0.70 up to 0.90

Z = -2.5 (-2.417 if ε_y = .00207) This point corresponds to the tension controlled strain limit of 0.005.

For this example points will be calculated in the compression controlled zone, (one with the column entirely in compression) (Z = 0.9, -0.5), the tension controlled zone (Z = -5.0) and the transition zone (Z = -1.1).

Notes		ACI-318 05 Reference
Compression Controlled zone	$\varepsilon_{s1} \geq \varepsilon_y$	Section 10.3.3
	Strain in reinforcement and concrete shall be assumed directly proportional to the distance from the neutral axis "c"	Section 10.2.2

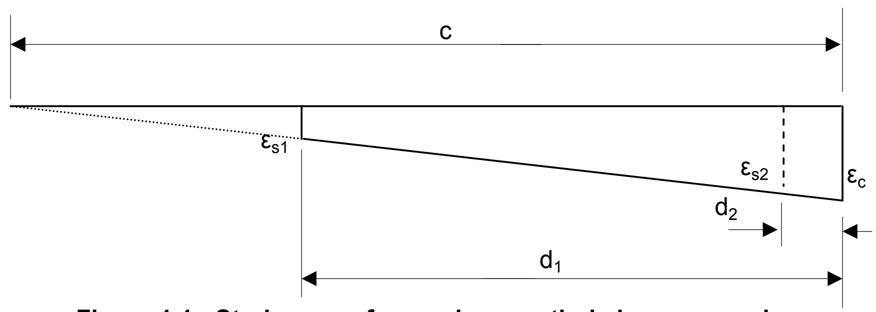


Figure 1.1 - Strain curve for a column entirely in compression

Given: d_1 = 9.50 in E_s = 29000 ksi
d_2 = 2.50 in A_{s1} = 1.58 in^2
A_{s2} = 1.58 in^2

Calculate ΦP_n & ΦM_n for point in compression contolled zone & column entirely in compression

Z = .9

$\varepsilon_c =$ 0.003 — **Section 10.2.3**

$$f_y = E_s \varepsilon_y \therefore \varepsilon_y = \frac{f_y}{E_s} = \frac{60ksi}{29000ksi}$$ **(Eq. 1-3)** — **Section 10.2.4**

$\varepsilon_y =$ 0.002069

Calculate ε_{s1} Strain in 1st row of steel

Using Equation 1-2, from previous page calculate ε_{s1}

$$\varepsilon_{s1} = .9(.002069)$$

$\boxed{\varepsilon_{s1} = 0.001862}$

As shown in Figure 1.1: "c" (distance from extreme compression fiber to neutral axis) can be calculated using similar triangles.

Calculate C

$$c = .003\left(\frac{d_1}{.003 - \varepsilon_{s1}}\right) = .003\left(\frac{9.5''}{.003 - .001862}\right)$$ **(Eq. 1-4)**

$\boxed{c = 25.05 \text{ in}}$

Calculate "a" (equivalent stress block)

$a = \beta_1 C$ **(Eq. 1-5)** $a \leq h$ "a' must be less than depth of column — **Section 10.2.7.1**

For f'c ≤ 4000 psi $\beta_1 = .85$ — **Section 10.2.7.3**
For f'c ≥ 4000 psi $\beta_1 = .85 - .05(f'_c - 4000)$ **(Eq. 1-6)**
$\beta_1 \geq .65$

$\beta_1 =$ 0.85 $a = .85(25.05'') = 21.29'' \geq 12''$

$\boxed{a = 12.000 \text{ in}}$

Notes		ACI-318 05 Reference

Calculate ε_{s2} Strain in 2nd row of steel

As shown in Figure 1.1: ε_{s2} can be calculated using similar triangles.

$$\varepsilon_{s2} = .003\left(\frac{C-d_2}{C}\right) = .003\left(\frac{25.05"-2.5"}{25.05"}\right) \quad \textbf{(eq. 1-7)}$$

$\boxed{\varepsilon_{s2} = \;\; 0.002701}$

Calculate stress in each row of steel (f_{s1} & f_{s2})

$$f_{sx} = \varepsilon_{sx} E_s \qquad f_{sx} \le f_y \qquad \textbf{(Eq. 1-8)}$$

