Analysis of Concrete Deep Beams with Fiber-Reinforced Polymer (FRP) Bars by Indeterminate Strut-and-Tie (IST) Method

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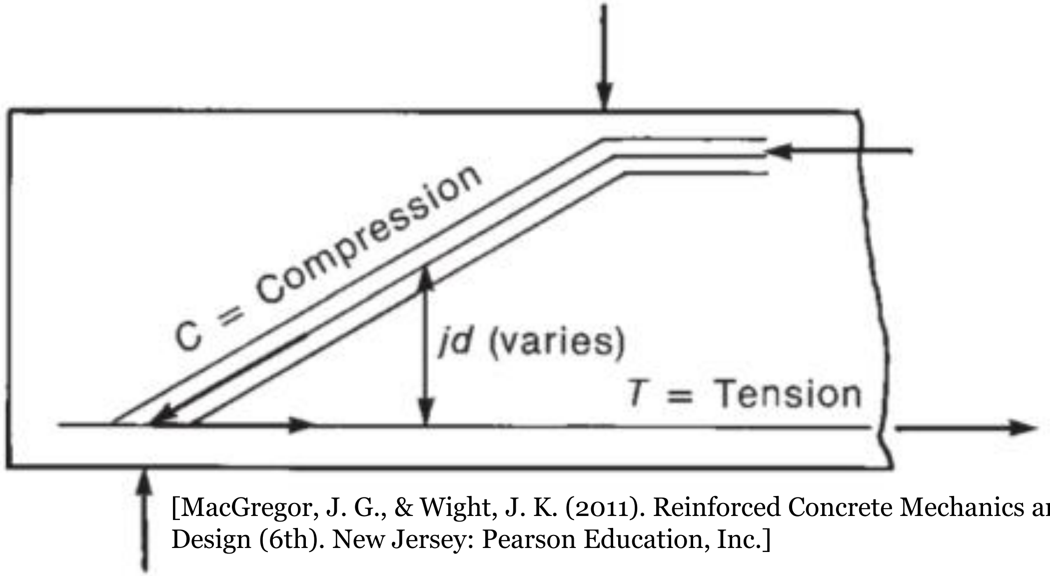
Overview

- Introduction, problem statements and background
- Specimens used for model development
- Elements of ST models
- Analysis process of the proposed Indeterminate ST method:
  - Elastic Analysis
  - Parameters to be determined
  - Failure Criteria
  - Overall Process
- Results
- Conclusions

Introduction

- FRP reinforcements:
  - light-weight, non-corrosive, linear elastic and brittle
  - Cannot yield → Concrete crushing preferred
- Deep beams:
  - Governed by arch action
  - Analyzed with strut-tie (ST) method
- Conventional ST method:
  - Based on steel yielding

    → Especially in analysis of statically indeterminate ST models (beams with complex reinforcement designs)



[MacGregor, J. G., & Wight, J. K. (2011). Reinforced Concrete Mechanics and Design (6th). New Jersey: Pearson Education, Inc.]

Background

- Analysis of deep beams reinforced by FRP bars:
  - Modelling of concrete struts becomes important
  - Reinforcement yielding cannot be assumed
- Current research and code provisions do not provide much guidance
- According to Krall and Polak (2014), Indeterminate ST method (IST method) may be the solution:
  - Elastic analysis for indeterminate ST models
  - Capable to incorporate concrete non-linear behavior
  - Needs modifications as it was developed for steel-reinforced members

Specimens

- Analyses conducted on following specimens

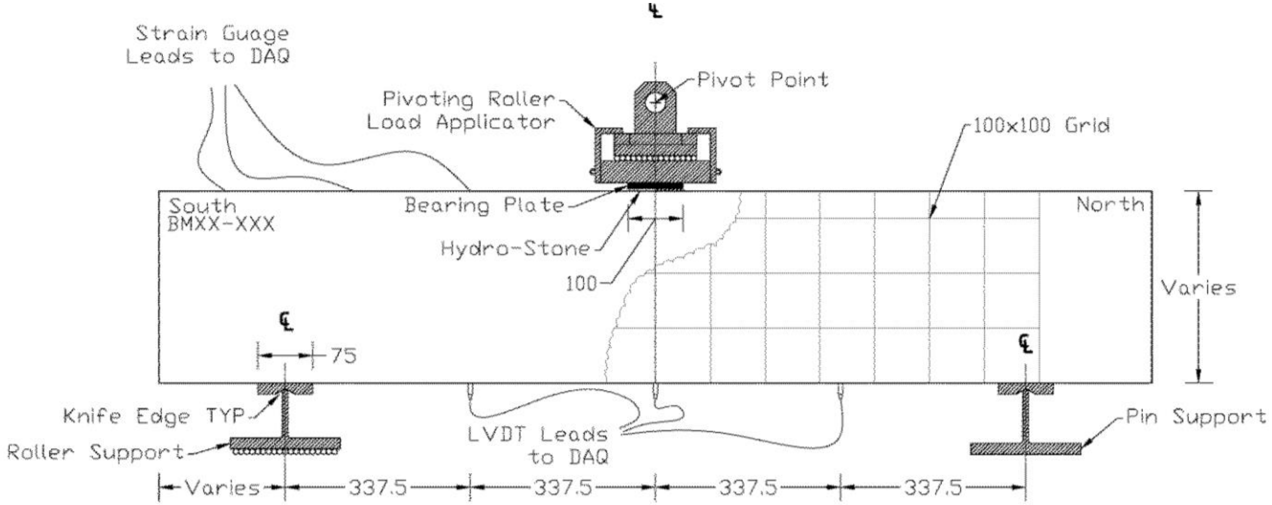
	Slenderness	Shear reinforcement	Loading conditions	Focus on
Krall and Polak (2014, 2019)	Deep beams $a/d = 2.5$	FRP stirrups (various ρ_v)	Three-point bending	The influence from shear reinforcement
Kim et al. (2014)	Deep beams a/d varies	No	Four-point bending	The influence from beam sizes, slenderness ratios, stiffness of longitudinal ties.

