Assessment of crack spacing and crack width formulations in RC elements externally strengthened with FRP materials

Cristina Barris, Francesca Ceroni & Alejandro Pérez Caldentey

AMADE ANALYSIS AND ADVANCED MATERIALS FOR STRUCTURAL DESIGN | Universitat de Girona
UNIVERSITÀ DEGLI STUDI DI NAPOLI PARTHENOPE
POLITÉCNICA

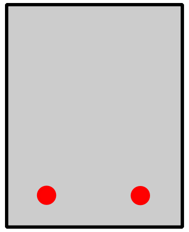
NEX FYFE ACMA MST BAR SIMPSON Strong-Tie MCTI TRANSPORTATION INNOVATION Missouri S&T MODJESKI and MASTERS IIFC International Institute for FRP in Construction

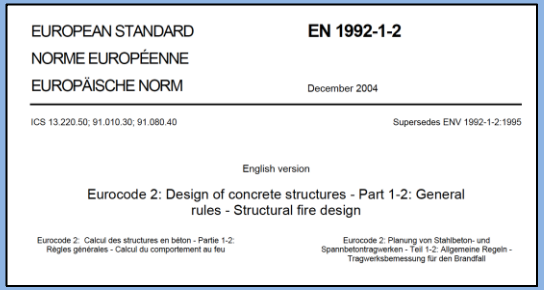
THE WORLD'S GATHERING PLACE FOR ADVANCING CONCRETE

aci CONCRETE CONVENTION

OUTLINE

❑ Introduction & Review on theoretical models on cracking





EUROPEAN STANDARD
NORME EUROPÉENNE
EUROPÄISCHE NORM

EN 1992-1-2

December 2004

ICS 13.220.50; 91.010.30; 91.080.40

Supersedes ENV 1992-1-2:1995

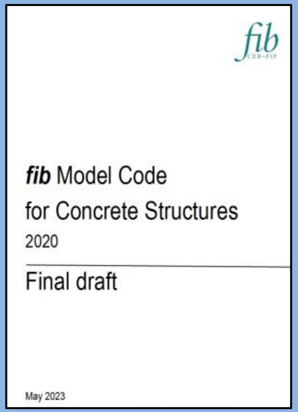
English version

Eurocode 2: Design of concrete structures - Part 1-2: General rules - Structural fire design

Eurocode 2: Calcul des structures en béton - Partie 1-2: Règles générales - Calcul du comportement au feu

Eurocode 2: Planung von Stahlbeton- und Spannbetontragwerken - Teil 1-2: Allgemeine Regeln - Tragwerksbemessung für den Brandfall

Eurocode 2 (2004)



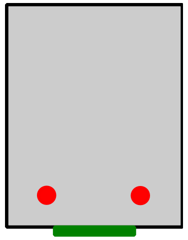
fib

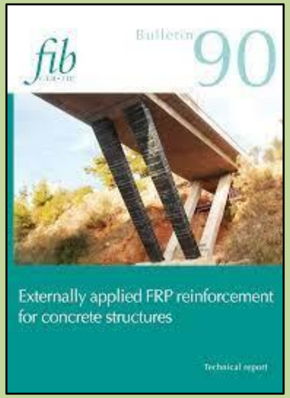
fib Model Code for Concrete Structures 2020

Final draft

May 2023

Model Code 2020 (approved 2023)





fib Bulletin 90 (2019)