Section 10.2.4

$$f_{s1} = .001862(29000ksi) \qquad f_{s2} = .002701(29000ksi) = 78.33 \ge 60$$

$\boxed{f_{s1} = \;\text{54 ksi} \qquad f_{s2} = \;\text{60 ksi}}$

Calculate force in each row of steel

Tension Steel: $F_x = f_{sx} A_{sx}$ **(Eq. 1-9)**

Compression Steel: $F_x = A_{sx}\left(f_{sx} - .85 f'_c\right)$ **(Eq. 1-10)**

(must subtract concrete stress when in compression since concrete will be replaced by steel)

$$F_1 = 1.58in^2\left(54ksi - .85(4ksi)\right) \qquad F_2 = 1.58in^2\left(60ksi - .85(4ksi)\right)$$

$\boxed{F_1 = \;\text{79.95 kips} \qquad F_2 = \;\text{89.43 kips}}$

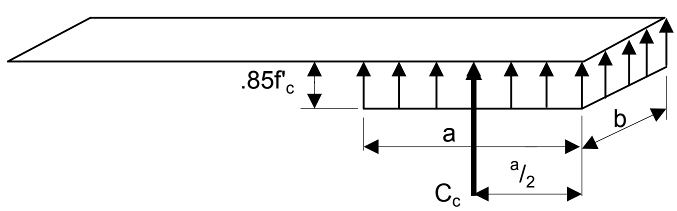


Figure 1.2 - Equivalent rectangular stress block diagram

Section 10.2.6
Section 10.2.7

An average stress of .85 f'_c is assumed uniformily distributed over an equivalent compression zone.

Section 10.2.7.1

Caclulate C_c (Concrete Compression Force)

Using Figure 1.2 - Caclulate C_c

$$C_c = .85 f'_c ab = .85(4ksi)(12")(12") \qquad \textbf{(Eq. 1-11)}$$

$\boxed{C_c = \;\text{489.60 kips}}$

Notes		ACI-318 05 Reference

Calculate P_n & M_n by applying forces to free body diagram

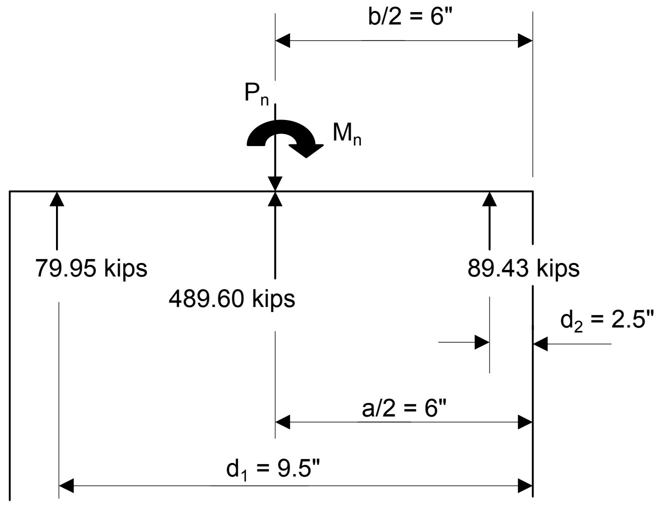


Figure 1.3: Column free body diagram for a "Z" of .9

Using Figure 1.3, Calculate P_n by summing vertical forces:

$$\Sigma F_v = 0 = 79.95kips + 489.60kips + 89.43kips - P_n$$

$\boxed{P_n = \text{ 658.98 kips}}$

Using Figure 1.3 from previous page, calculate M_n by summing moments about steel in line d_1: (counterclockwise being positive moment)

$$\Sigma M = 0 = \frac{489.6kips(3.5'')}{12''/1'} + \frac{89.43kips(7'')}{12''/1'} - \frac{658.98kips(3.5'')}{12''/1'} - M_n$$

Moment arms will be in inches, must convert to feet for desired Units.