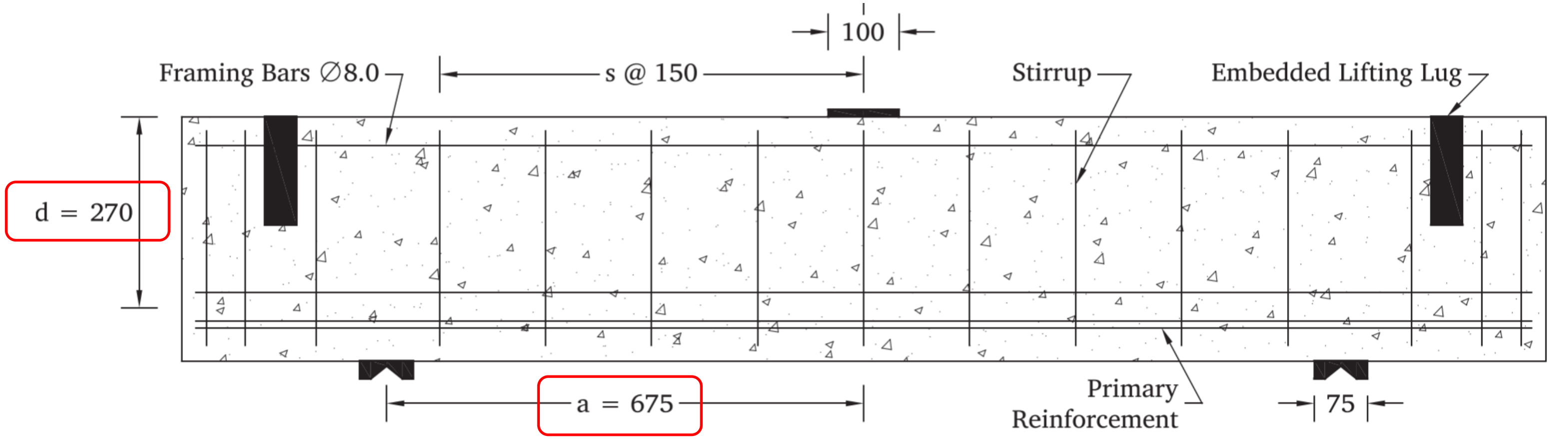
a/d: Shear span to depth ratio

ρ_v: Shear reinforcement ratio

Specimens

- Krall and Polak (2019): Six beams used for the analyses



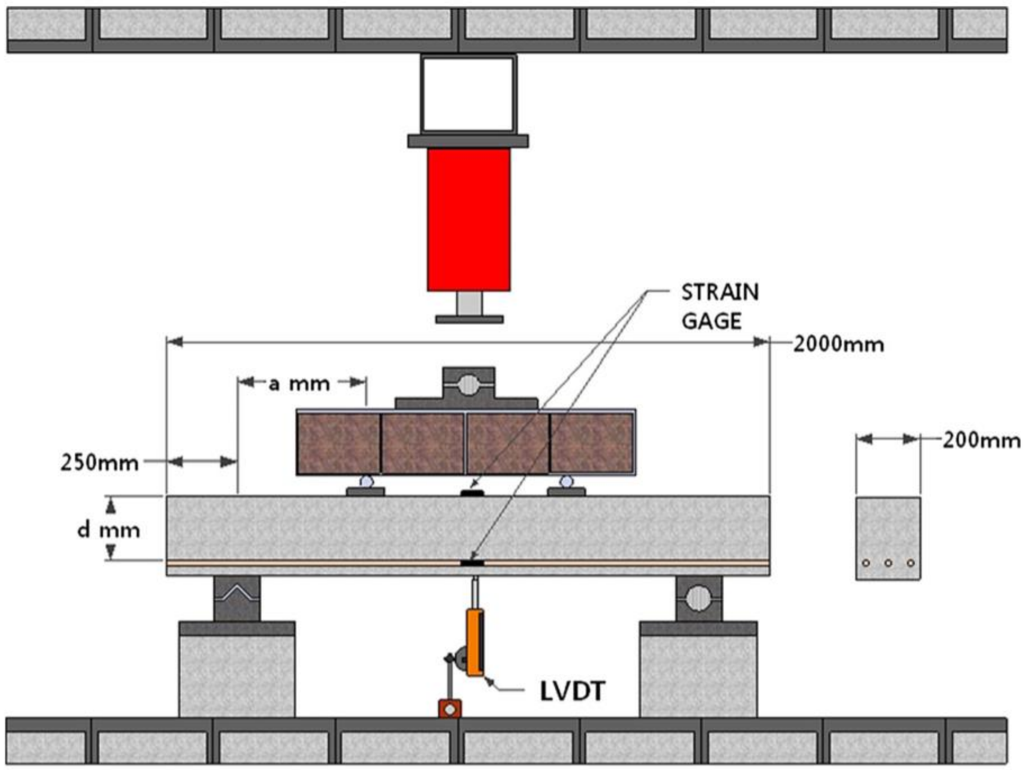


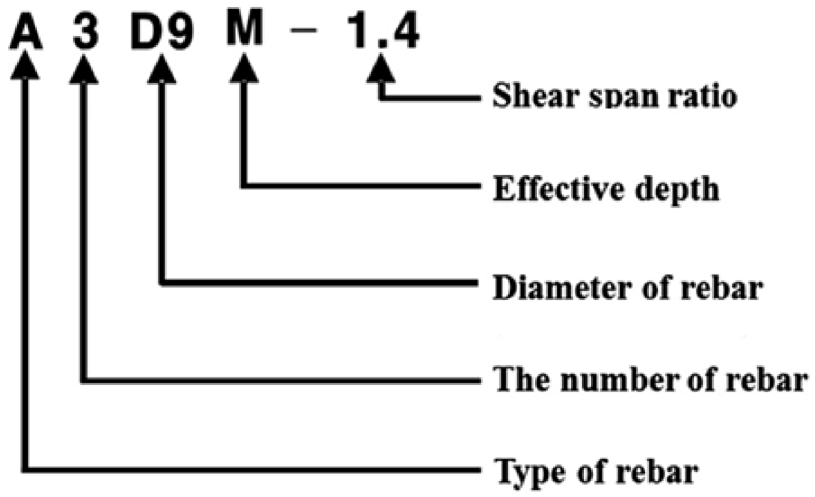
BM12-150 → Spacing between stirrups (mm);
INF: no stirrup;
s230: larger stirrups @ 230mm

→ Longitudinal rebar diameter (mm)

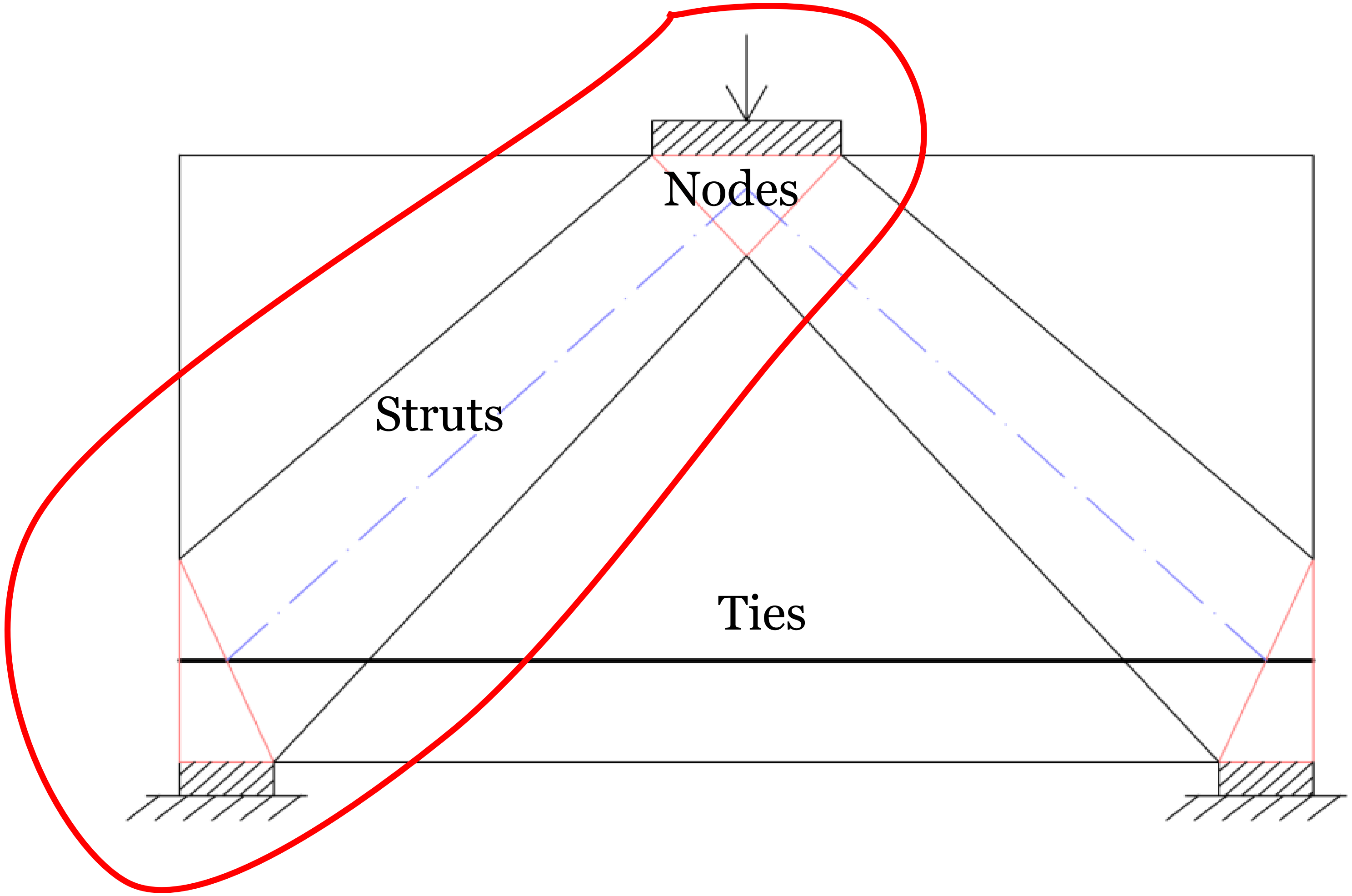
Specimens

- Kim et al. (2014). Six beams used for the analyses.