❑ Experimental database & comparison with existing formulation

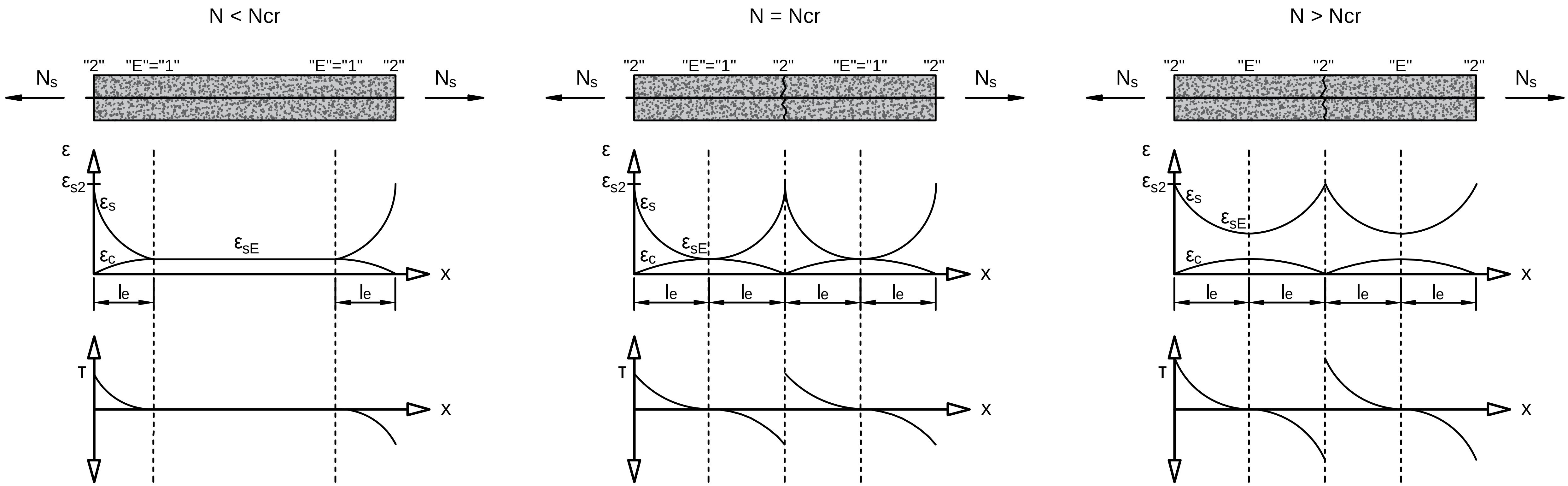
❑ Summary and concluding remarks

❑ Future works

CRACKING PROCESS OF A RC ELEMENT

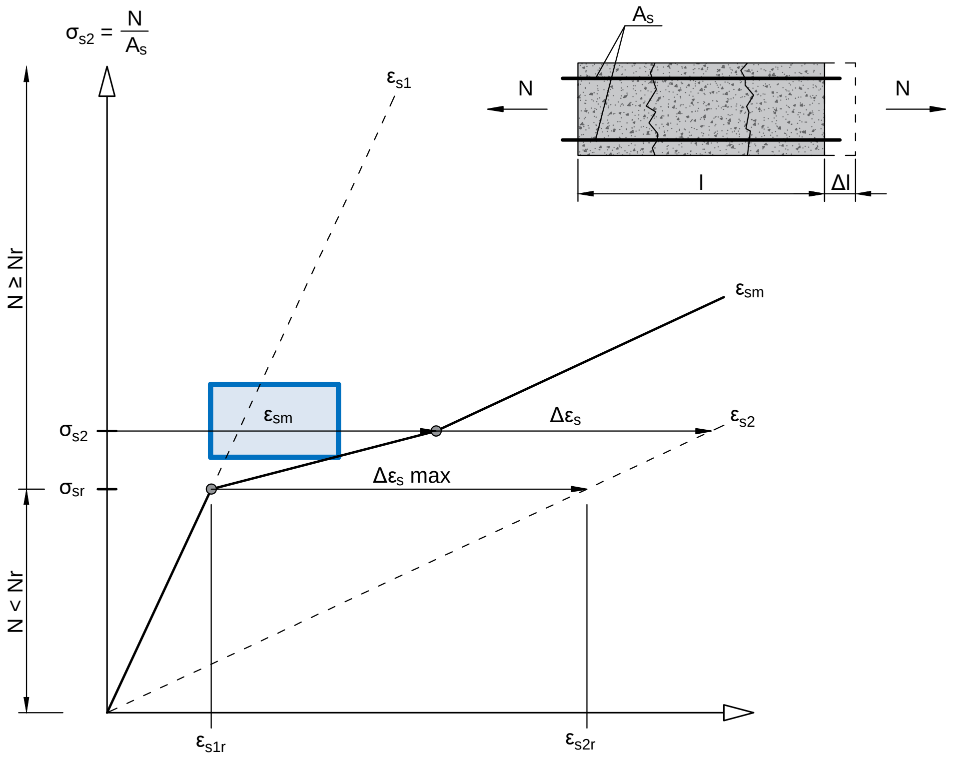
Cracking process of a RC tie without FRP strengthening:

Non-cracked state | Crack formation phase | Stabilised cracking phase



CRACKING PROCESS OF A RC ELEMENT

Tension-stiffening effect:

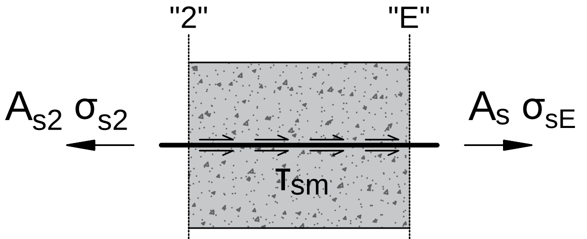


General expression for crack width:

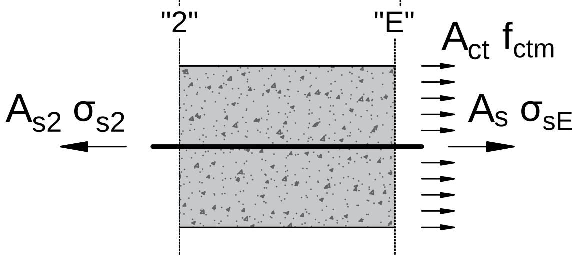
$$w_m = s_{rm} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

Transfer length, l_e, obtained from equilibrium of forces:

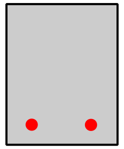
- At the reinforcement along l_e:



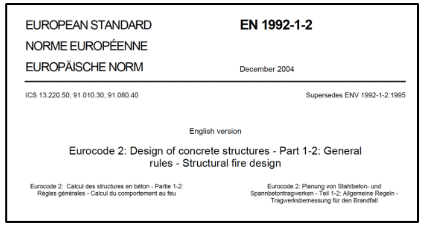
- Between the cracked and no-slip section:

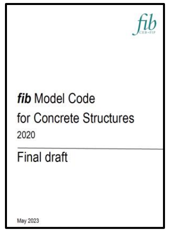


$$l_e = \frac{A_{c,ef} \cdot f_{ctm}}{\tau_{sm} \cdot u_s} = \frac{f_{ctm}}{\tau_{sm} \cdot \rho_{s,ef}} \cdot \frac{A_s}{u_s} = \frac{f_{ctm}}{\tau_{sm} \cdot \rho_{s,ef}} \cdot \frac{\pi\phi^2}{4\pi\phi} = 0.25\,\frac{f_{ctm} \cdot \phi_s}{\tau_{sm} \cdot \rho_{s,ef}}$$