$\boxed{M_n = \text{ 2.82 kip-ft}}$

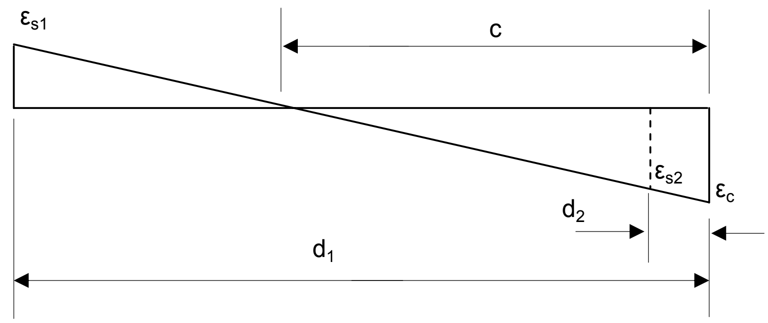
Calculate ΦP_n & ΦM_n

$\phi = .65$ — **Section 9.3.2.2**

$\phi P_n = (.65)658.98$ $\qquad$ $\phi M_n = (.65)2.82$

$\phi P_n =$ 428.33 kips
$\phi M_n =$ 1.83 kip-ft

<u>Point on curve for "Z" = .9</u>

Notes		ACI-318 05 Reference
		
	Figure 1.4 - Strain curve for a column in compression & tension	
Calculate ΦP_n & ΦM_n (column in compression & tension)	Given: $d_1 = 9.50$ in, $d_2 = 2.50$ in, $E_s = 29000$ ksi, $A_{s1} = 1.58$ in^2, $A_{s2} = 1.58$ in^2	
	Z = -.5	
	$\varepsilon_c = 0.003$	
Calculate ε_{s1} Strain in 1st row of steel	$\varepsilon_y = 0.002069$	
	$\varepsilon_{s1} = -.5(.002069)$ $\boxed{\varepsilon_{s1} = -0.001034}$	**(Eq. 1-2)**
Calculate C	$c = .003\left(\frac{9.5''}{.003-(-.001034)}\right)$	**(Eq. 1-4)**
	$\boxed{c = 7.06 \text{ in}}$	
	$\beta_1 = 0.85$	
Calculate "a"	$a = .85(7.06'') = 6.00'' \leq 12''$	**(Eq. 1-5)**
	$\boxed{a = 6.004 \text{ in}}$	
Calculate ε_{s2} Strain in 2nd row of steel	$\varepsilon_{s2} = .003\left(\frac{7.06''-2.5''}{7.06''}\right)$	**(eq. 1-7)**
	$\boxed{\varepsilon_{s2} = 0.001938}$	
Calculate stress in each row of steel (f_{s1} & f_{s2})	$f_{s1} = -.001034(29000ksi)$ $f_{s2} = .001938(29000ksi)$	**(Eq. 1-8)**
	$\boxed{f_{s1} = -30.00 \text{ ksi} \quad f_{s2} = 56.21 \text{ kips}}$	
Calculate force in each row of steel	$F_1 = 1.58in^2(-30ksi)$	**(Eq. 1-9)**
	$F_2 = 1.58in^2(56.21ksi - .85(4ksi))$	**(Eq. 1-10)**
	$\boxed{F_1 = -47.40 \text{ kips} \quad F_2 = 83.44 \text{ kips}}$	

Notes		ACI-318 05 Reference
Caclulate C_c	$C_c = .85(4ksi)(6.004")(12")$ **(Eq. 1-11)**	**Section 10.2.6**
	$C_c =$ 244.98 kips	

Calculate P_n & M_n by applying forces to free body diagram

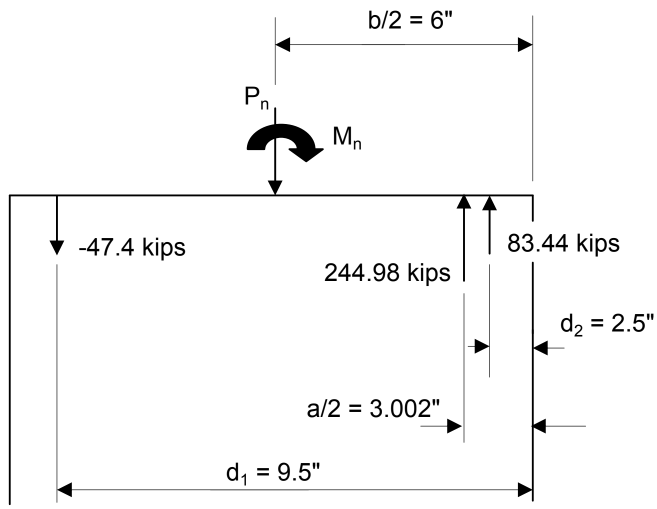


Figure 1.5: Column free body diagram for a "Z" of -.5

Tensile strength of concrete shall be neglected in axial and flexural calculations of reinforced concrete. **Section 10.2.5**