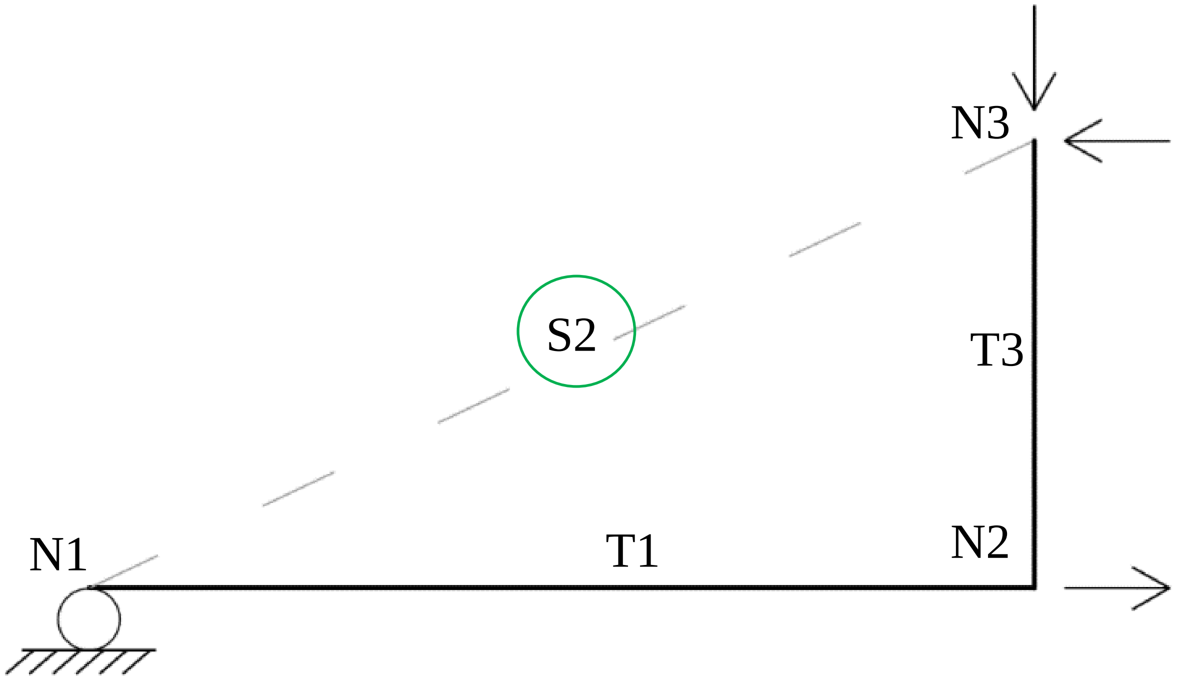


ST Models - Elements



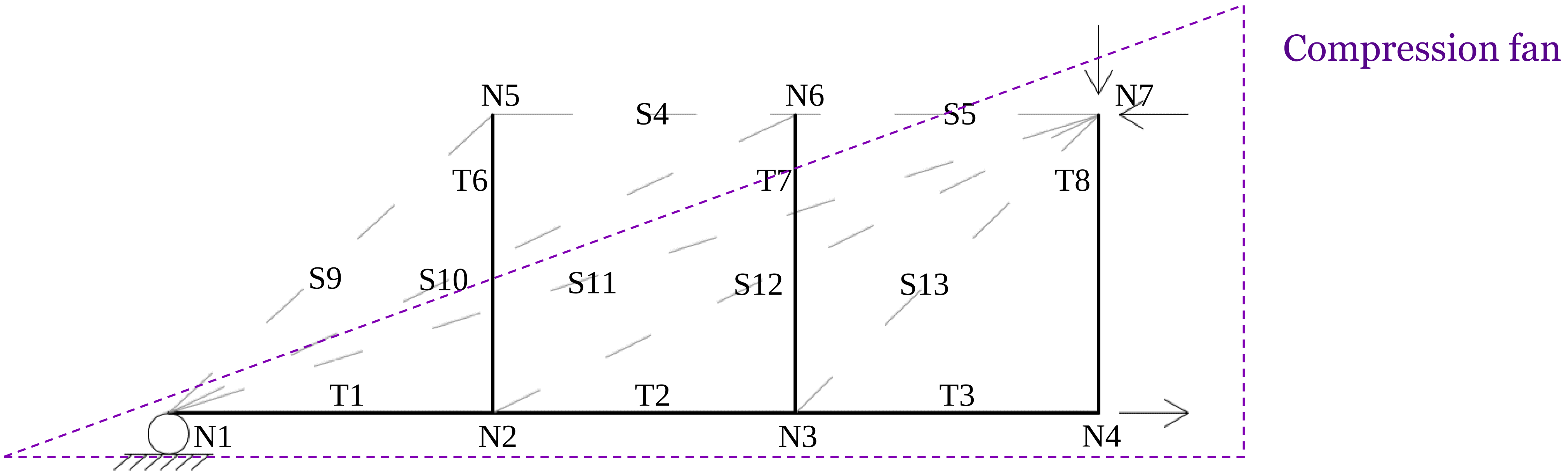
ST Models – Beams without Stirrups

- The typical triangular model
- One load path; failure occurs when S2 fails



ST Models – Beams with Stirrups

- Whole section as compression fan
- Changes with number of stirrup
- Load paths depend on inclined struts inside compression fan



IST Method – Elastic Analysis

- The elastic analysis is used to compute internal forces in each member using the assembled stiffness matrix.
- Stiffnesses of elements:

  – Stiffness of members: **cross-sectional area** x **elastic modulus**

  – **FRP ties**: constant elastic modulus, constant area

  – **Concrete struts:**

    –tangential elastic modulus from softened concrete models,

    –cross-sectional area equal to strut width (w_s) times beam width;

Concrete Struts– Softened Concrete Model

- Softened concrete model:
  – Hognestad Parabola (Hognestad, 1951)
  – Softened according to Pang and Hsu (1995)
  – Tangential modulus as the derivatives:

$$E = \frac{2 \cdot f_c'}{\varepsilon_0}\left(1 - \frac{\varepsilon_c}{\zeta \varepsilon_0}\right)$$

Softening factor

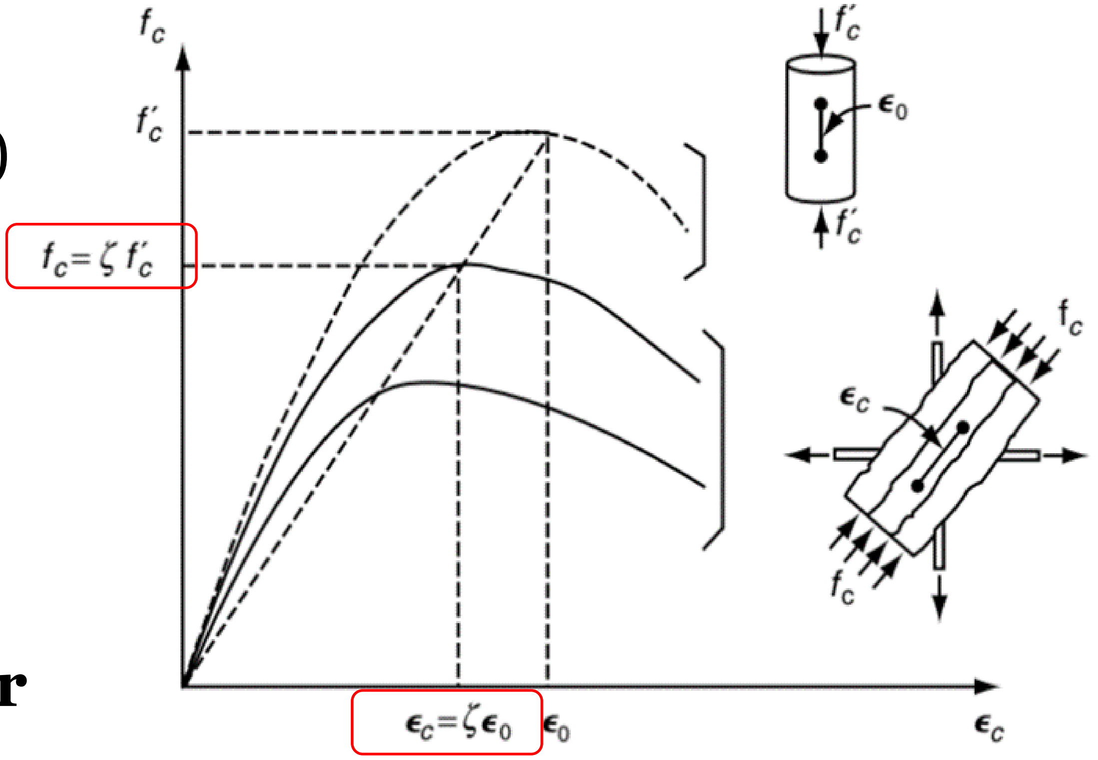


Figure from Pang and Hsu (1995)

Softening Factors (ζ) – Existing Approaches

- **ACI 318-19**
  - $\zeta_{ACI} = 0.6375$ [with stirrups]; $\zeta_{ACI} = 0.34$ [without stirrups]
- **Nehdi et al. (2008):**
  - $\beta_s = 0.68 - 0.012\left(\frac{a}{d}\right)^4$ for $\left(E_f \rho_f\right)^{1/3} \leq 10$ $\quad k = \max\left(\frac{250+d}{550},\ 1.0\right)$