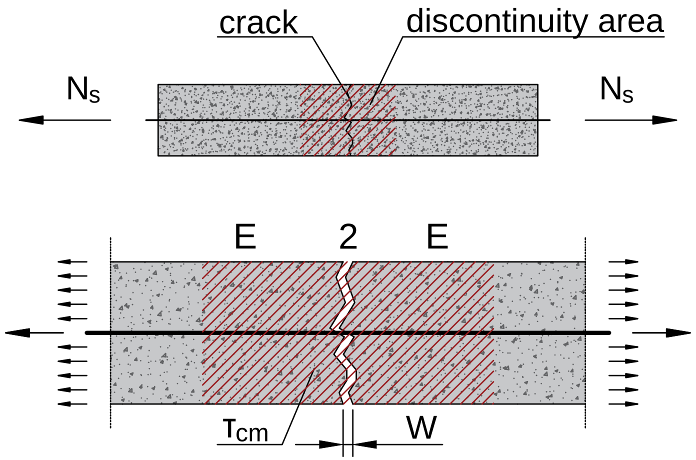
REVIEW ON THEORETICAL MODELS: EUROCODE 2 (2004) & MODEL CODE 2020

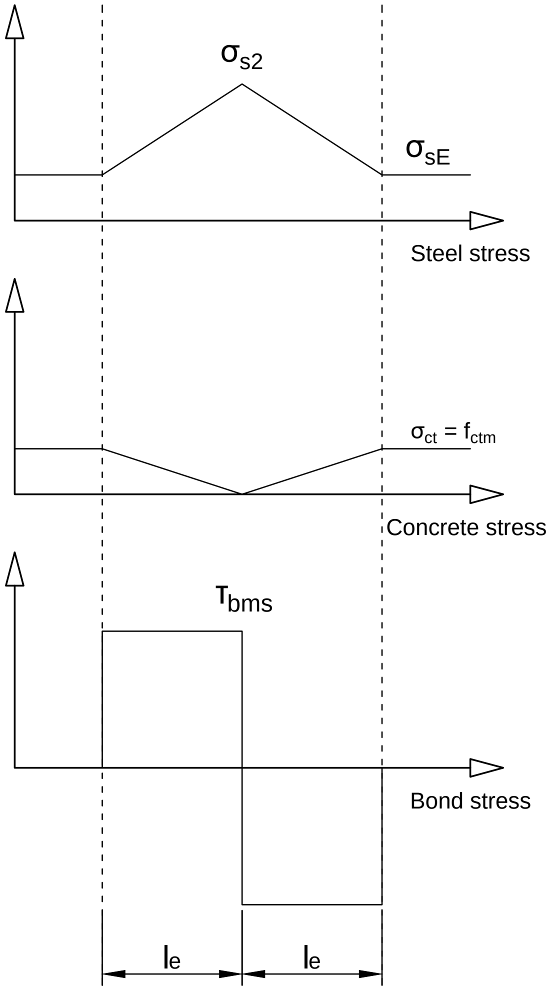


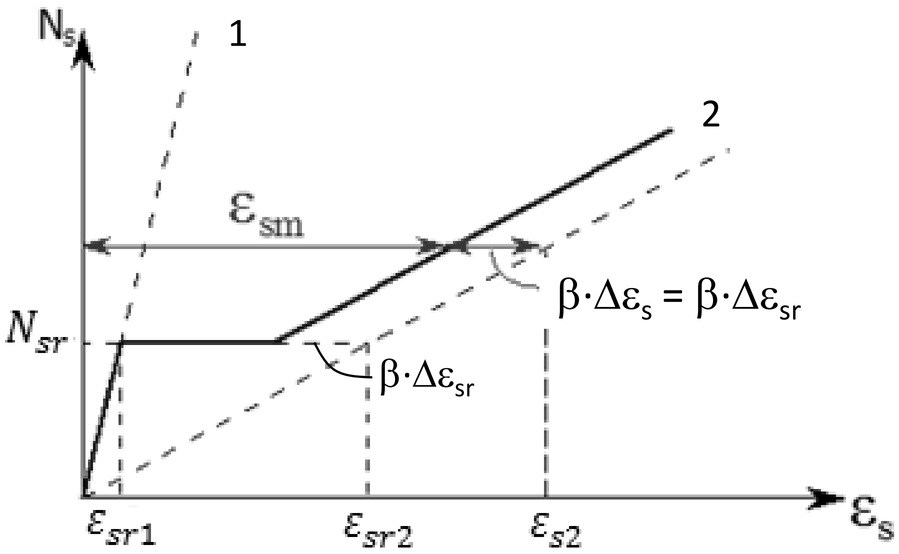


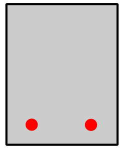
Characteristic crack width: $w_k = k \cdot s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$

Main assumptions: i) equilibrium of forces and constant shear stress contribution along the transfer length, ii) constant tension-stiffening contribution.

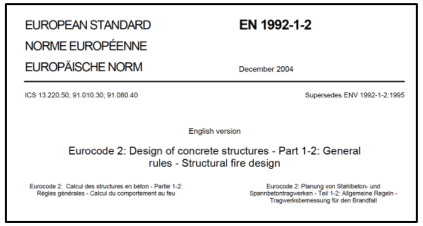








REVIEW ON THEORETICAL MODELS: EUROCODE 2 (2004)


EUROPEAN STANDARD — EN 1992-1-2
NORME EUROPÉENNE
EUROPÄISCHE NORM — December 2004

ICS 13.220.50; 91.010.30; 91.080.40 — Supersedes ENV 1992-1-2:1995

English version

Eurocode 2: Design of concrete structures - Part 1-2: General rules - Structural fire design

Eurocode 2: Calcul des structures en béton - Partie 1-2: Règles générales - Calcul du comportement au feu

Eurocode 2: Planung von Stahlbeton- und Spannbetontragwerken - Teil 1-2: Allgemeine Regeln - Tragwerksbemessung für den Brandfall

Characteristic value of crack width:

$$w_k = s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

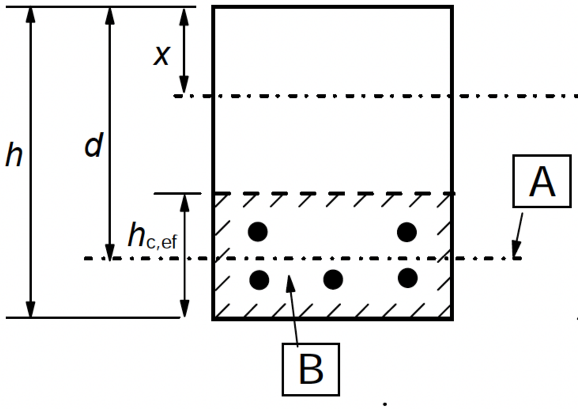
Maximum crack spacing:

$$s_{r,max} = k_3 \cdot c + k_1 \cdot k_2 \cdot k_4 \frac{\phi_s}{\rho_{s,ef}}$$

Cover — Shear stress transfer (constant distribution of shear stresses)

- k_1: bond properties (well stablished for steel reinforcement: 0.8 good bond / 1.6 plain bar)
- k_2: strain distribution (1.0 tensile / 0.5 bending)
- k_3 and k_4: empirically callibrated parameters (3.4 & 0.425, respectively)
- $\rho_{s,ef}$: effective reinforcement ratio, based on an effective area of concrete in tension:

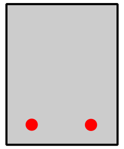
$$A_{c,ef} = b \cdot min(2.5c, \frac{h-x}{3}, 0.5h)$$



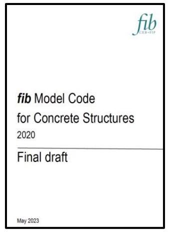
Mean strain difference:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - k_t \cdot \frac{f_{ctm}}{\rho_{s,ef}} (1 + \alpha_s \cdot \rho_{s,ef})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s}$$

- Obtained from equilibrium of forces between cracked and no-slip section
- Assumes constant tension-stiffening contribution



REVIEW ON THEORETICAL MODELS: MODEL CODE 2020



Characteristic value of crack width: $$w_k = k_{1/r} \cdot s_{r,max} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

- Effect of curvature: $k_{1/r} = \frac{h-x}{d-x}$

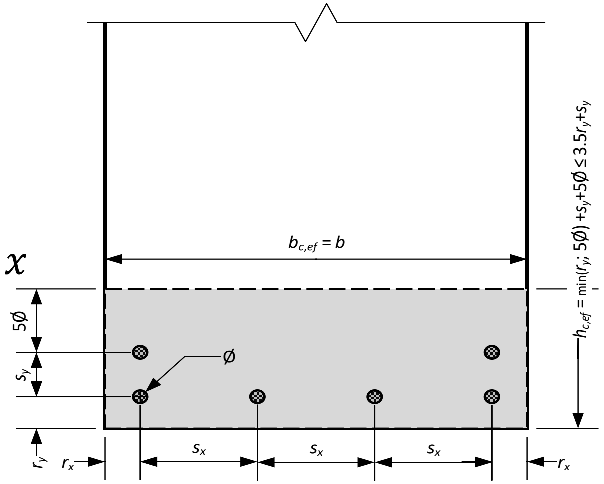
Maximum crack spacing:

Cover | Shear stress transfer (constant distribution of shear stresses)

$$s_{r,max} = \beta \cdot \left(k_c \cdot c + k_{\phi/\rho} \cdot k_{fl} \cdot k_b \cdot \frac{f_{ctm} \cdot \phi_s}{\tau_{bms} \cdot \rho_{s,ef}} \right)$$

- β: conversion factor from mean to characteristic value (1.7 stabilized cracking stage)
- k_c: empirical factor (=1.5); $k_{\phi/\rho}$: influence of bond (=0.25); k_b: casting factor (0.9/1.2 for good/bad conditions)
- k_{fl}: stress distribution before cracking: $k_{fl} = \frac{1}{2}\left(1 + \frac{h - x_g - h_{c,ef}}{h - x_g}\right)$
- $\rho_{s,ef}$: effective reinforcement ratio, based on an effective area of concrete in tension:

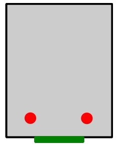
$$A_{c,ef} = b \cdot h_{c,ef};\ h_{c,ef} = min(r_y + 5\phi_s; 10\phi_s; 3.5r_y) + (n_l - 1) \cdot s_y \leq h - x$$



Mean strain difference:

$$\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_s - \beta_{TS} \cdot \sigma_{sr,ef}}{E_s} \geq \frac{\sigma_s}{E_s}(1 - \beta_{TS})$$

$$\sigma_{sr,ef} = \frac{f_{ctm}}{\rho_{s,ef}} \cdot (1 + \alpha_s \cdot \rho_{s,ef})$$