Using Figure 1.5 , calculate P_n by summing vertical forces:

$$\Sigma F_v = 0 = -47.4kips + 244.98kips + 83.44kips - P_n$$

$P_n =$ 281.02 kips

Using Figure 1.5, calculate M_n by summing moments about steel in line d_1:
(counterclockwise being positive moment)

$$\Sigma M = 0 = \frac{244.98kips(6.5")}{12"/1'} + \frac{83.44kips(7")}{12"/1'} - \frac{281.02kips(3.5")}{12"/1'} - M_n$$

$M_n =$ 99.41 kip-ft

Calculate ΦP_n & ΦM_n

$\phi = .65$

$\phi P_n = (.65)281.02$ $\phi M_n = (.65)99.41$

$\phi P_n =$ 182.67 kips
$\phi M_n =$ 64.62 kip-ft

Point on curve for "Z" = -.5

Notes		ACI-318 05 Reference
Transition Zone	$.002 \le \varepsilon_{s1} \le .005$	
Calculate ΦP_n & ΦM_n (column in compression & tension)	Given: d_1 = 9.50 in; d_2 = 2.50 in; E_s = 29000 ksi; A_{s1} = 1.58 in^2; A_{s2} = 1.58 in^2	
	$\varepsilon_c =$ 0.003	
Calculate ε_{s1} Strain in 1st row of steel	$\varepsilon_y =$ 0.002069 $\varepsilon_{s1} = -1.1(.002069)$ $\boxed{\varepsilon_{s1} = \text{5.401961}}$	
Calculate C	$c = .003\left(\dfrac{9.5''}{.003-(-.002276)}\right)$ $\boxed{c = \text{5.40 in}}$	(Eq. 1-4)
Calculate "a" (equivalent stress block)	$\beta_1 =$ 0.85 $a = .85(5.4'') = 4.59'' \le 12''$ $\boxed{a = \text{4.590 in}}$	(Eq. 1-5)
Calculate ε_{s2} Strain in 2nd row of steel	$\varepsilon_{s2} = .003\left(\dfrac{5.40''-2.5''}{5.40''}\right)$ $\boxed{\varepsilon_{s2} = \text{0.001612}}$	(eq. 1-7)
Calculate stress in each row of steel (f_{s1} & f_{s2})	$f_{s1} = -.002276(29000ksi) = -66 \ge 60ksi$ $f_{s2} = .001612(29000ksi)$ $\boxed{f_{s1} = \text{-60.00 ksi} \quad f_{s2} = \text{46.75 kips}}$	(Eq. 1-8) (Eq. 1-8)
Calculate force in each row of steel	$F_1 = 1.58in^2(-60ksi)$ $F_2 = 1.58in^2(46.75ksi - .85(4ksi))$ $\boxed{F_1 = \text{-94.80 kips} \quad F_2 = \text{68.49 kips}}$	(Eq. 1-9) (Eq. 1-10)
Caclulate C_c	$C_c = .85(4ksi)(4.59'')(12'')$ $\boxed{C_c = \text{187.27 kips}}$	(Eq. 1-11)

Notes		ACI-318 05 Reference

Calculate P_n & M_n by applying forces to free body diagram

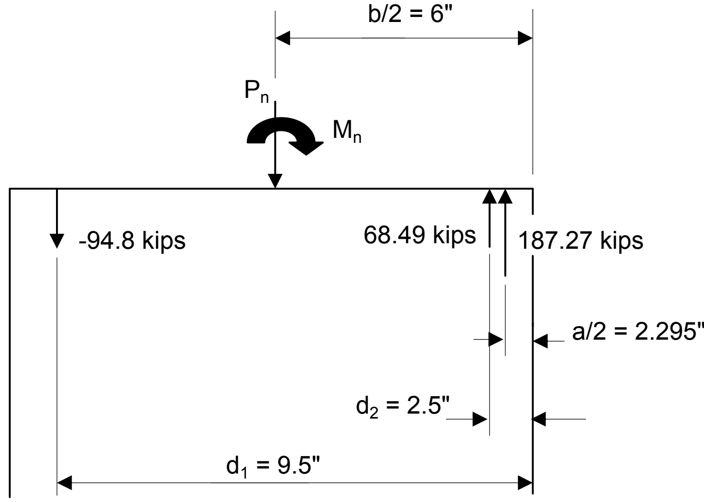


Figure 1.6: Column free body diagram for a "Z" of -1.1

Using Figure 1.6, calculate P_n by summing vertical forces:

$$\Sigma F_v = 0 = -94.8kips + 68.49kips + 187.27kips - P_n$$

$\boxed{P_n = \text{160.96 kips}}$

Using Figure 1.6 calculate M_n by summing moments about steel in line d_1:
(counterclockwise being positive moment)

$$\Sigma M = 0 = \frac{187.27kips(7.205'')}{12''/1'} + \frac{68.49kips(7'')}{12''/1'} - \frac{160.96kips(3.5'')}{12''/1'} - M_n$$

$\boxed{M_n = \text{105.45 kip-ft}}$

Calculate ΦP_n & ΦM_n

Figure R9.3.2

$$\phi = .65 + (\varepsilon_{s1} - .002)\left(\frac{250}{3}\right) = .65 + (.002276 - .002)\left(\frac{250}{3}\right) = .673$$

$$\phi P_n = (.673)160.96 \qquad \phi M_n = (.673)105.45$$

$\phi P_n =$ 108.33 kips
$\phi M_n =$ 70.97 kip-ft

<u>Point on curve for "Z" = -1.1</u>

Notes		ACI-318 05 Reference
Tension Controlled Zone	$\varepsilon_{s1} \geq .005$	
Calculate ΦP_n & ΦM_n for point in Tension controlled zone & column in compression & tension	Given: d_1 = 9.50 in, d_2 = 2.50 in, E_s = 29000 ksi, A_{s1} = 1.58 in^2, A_{s2} = 1.58 in^2	
	Z = -5	
	$\varepsilon_c =$ 0.003	
Calculate ε_{s1} Strain in 1st row of steel	$\varepsilon_y =$ 0.002069	
	$\varepsilon_{s1} = -5(.002069)$ $\varepsilon_{s1} =$ -0.010345	**(Eq. 1-2)**
Calculate C	$c = .003\left(\dfrac{9.5''}{.003-(-.010345)}\right)$	**(Eq. 1-4)**
	$c =$ 2.14 in	
	$\beta_1 =$ 0.85	
Calculate "a"	$a = .85(2.14'') = 1.819'' \leq 12''$	**(Eq. 1-5)**
	$a =$ 1.819 in	
Calculate ε_{s2} Strain in 2nd row of steel	$\varepsilon_{s2} = .003\left(\dfrac{2.14''-2.5''}{2.14''}\right)$	**(Eq. 1-7)**
	$\varepsilon_{s2} =$ -0.000505	
Calculate stress in each row of steel (f_{s1} & f_{s2})	$f_{s1} = -.010345(29000ksi) = -300 \geq 60ksi$	**(Eq. 1-8)**
	$f_{s2} = -.000505(29000ksi)$	**(Eq. 1-8)**
	$f_{s1} =$ -60.00 ksi $f_{s2} =$ -14.65 kips	
Calculate force in each row of steel	$F_1 = 1.58in^2(-60ksi)$ $F_2 = 1.58in^2(-14.65ksi)$	**(Eq. 1-9)**
	$F_1 =$ -94.80 kips $F_2 =$ -23.15 kips	
Caclulate C_c	$C_c = .85(4ksi)(1.819'')(12'')$	**(Eq. 1-11)**
	$C_c =$ 74.22 kips	