    $\beta_s = 0.75 - 0.01\left(\frac{a}{d}\right)^4$ for $\left(E_f \rho_f\right)^{1/3} > 10$ $\quad \zeta_{Nd} = 0.85 k \beta_s$
- **CSA S806-12** :

  $$\zeta_{CSA} = \frac{1}{0.8 - 0.34^{\varepsilon_1}/\varepsilon_0} \leq 0.85$$

  $$\varepsilon_1 = \varepsilon_F + (\varepsilon_F - \varepsilon_s)\cot^2\theta_s$$

- **Risk of overestimating the strength**
- **Predicting incorrect failure mode**
- **Not reflecting strength increase with increasing shear reinforcement ratios**

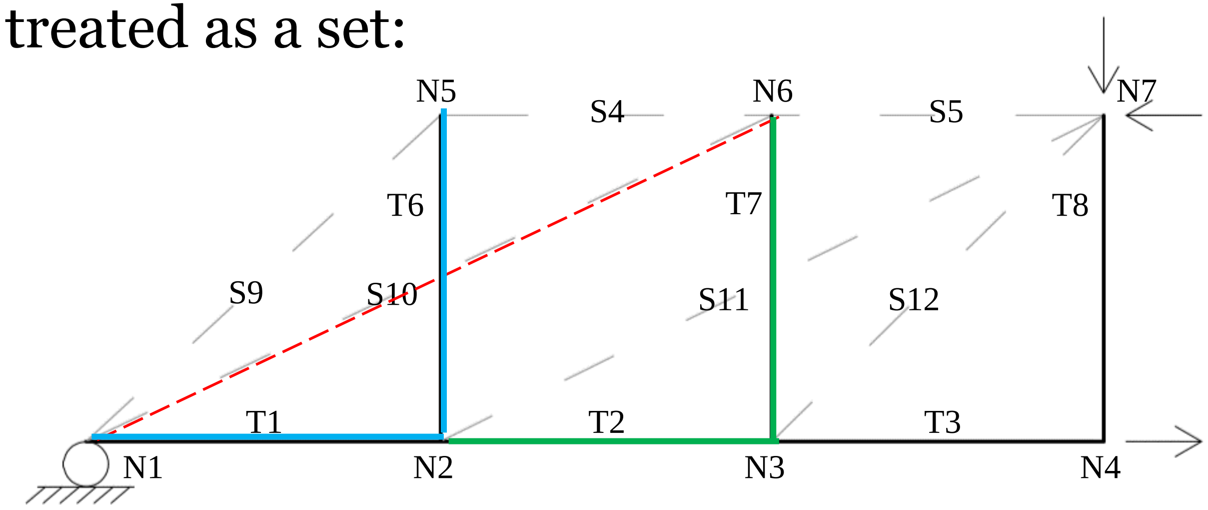
Proposed Softening Factor (ζ)

– Based on the modified compression field theory from Vecchio and Collins (1986)

– Uses an alternative form for the equation from Mohr's circle

$$\zeta_{new} = \frac{1}{0.8 - 0.34\,{}^{\varepsilon_1}/_{\varepsilon_0}} \leq 0.85 \qquad \varepsilon_1 = \max(\varepsilon_f + \varepsilon_v) - \varepsilon_s$$

- $(\varepsilon_f + \varepsilon_v)$ is treated as a set:



ε_1: Principal tensile strain

ε_f: Strain in horizontal ties

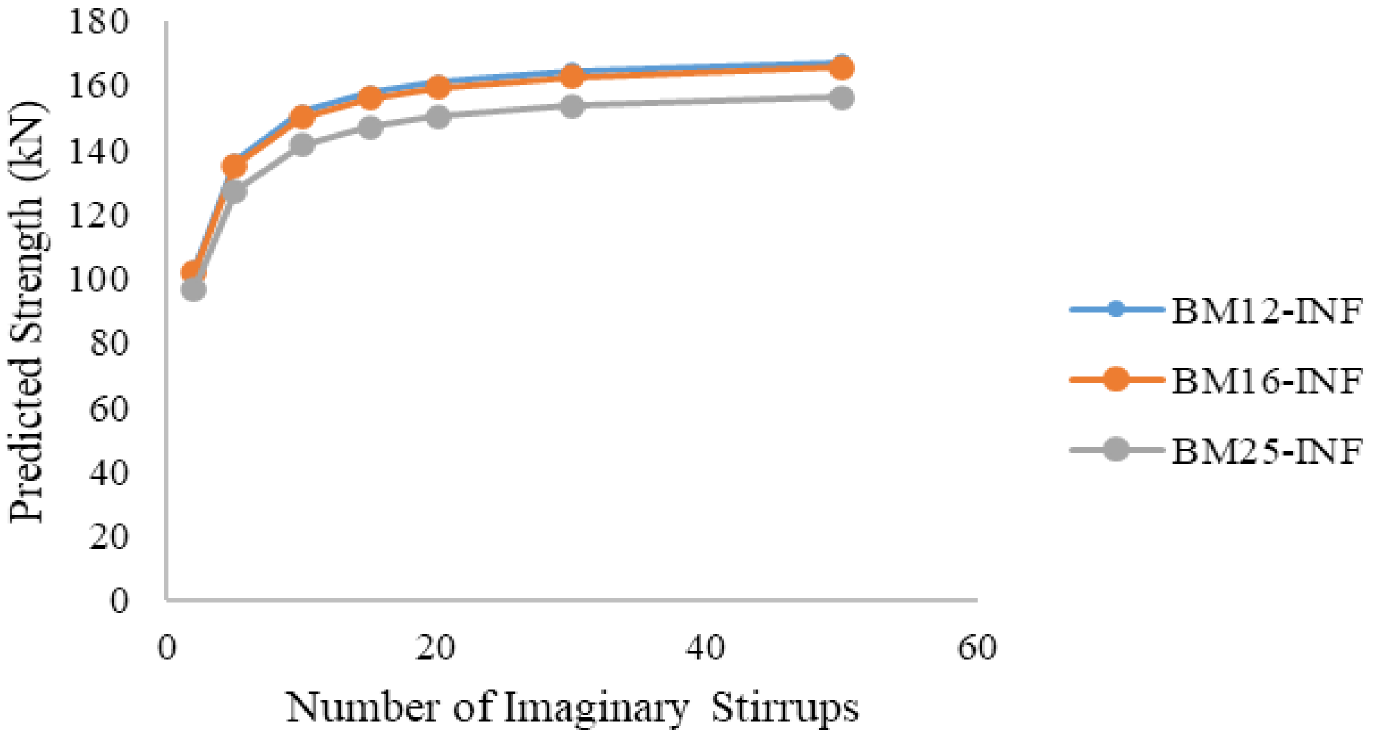
ε_v: Strain in vertical ties

ε_0: Strain @ compressive strengt (-)

ε_s: Strain in strut (-)

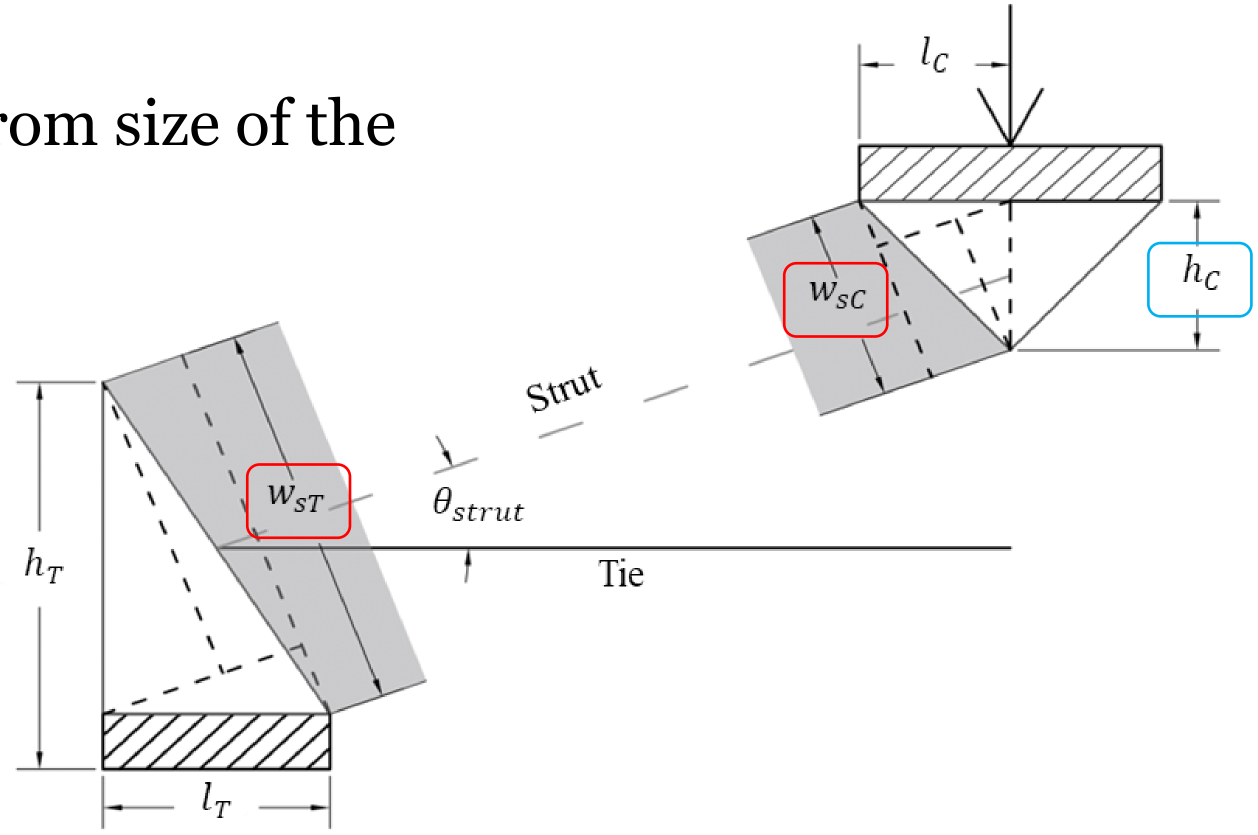
Proposed Softening Factor (ζ) -beams without stirrups