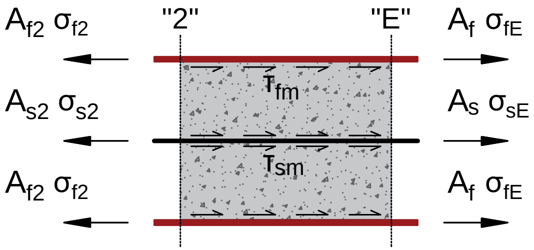
CRACKING PROCESS OF A RC ELEMENT STRENGTHENED WITH FRP

General expression for crack width:

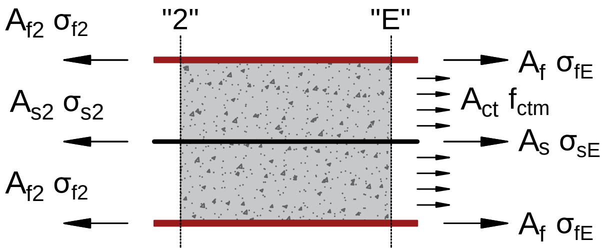
$$w_m = s_{rm} \cdot (\varepsilon_{sm} - \varepsilon_{cm})$$

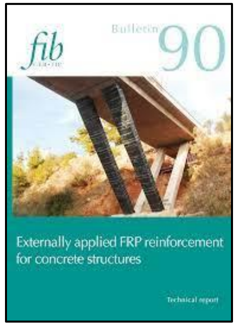
Transfer length, l_e, obtained from equilibrium of forces:

- At both reinforcements along l_e:



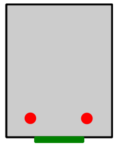
- Between the cracked and no-slip section:



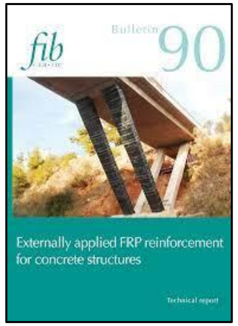


- Model 1: mechanical model based on assuming crack width as 2·slip, known bond-slip laws between reinforcements & concrete (iterative approach).

- Model 2: simplified approach based on empirical adjustment of Eurocode 2 (2004) approach.



REVIEW ON THEORETICAL MODELS: FIB BULLETIN 90 (2019). SIMPLIFIED APPROACH



Characteristic value of crack width: $w_k = s_{r,max} \cdot (\varepsilon_{fm} - \varepsilon_{cm})$

Maximum crack spacing: $$s_{r,max} = 1.6\left(20 + 4 \cdot \frac{A_{c,ef}{}^{0.5} \cdot \emptyset_s}{A_s{}^{0.75} + \left(\frac{A_f \cdot E_f}{E_s}\right)^{0.75}}\right)$$

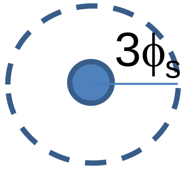
Cover — Shear stress transfer (empirically adjusted)

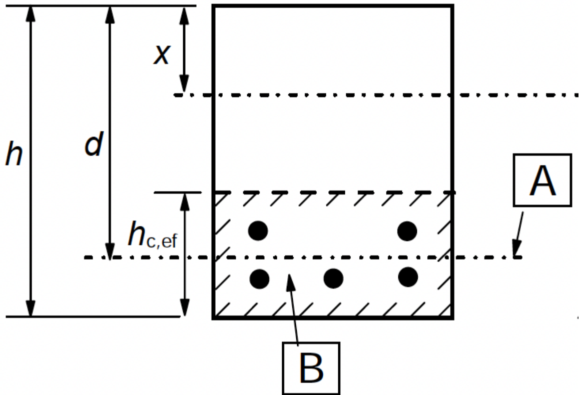
Mean strain difference: $$\varepsilon_{fm} - \varepsilon_{cm} = \varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_{s2} - k_t \cdot \frac{f_{ctm}}{\rho_{ef,eq}}(1 + \alpha_s \cdot \rho_{ef,eq})}{E_s} \geq 0.6\frac{\sigma_{s2}}{E_s}$$

- Equivalent effective ratio: $\rho_{ef,eq} = \frac{A_s}{A_{c,ef}} + \frac{E_f}{E_s}\frac{A_f}{A_{c,ef}}$

$A_{c,ef}$:

- Beams: $A_{c,ef} = b \cdot min(2.5c, \frac{h-x}{3}, 0.5h)$
- Ties: area surrounding $3\phi_s$





EXPERIMENTAL DATABASE. DATA

	Beams	Ties
N. Elements	36 elements: • 19 wet lay-up EBR • 6 pre-cured EBR • 7 pre-cured NSM • 4 NSM rods	31 wet lay-up EBR
Cross-section dimensions	100 x 150 mm ÷ 400 x 250 mm	100 x 100 mm ÷ 200 x 200 mm
Concrete average compressive strength	17 ÷ 45 MPa	30 ÷ 92 MPa
Steel reinforcement ratio	0.54% ÷ 2.87%	0.50% ÷ 2.01%.
FRP arrangement	80÷300 x 0.167 mm sheets 240 x 1.4 mm laminates 10 x 1.4-3.0 mm strips d50 mm rods	100÷150 x 0.10÷0.11 mm sheets
FRP modulus of elasticity	65.6 ÷ 230 GPa.	57 ÷ 267 GPa
Pre-loaded element?	-	9 specimens

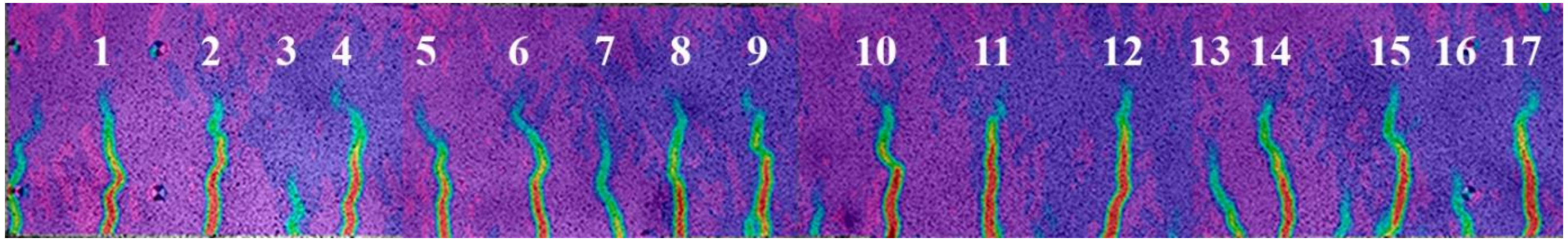
EXPERIMENTAL DATABASE. CRACK MEASUREMENT

General observations:

- The **service load** was defined in all cases at the stage where internal steel reinforcement reached 80% of its yield strength.
- **Shrinkage strain** was not reported in any of the experiments because it was not measured.