Notes		ACI-318 05 Reference

Calculate P_n & M_n by applying forces to free body diagram

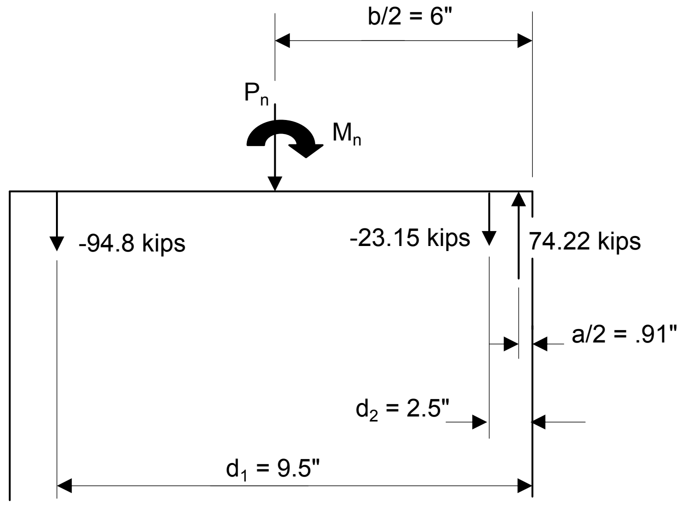


Figure 1.7: Column free body diagram for a "Z" of -5

Using Figure 1.7, calculate P_n by summing vertical forces:

$$\Sigma F_v = 0 = -94.8kips - 23.15kips + 74.22kips - P_n$$

$\boxed{P_n = \text{-43.73 kips}}$

Using Figure 1.7 calculate M_n by summing moments about steel in line d_1:
(counterclockwise being positive moment)

$$\Sigma M = 0 = \frac{74.22kips(8.59'')}{12''/1'} - \frac{23.15kips(7'')}{12''/1'} - \frac{-43.73kips(3.5'')}{12''/1'} - M_n$$

$\boxed{M_n = \text{52.38 kip-ft}}$

$\phi = .9$ — **Section 9.3.2.2**

Calculate ΦP_n & ΦM_n

$\phi P_n = (.9)(-43.73)$ $\qquad \phi M_n = (.9)52.38$

$\phi P_n =$ -39.36 kips $\phi M_n =$ 47.14 kip-ft	**<u>Point on curve for "Z" = -5</u>**

Notes		ACI-318 05 Reference

Calculate ΦP_{nt}

When column is entirely in tension the designer shall assume the concrete in the column will not contribute to tension strength, only reinforcement shall resist tension.

Section 10.2.5

$$P_{nt} = -A_{st} f_y$$

(Area of 1 - #8 bar = .79 in^2)

A_{st} = (4 * .79) = 3.16 in^2

$$P_{nt} = 3.16 in^2 (60 ksi)$$

P_{nt} = -189.60 kips

Find Φ

Φ = 0.9

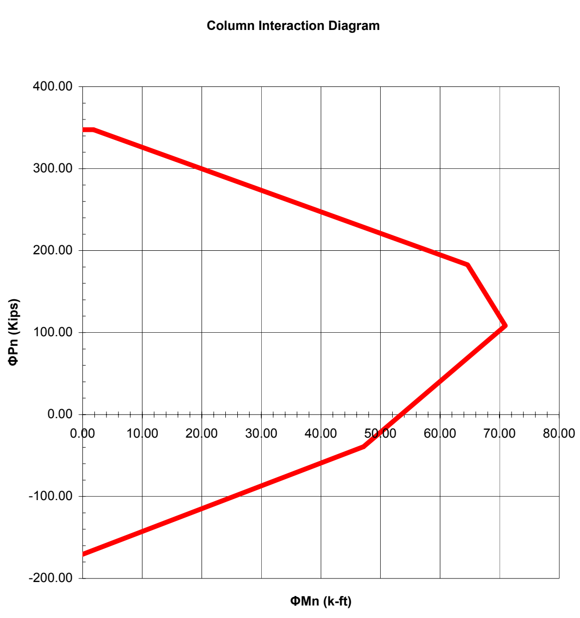
Section 9.3.2.2

$$\phi P_{nt} = .9(-189.6 kips)$$

$\boxed{\mathbf{\Phi P_{nt} = -170.64\ kips}}$

Draw interaction diagram using points calculated.

ε_{s1}	ΦP_n	ΦM_n
0.002069	347.60	0.00
0.0018621	347.60	1.83
-0.001034	182.67	64.62
-0.002276	108.33	70.97
-0.010345	-39.36	47.14
	-170.64	0.00



Notes		ACI-318 05 Reference
	Finally an actual iteraction diagram will be shown with using more values of "Z", to get a more detailed curve.