- Cannot calculate softening factor without knowing ε_v
- ε_v is strain in vertical stirrup
- Truss consisting ***imaginary ties*** (with stiffness close to zero) is proposed to find ε_v
  - $P_{predict} = C_{strut@failure} \sin\theta_s$
- **Analysis with 5 imaginary ties is proposed**
  - conservative



Concrete Strut Widths

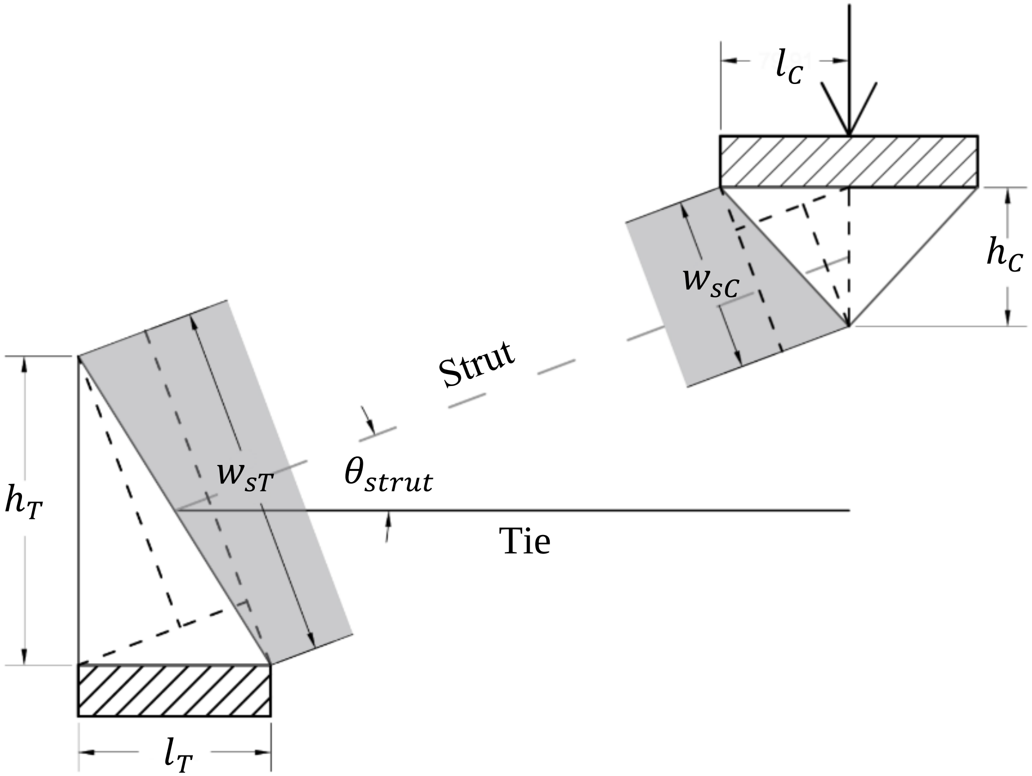
- Widths of the struts (w_s) are calculated from size of the nodes and the incline of the strut
- Problems:
  - Two nodes → two values
  - How to compute h_C
- Solutions
  - w_s can be average or smaller value (Conservative)
  - Need to propose a new method to determine h_C without assuming tie yielding

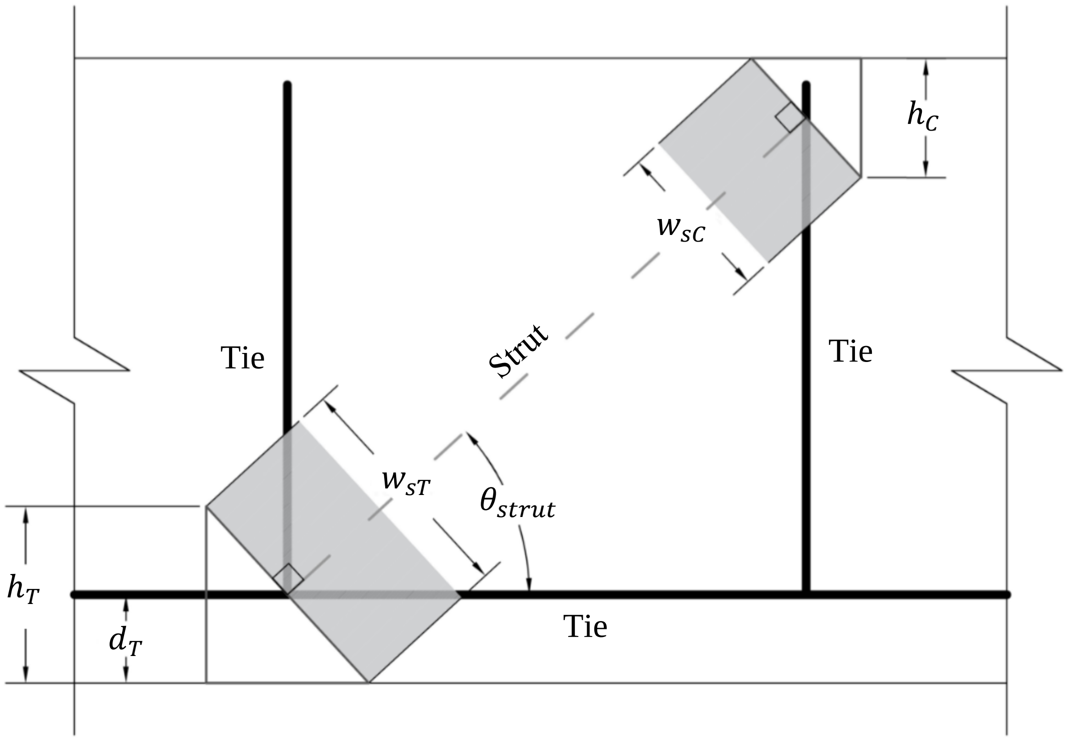


Width of the struts

$$w_{sT} = \begin{cases} h_T \cos\theta_{strut} + l_T \sin\theta_{strut}, & \textit{with loading plates} \\ h_T \sec\theta_{strut}, & \textit{without loading plates} \end{cases}$$