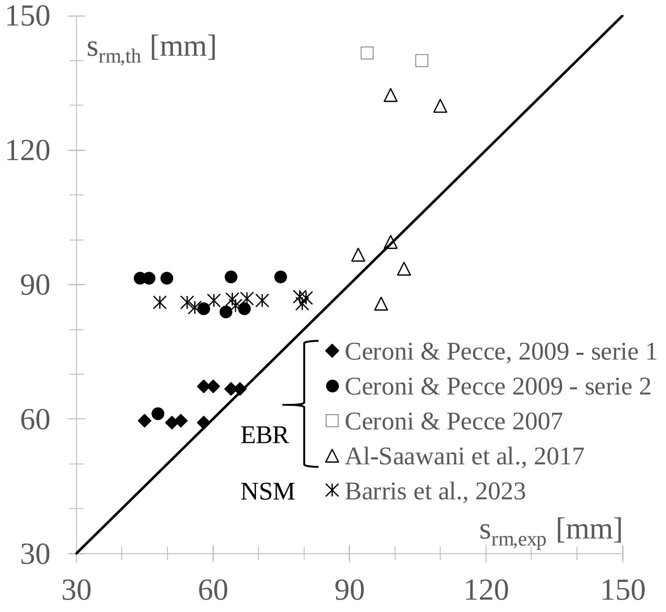
Crack measurement:

- At the level of steel reinforcement and at the lateral side of the beam
- At pure bending zone (Beams) / Along the element (Ties)
- Mean, maximum and minimum value
- Optical means (manual or DIC)

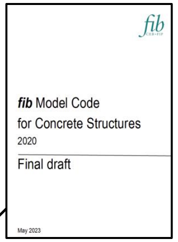


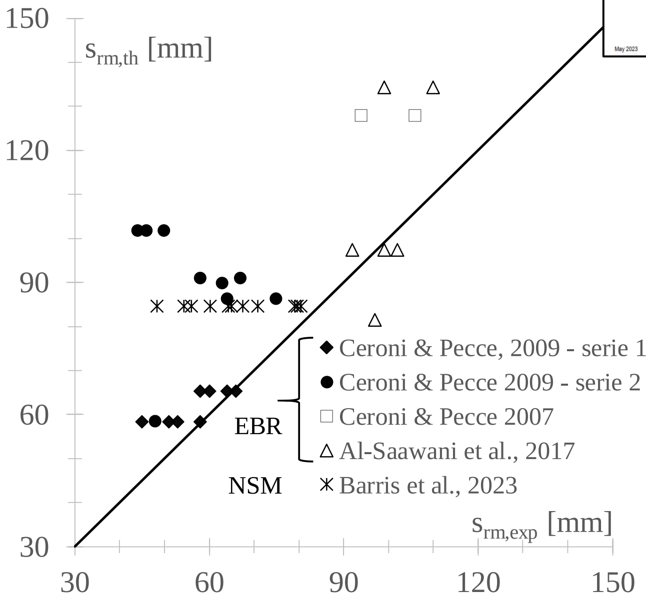
RESULTS AND COMPARISON. CRACK SPACING – BEAMS:

Eurocode 2

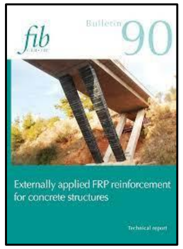


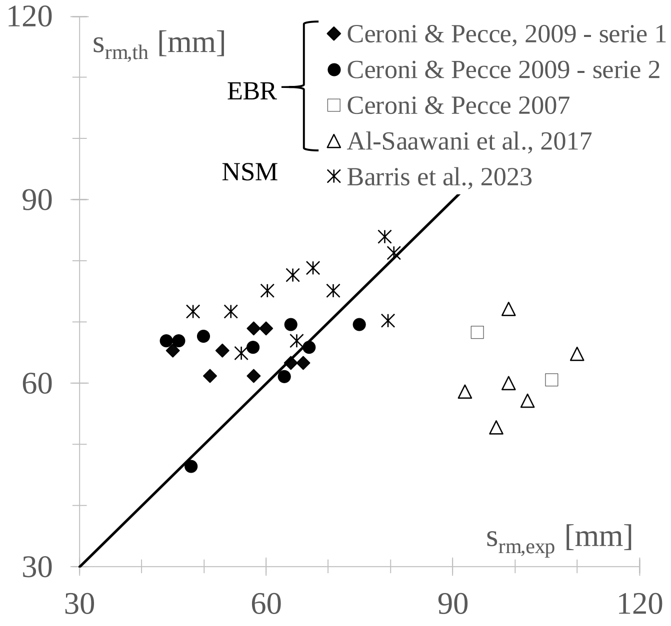
Model Code 2020





fib Bulletin 90





$s_{rm,th}$/ $s_{rm,exp}$	Beams		
	EC2	MC 2020	*fib* B90
Mean	1.30	1.30	1.03
St. dev.	0.28	0.34	0.28
CoV	22%	26%	27%

- EC2 and MC2020 overpredict, by 30%, crack spacing (additional stiffness & shear bond transfer provided by external reinforcement not provided)
- *fib* B90 provides good agreement, both for EBR (calibrated) & NSM.
- High dispersion of results.

POLITÉCNICA UdG

RESULTS AND COMPARISON. CRACK SPACING – TIES:

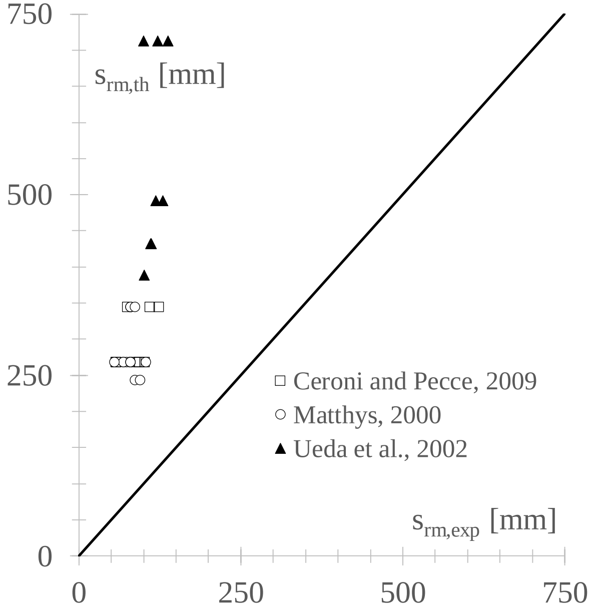
Eurocode 2


English version

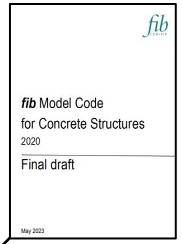
Eurocode 2: Design of concrete structures - Part 1-2: General rules - Structural fire design

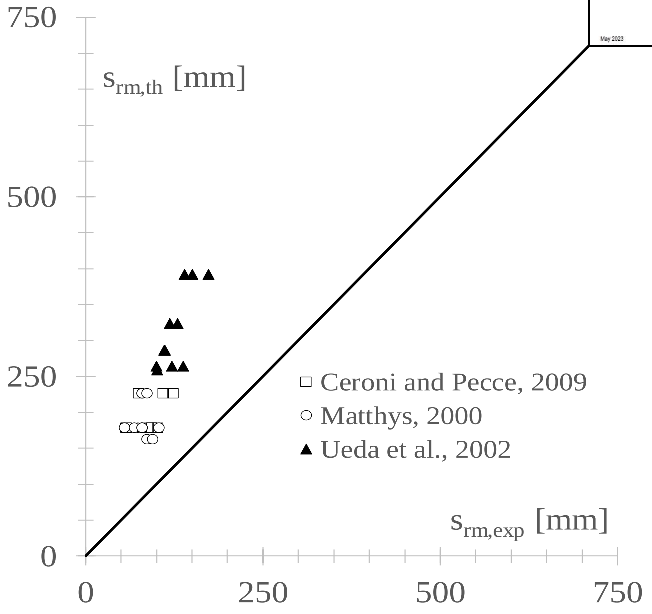
Eurocode 2: Calcul des structures en béton - Partie 1-2: Règles générales - Calcul du comportement au feu