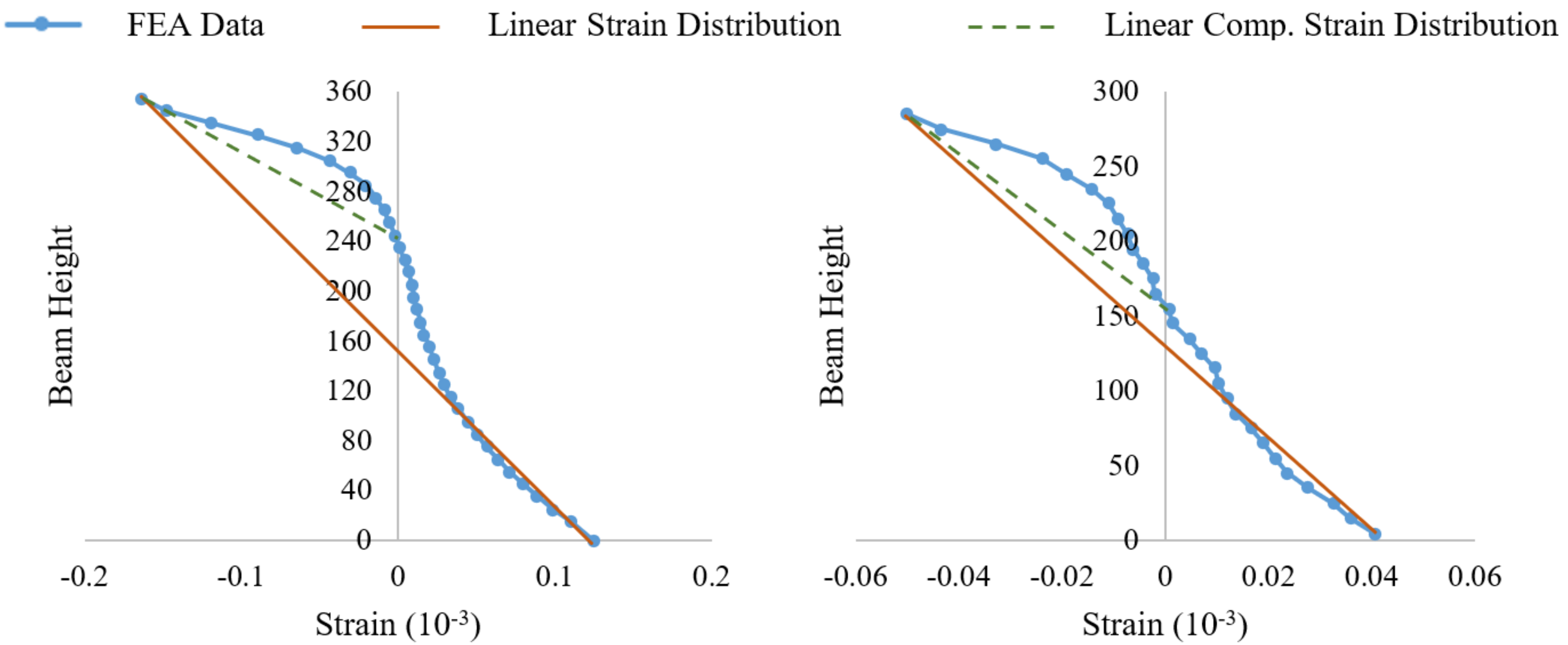
$$w_{sC} = \begin{cases} h_C \cos\theta_{strut} + l_C \sin\theta_{strut}, & \textit{with loading plates} \\ h_C \sec\theta_{strut}, & \textit{without loading plates} \end{cases}$$





Determination of h_C

- Based on FEA results:
  - Strain profile is assumed to be linear in compression part only
- Combined



a) under three-point bending

b) under four-point bending

compressive strength

compressive strength

outermost comp. fibre

reinforcement ratio

- Compute c

dth

- Hence,

$3\varepsilon_0$ -

$\varepsilon_{Top} = ——$

$$h_c = 2\left(c - \bar{y}_{from\_N.A.}\right) = \frac{4\varepsilon_0 - \varepsilon_{Top}}{6\varepsilon_0 - 2\varepsilon_{Top}}c$$

IST Method – Failure Criteria

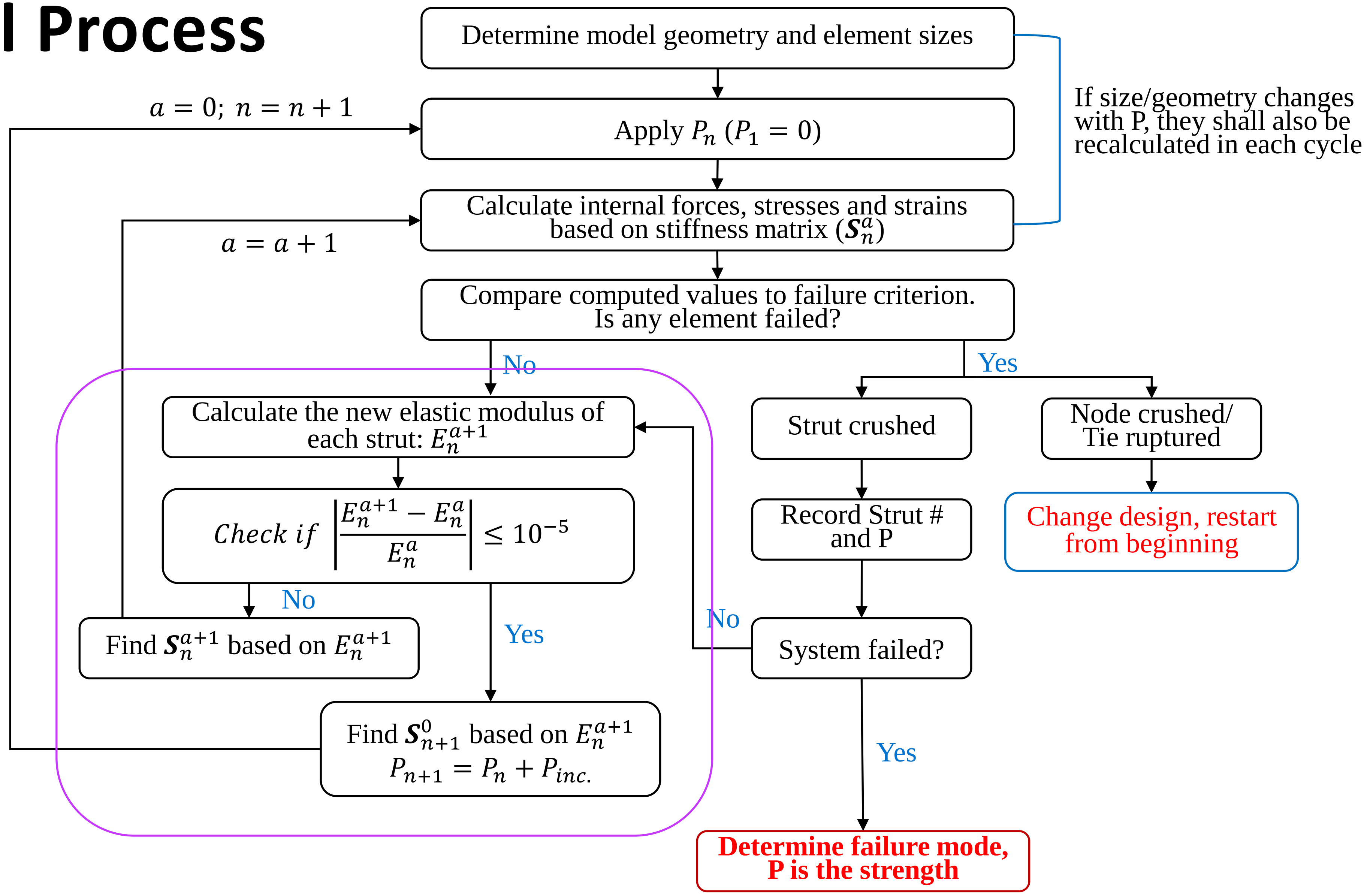
Tie Rupture	Node Crushing	Strut Crushing
• Rupture strength ✕ - Brittle	• Limited nodal strength ✕ - Localized	• $E \rightarrow 0$ • Failed struts: $E = 1\% E_0$ ✓

Strut strength modelling affects strength prediction

- Statically indeterminate models:
  - Crushing of enough struts makes the model unstable
- System failure:
  - Shear Failure: Crushing of inclined struts
  - Flexural Failure: Crushing of horizontal struts
  - Combined Failure: Crushing of both types of struts
  - Both shear failure and combined failure predicts shear strength