Eurocode 2: Planung von Stahlbeton- und Spannbetontragwerken - Teil 1-2: Allgemeine Regeln - Tragwerksbemessung für den Brandfall

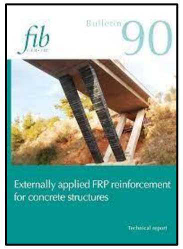


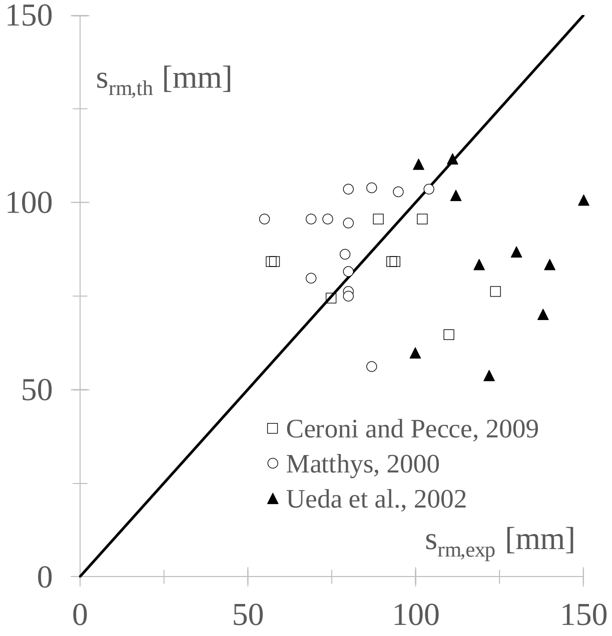
Model Code 2020





fib Bulletin 90





$s_{rm,th}$/ $s_{rm,exp}$	Ties		
	EC2	MC 2020	*fib* B90
Mean	3.97	2.39	0.96
St. dev.	1.07	0.43	0.31
CoV	27%	18%	33%

- Experimental crack spacing is largely overestimated by EC2 & MC2020.
- *fib* B90 provides closer predictions.
- High scatter of results.

RESULTS AND COMPARISON. CRACK WIDTH – BEAMS:

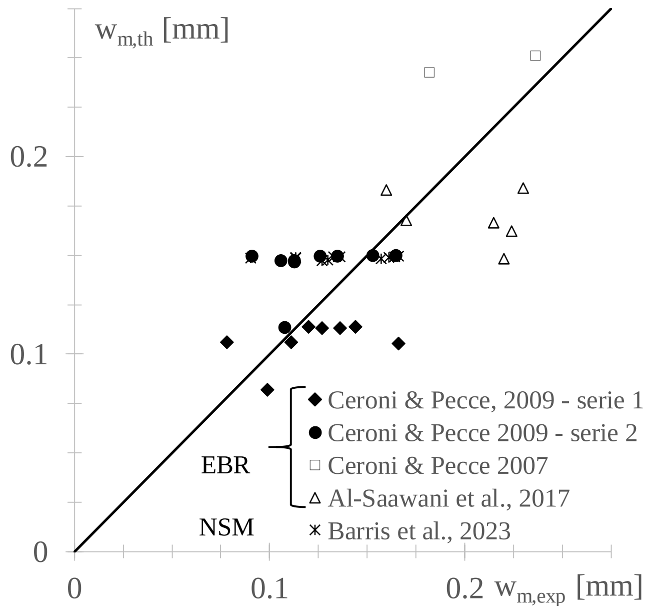

English version

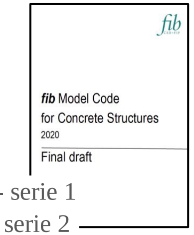
Eurocode 2: Design of concrete structures - Part 1-2: General rules - Structural fire design

Eurocode 2: Calcul des structures en béton - Partie 1-2: Règles générales - Calcul du comportement au feu

Eurocode 2: Planung von Stahlbeton- und Spannbetontragwerken - Teil 1-2: Allgemeine Regeln - Tragwerksbemessung für den Brandfall

Eurocode 2

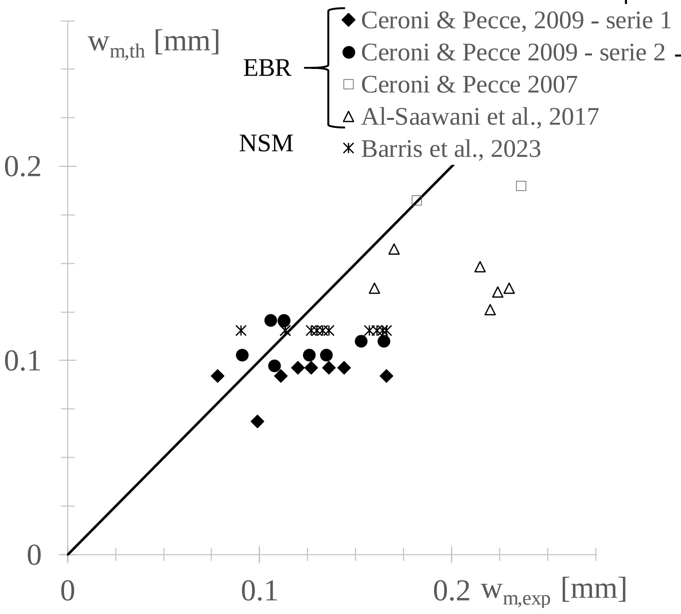



fib

fib Model Code for Concrete Structures 2020

Final draft

Model Code 2020



$w_{rm,th}$/ $w_{rm,exp}$	Beams	
	EC2	MC 2020
Mean	1.06	0.84
St. dev.	0.25	0.18
CoV	24%	22%

- EC2 gives the best approach, while MC2020 provides unsafe underestimation to safely predict crack width (in both cases, despite not considering the effect of external reinforcement).
- Two possible explanations:
  - Shrinkage strains at early stages
  - Theoretical expression of $(\varepsilon_{sm} - \varepsilon_{cm})$ does not consider the effect of FRP → for a given load level: $(\varepsilon_{sm} - \varepsilon_{cm})_{th}$ ↓ → $w_{rm,th}$ ↓

RESULTS AND COMPARISON. CRACK WIDTH – BEAMS:

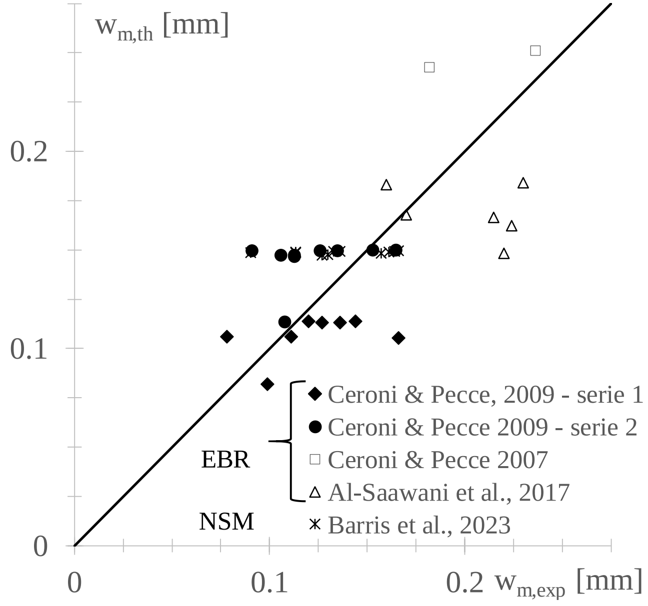

English version

Eurocode 2: Design of concrete structures - Part 1-2: General rules - Structural fire design

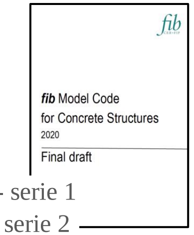
Eurocode 2: Calcul des structures en béton - Partie 1-2: Règles générales - Calcul du comportement au feu

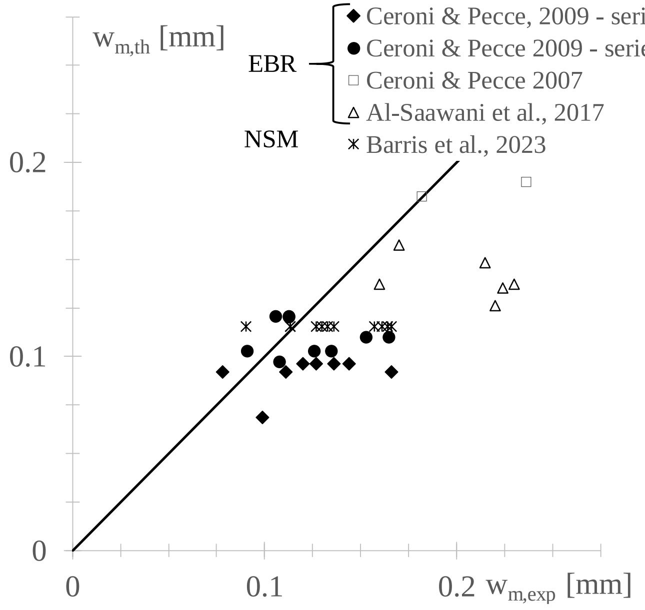
Eurocode 2: Planung von Stahlbeton- und Spannbetontragwerken - Teil 1-2: Allgemeine Regeln - Tragwerksbemessung für den Brandfall

Eurocode 2

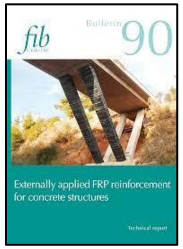


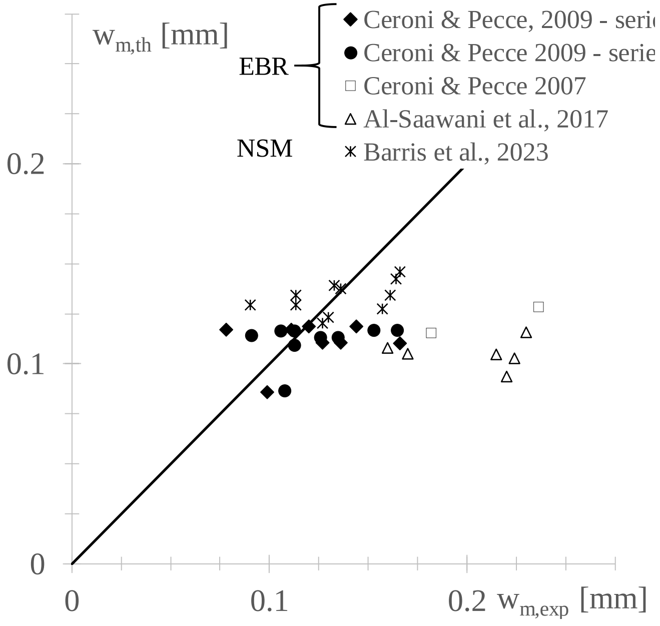
Model Code 2020





fib Bulletin 90





$w_{rm,th}/w_{rm,exp}$	Beams		
	EC2	MC 2020	*fib* B90
Mean	1.06	0.84	0.87
St. dev.	0.25	0.18	0.25
CoV	24%	22%	29%

- *fib* B90 underestimates by 13% the experimental crack width.
- Effect of shrinkage strains in the experimental measurements.

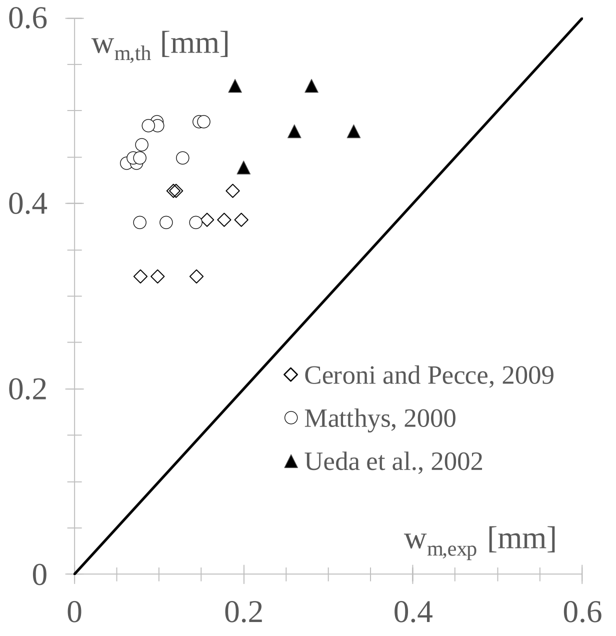
POLITÉCNICA

UdG

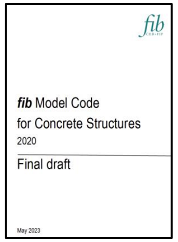
RESULTS AND COMPARISON. CRACK WIDTH – TIES:

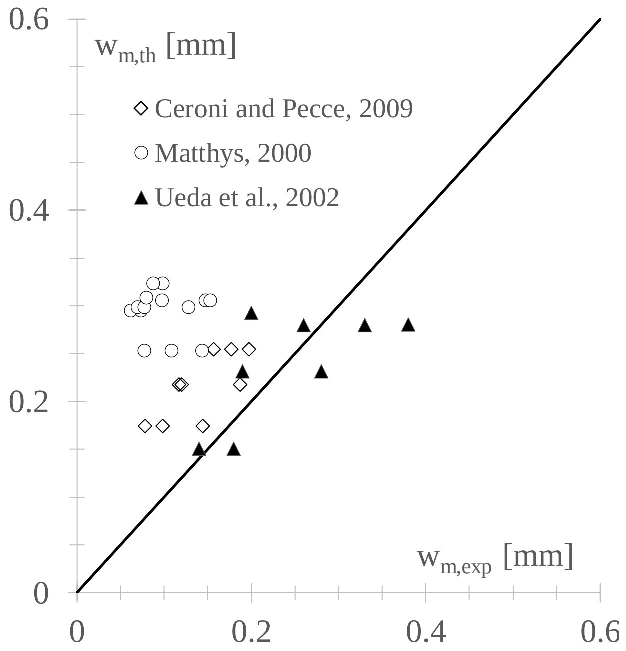
Eurocode 2


English version
Eurocode 2: Design of concrete structures - Part 1-2: General rules - Structural fire design
Eurocode 2: Calcul des structures en béton - Partie 1-2: Règles générales - Calcul du comportement au feu
Eurocode 2: Planung von Stahlbeton- und Spannbetontragwerken - Teil 1-2: Allgemeine Regeln - Tragwerksbemessung für den Brandfall