ST models also affects the analysis

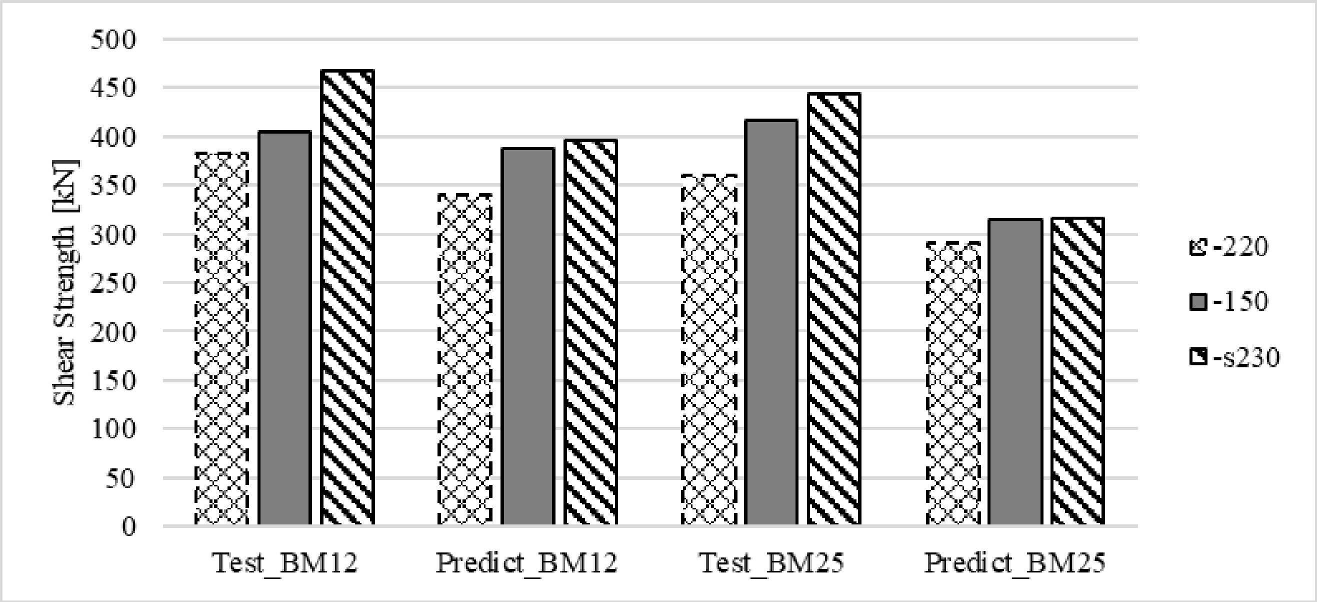
Overall Process

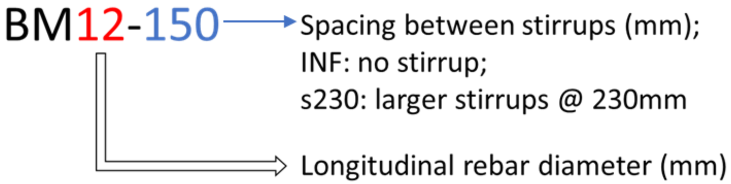


Results – Specimens with stirrups

Specimen	P_{test} (kN)	$P_{predict}$(kN)	Failure Mode	$P_{predict}/P_{test}$
BM12-220	382.4	341	Shear	0.89
BM12-150	405.2	387	Combined	0.96
BM12-s230	466.9	397	Shear	0.85
BM25-220	360.1	291	Shear	0.81
BM25-150	415.8	314	Combined	0.75
BM25-s230	444	316	Shear	0.71

[1 kN = 0.2248 kip]

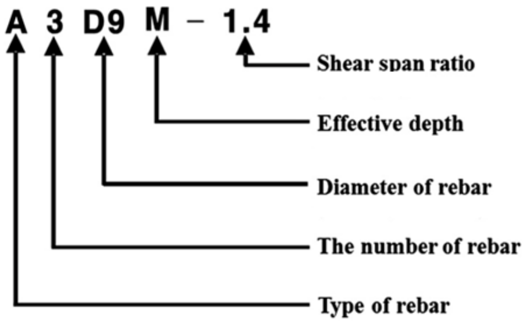




Strength increase with increased shear reinforcement ratio

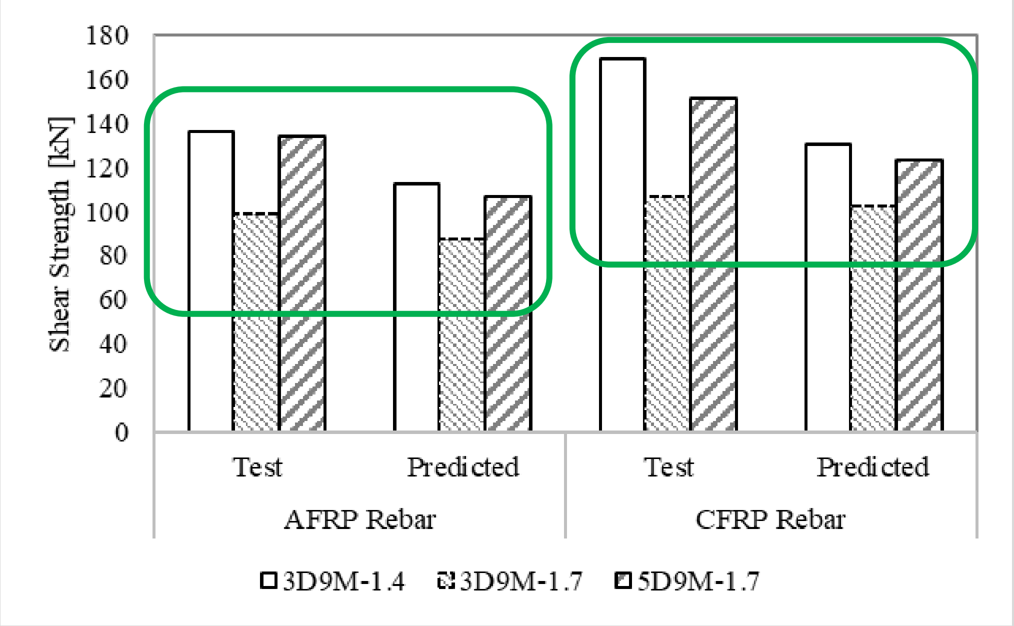
Lower-bound estimation

Results – Specimens without stirrups



Specimen	P_{test} (kN)	$P_{predict}$(kN)	$P_{predict}/P_{test}$
A3D9M-1.4	136.1	112	0.83
A3D9M-1.7	99.0	88	0.89
A5D9M-1.7	134.0	107	0.80
C3D9M-1.4	169.3	131	0.77
C3D9M-1.7	106.5	102	0.96
C5D9M-1.7	151.4	123	0.81

[1 kN = 0.2248 kip]



Similar strength trends

Lower-bound estimation

Conclusions

- **The proposed IST method includes the following components:**
  - **The softened Hognestad Parabola;**
  - **The proposed Softening factor formulation;**
  - **The proposed method to compute the compression block depth (h_c)**

- The IST method can be used to design and analyze shear strengths of FRP reinforced concrete deep beams with and without stirrups with relatively accurate results with proper strength increase/decrease trends.

Recommendations

- The proposed IST method is based on a limited number of specimens with stirrups due to the lack of experimental data, thus it shall be further verified on more specimens with different designs.
- The proposed IST method should be tested on other D-regions to determine its limitations and to further develop the methodology.