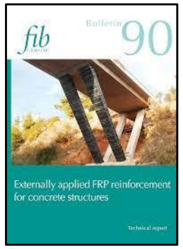


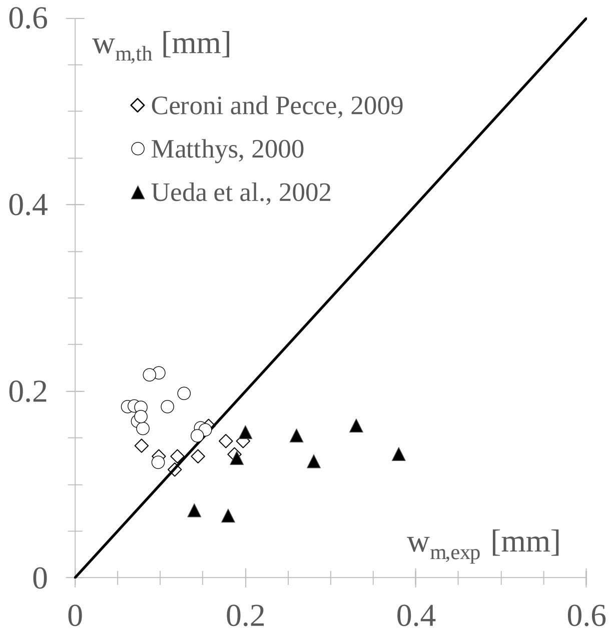
Model Code 2020





fib Bulletin 90





$w_{rm,th}/w_{rm,exp}$	Ties		
	EC2	MC 2020	*fib* B90
Mean	3.64	2.16	1.30
St. dev.	1.54	1.17	0.75
CoV	42%	54%	57%

- ❑ Very high scatter of results.
- ❑ EC2 & MC2020: Similar trends than in beams: lower predicted values (although still overestimating experimental results).
- ❑ *fib* B90: overestimation of 30%

POLITÉCNICA UdG

RESULTS AND COMPARISON

Assessment of shrinkage strain in crack width:

- Assuming a steel yield stress of 500 MPa, Young's modulus of 200 GPa & Service load @80% of steel yield stress, the steel strain at the crack equals to: ε_s = 0.8·500/200000=2000 με .
- Assuming a free shrinkage strain 300÷500 με & k_t = 0.60, the **relative increase in the measured crack width** would be (2300÷2500)/2000 = **1.15÷1.25**.
- This amplifying factor can explain the gap observed between the prediction of crack width provided by *fib* Bulletin 90 for beams.
- For codes not considering FRP presence, this factor may explain the reduction of overprediction in EC2 and underestimation of crack width for MC 2020.
- For the case of ties, the large scatter of results hinders any comparison.

CONCLUSIONS (1/3)

GENERAL COMMENTS:

- Very **few experimental results** concerning cracking are available in literature. An experimental database made only of 67 elements, 56 strengthened with EBR and 11 strengthened with NSM has been collected and analysed.

- A generally **large scatter of results** is obtained in comparisons between theoretical and experimental values for both crack spacing and crack width.

CRACK SPACING:

- Generally **overestimated by EC2-2004 and MC2020** provisions, especially for ties, since the additional tension stiffening effect of the external FRP reinforcement is not taken into account in the formulation.

- ***fib* Bulletin 90 formulation provides a closer prediction** in terms of average ratio of the predicted-to-the experimental crack spacing, for both ties and beams.

CONCLUSIONS (2/3)

CRACK WIDTH:

- May be **more affected by scatter** in comparison with crack spacing (dependency on the accuracy of the instruments, the load levels, the place where they are measured, but mainly the **unknown level of shrinkage** occurring in the specimens before the test).

- Comparison with ***fib* Bulletin 90**: while in beams it is around 15% underestimated, probably because of the shrinkage strains, in ties it is overestimated.

- Comparison with **EC2 and MC2020**: the distances between the theoretical values of crack width and the experimental ones are lesser, for both beams and ties. Possible reasons:

  - Rate of crack width related to the shrinkage strains, certainly present in the experimental measurements, but not considered in the theoretical formulations.

  - FRP reinforcement not considered in the effective percentage $\rightarrow (\varepsilon_{sm} - \varepsilon_{cm})_{calc} \downarrow \rightarrow w_{k,calc} \downarrow$.

CONCLUSIONS (3/3)

INFLUENCE OF SHRINKAGE STRAIN:

- Is it reliable to calibrate the mechanical models on both experimental values of crack spacing and crack width?

- … Or it is more correct to use the only experimental values of crack spacing and use the mechanical models to predict the crack width?

FUTURE WORKS:

- There is a need for a **new model taking into account the contribution of both internal and external reinforcement**.

- **More experimental data** is need to carry out reliable calibrations of the models.

FUTURE WORKS (BASED ON MODEL CODE 2020 APPROACH)

- New crack spacing equation, considering a new transfer length that assumes a constant shear stress distribution @ steel-concrete and @ FRP-concrete interfaces: $l_e = \frac{f_{ctm}}{4\tau_{sm}} \cdot \frac{\phi_s}{\rho_{s+FRP,ef}}$

- ... with an equivalent reinforcement ratio: $\rho_{s+FRP,ef} = \frac{A_s + \xi_1^2 A_f}{A_{ct,ef}}$

- ... that is adapted to EBR and NSM configurations:
  - EBR: $\xi_1^2 = \frac{\tau_{bmf}}{\tau_{bms}} \cdot \frac{\phi_s}{4t_f}$
  - NSM: $\xi_1^2 = \frac{\tau_{bmf}}{\tau_{bms}} \cdot \frac{\phi_s}{2t_f}$

- New crack spacing: $s_{rmax} = \beta_w \cdot (k_c \cdot c + k_{\phi/\rho} \cdot k_{fl} \cdot k_b \frac{f_{ctm} \cdot \phi_s}{\tau_{bms} \cdot \rho_{s+FRP,ef}})$

- New mean strain difference: $\varepsilon_{sm} - \varepsilon_{cm} = \frac{\sigma_{s2} - \beta \cdot \frac{f_{ctm}}{\rho_{s+FRP,eff}} (1 + \rho_{s+FRP,eff} \cdot \alpha_s \ )}{E_s}$

POLITÉCNICA UdG

Assessment of crack spacing and crack width formulations in RC elements externally strengthened with FRP materials

Cristina Barris, Francesca Ceroni & Alejandro Pérez Caldentey

THANK YOU!