THANK YOU

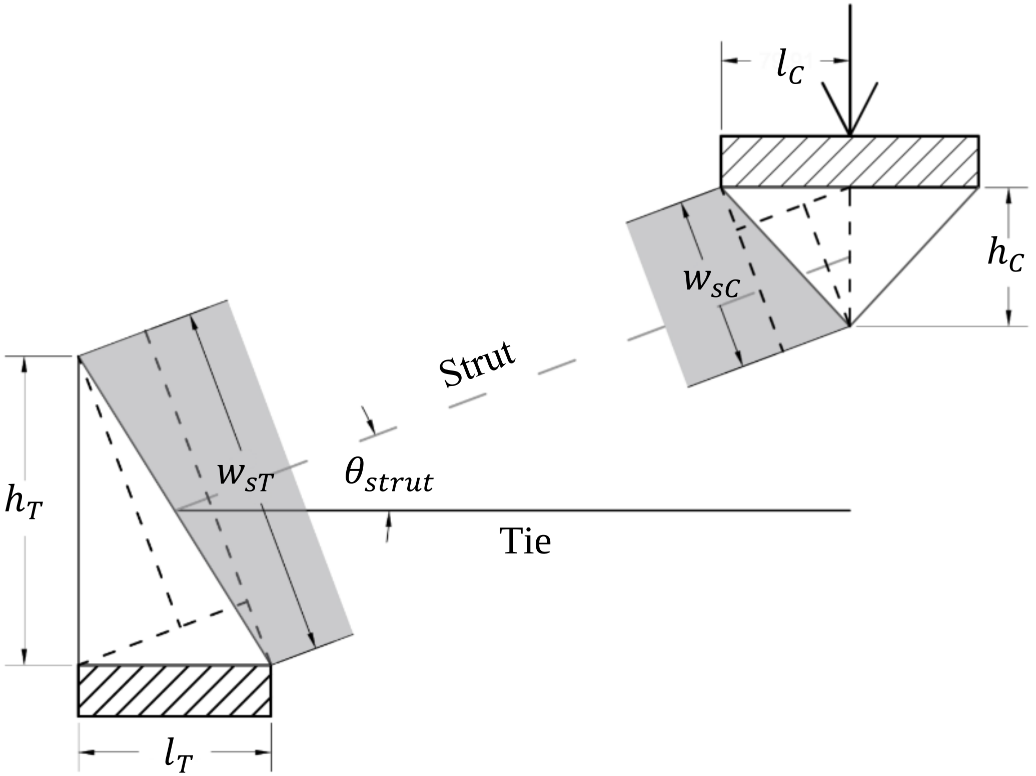
NSERC
CRSNG

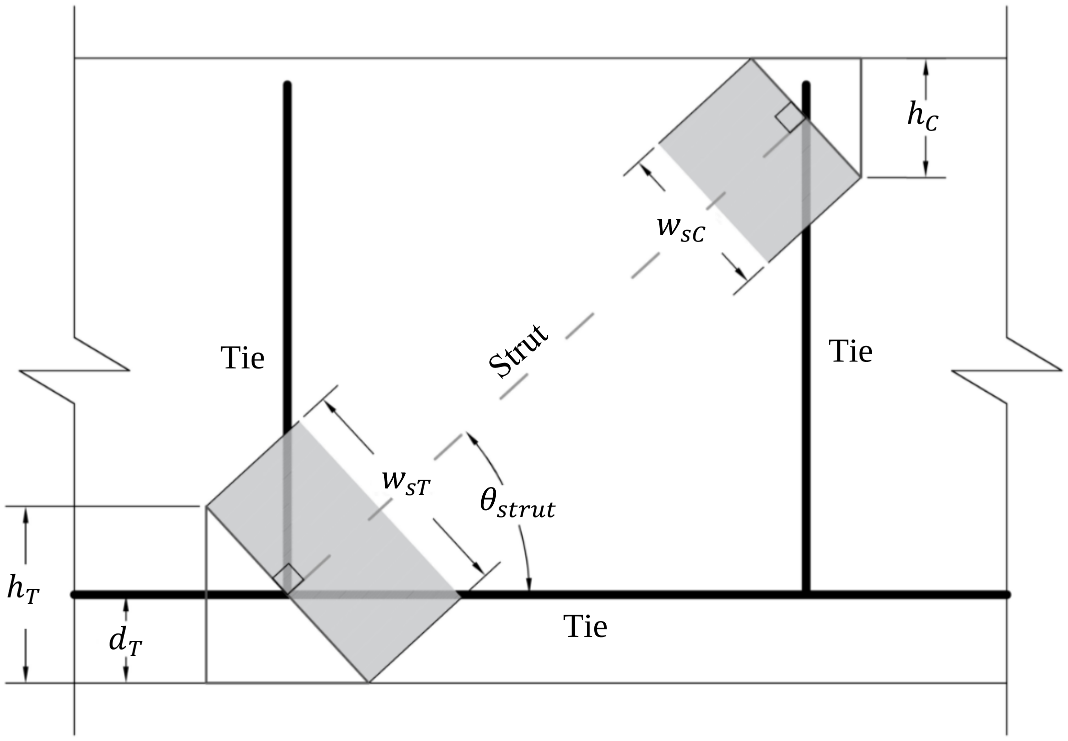
ADDITIONAL SLIDES

Width of the struts

$$w_{sT} = \begin{cases} h_T \cos\theta_{strut} + l_T \sin\theta_{strut}\,, & \textit{with loading plates} \\ h_T \sec\theta_{strut}\,, & \textit{without loading plates} \end{cases}$$

$$w_{sC} = \begin{cases} h_C \cos\theta_{strut} + l_C \sin\theta_{strut}\,, & \textit{with loading plates} \\ h_C \sec\theta_{strut}\,, & \textit{without loading plates} \end{cases}$$

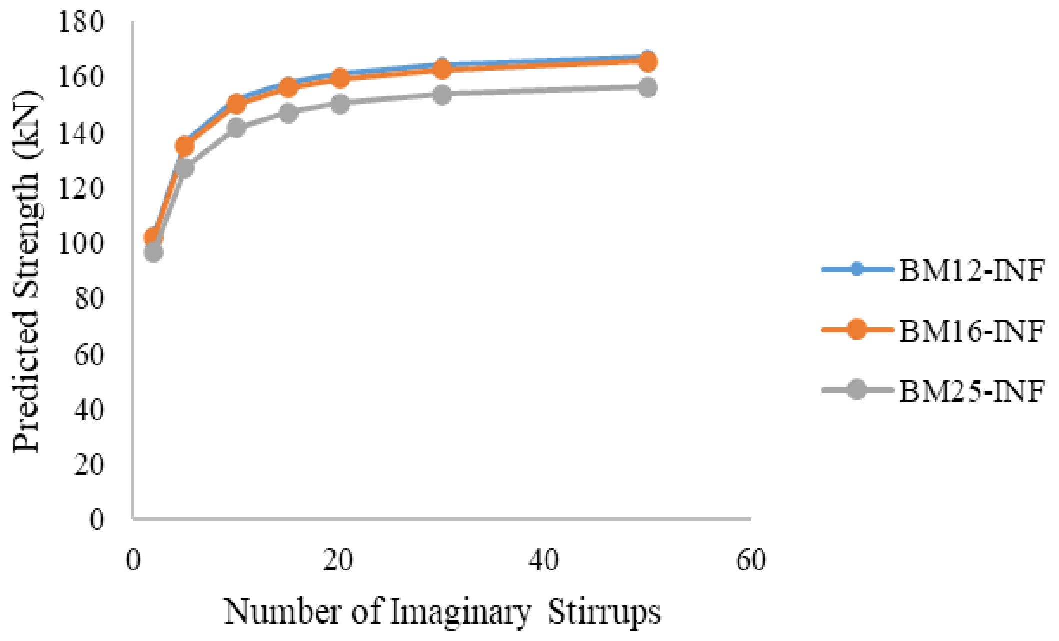




Proposed ζ- Imaginary ties and Convergence

- It is incapable of analyzing beams without knowing ε_v:
  - The case of beams without stirrups
- Truss consisting imaginary ties (with nearly no stiffness) can be utilized to find ε_v
  - $P_{predict} = C_{strut@failure} \sin\theta_s$
  - $P_{predict}$ increase with more ties, but converges
- Analysis with 5 imaginary ties is proposed

→ 1. Conservative 2. Save time

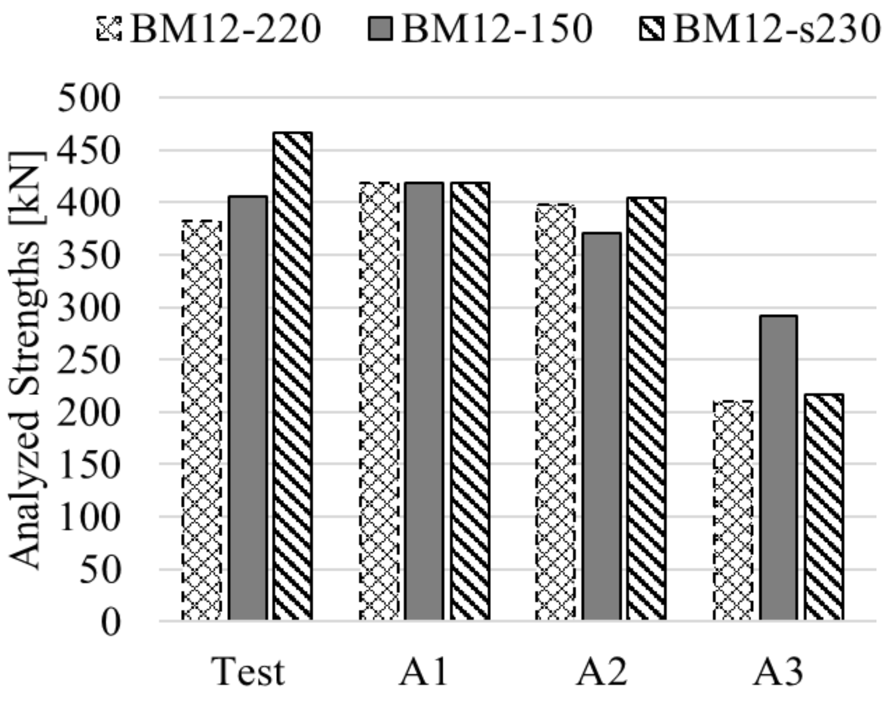


ε_v: Strain in vertical ties

θ_s: Angle of the strut

Problems on existing ζ approaches

BM12-150 → Spacing between stirrups (mm);
INF: no stirrup;
s230: larger stirrups @ 230mm
→ Longitudinal rebar diameter (mm)



Specimen	P_{test} (kN)	$P_{predict}$ (kN) with A1	A2	A3
BM12-220	382.4	**419**	**398**	210
BM12-150	405.2	**419**	370	291
BM12-s230	466.9	418	405	217
		Predicted Failure Modes		
BM12-220		**Flexure**	Shear	Shear
BM12-150		**Flexure**	Shear (C)	Shear
BM12-s230		Combine	Shear	Shear

Bold results are unconservative

- A1 based on ACI 318-19
- A2 based on Nehdi et al. (2008)
- A3 based on CSA S806-12, and

FEA results for h_C assumptions

FEA results for h_C assumptions