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Another Look at Cracking and Crack Control in Reinforced Concrete

by Robert J. Frosch

The ACI Building Code requires the control of flexural cracking in reinforced concrete structures. Because of durability concerns, the use of thicker concrete covers is rapidly increasing. However, currently used crack control methods that are based strictly on statistical reasoning become unworkable with the use of thick covers. This study investigates the development of the crack control provisions and the crack width equation on which those provisions are based, explores the use of the crack width equation for the calculation of large covers, and presents a new formulation of the equation for calculating crack width that is based on the physical phenomenon. Use of that equation is supported by an evaluation of existing test data; based on that equation, a design recommendation is presented for the control of cracking that addresses the use of both coated and uncoated reinforcement.

Keywords: concrete durability; crack width and spacing; cracking (fracturing).

INTRODUCTION

Since 1971, the ACI Building Code has required control of flexural cracking in reinforced concrete structures through the use of the z-factor method.[1] The z-factor approach is a modified form of the Gergely-Lutz[2] crack width equation that was developed from a statistical evaluation of experimental crack width data.

Currently, the use of thicker concrete covers is increasing because research and experience have indicated that the use of thicker covers, as well as high-performance concrete, can increase durability. With the use of thicker covers, however, the z-factor method becomes practically unworkable. The method indicates that the increased cover is detrimental to crack control and results in bar spacings that are not practical in construction.

RESEARCH SIGNIFICANCE

To allow for the design and construction of more durable concrete structures, it is necessary to answer the following questions: Is it possible to increase cover while still providing reasonable crack control? Since the Gergely-Lutz equation was based on specific experimental data, are the equation and resulting z-factor approach valid for thicker concrete covers?

It is also important to address the effect that epoxy-coated reinforcement has on cracking.

PREVIOUS RESEARCH

Gergely and Lutz[2] performed a statistical evaluation of experimental cracking data. That study resulted in the well-known Gergely-Lutz equation for the calculation of crack widths. The data used in the study included test results from Hognestad,[3] Kaar and Mattock,[4] Kaar and Hognestad,[5] Clark,[6] and Rüsch and Rehm.[7] Because different measurement methods were used by the various investigators, crack widths were recorded at two primary locations, the bottom tension surface and the side face at the level of the reinforcement. The study included 612 observations for bottom cracks and 355 observations for side cracks. According to Gergely and Lutz,[2] "the maximum crack width measured by an investigator at a certain stress level is considered statistically as an observation. If the maximum crack width is measured at six stress levels, there will be six observations resulting from this one beam test."

Since the study was intended for U.S. practice, the primary focus was on test results with bars meeting the ASTM A 305 standard on bar deformations. Since the Rüsch and Rehm investigation was conducted in Germany, the bars in that test series did not conform to the ASTM A 305 standard; therefore, Gergely and Lutz included only beams from that investigation that had bars with deformations similar to those of U.S. bars. In addition, a few beams with bars having a deformation spacing somewhat greater than indicated in the ASTM specification were also included.

Based on the statistical analysis of the test data, different equations were developed for crack widths occurring on the side face at the level of steel and for those at the bottom (tension) face. Since the crack width of primary interest is at the bottom surface, the crack width equation ACI bases the crack control provisions is the simpler form for bottom face cracking presented by Gergely and Lutz[2]

$$w_b = 0.076\beta f_s \sqrt[3]{d_c A} \quad (1)$$

where

w_b = maximum bottom crack width, 0.001 in.;
β = ratio of distances to neutral axis from extreme tension fiber and from centroid of reinforcement;
f_s = steel stress calculated by elastic crack section theory, ksi
d_c = bottom cover measured from center of lowest bar, in.; and
A = average effective concrete area around reinforcing bar, having same centroid as reinforcement, in.^{2}

As the concrete cover d_c is of particular interest, it is important to note the ranges of covers considered in the statistical evaluation for bottom face cracking (Table 1). While the maximum cover was 3.31 in. from the Hognestad test data, it should be noted that only three test specimens had covers greater than 2.5 in.

Other investigators have developed similar crack width equations, including Kaar and Mattock,[4] who developed a well-known expression in which the variables are defined the same as above

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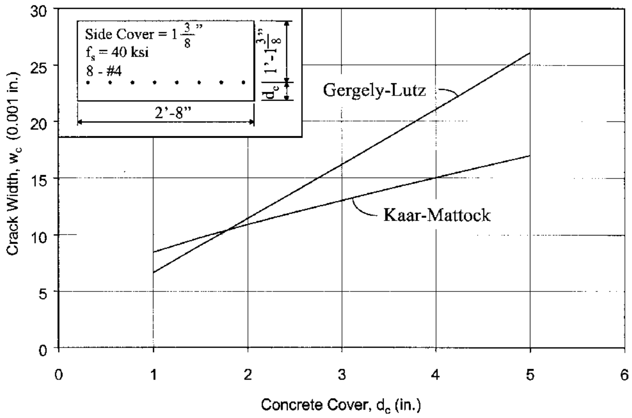

Fig. 1—Comparison of crack width equations.

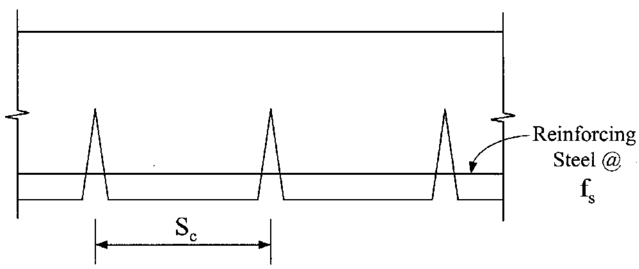

Fig. 2—Cracked section.

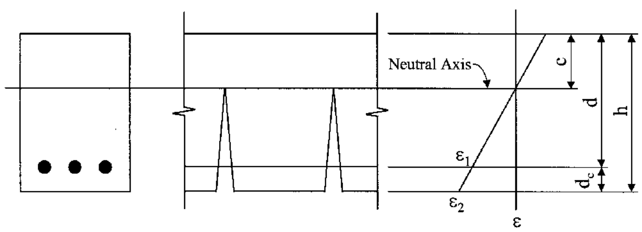

Fig. 3—Strain gradient.

$$w_b = 0.115\beta f_s \sqrt[4]{A} \quad (2)$$

The Kaar and Mattock equation was developed from a curve fit of limited data primarily from Hognestad[3] and Kaar-Mattock.[4] That data was also included in the statistical study by Gergely and Lutz.

While both equations presented fit the data reasonably well, it is reasonable to question the validity of the equations outside the range of data considered. To examine their validity outside that range, both equations were used to compute the crack widths for a beam section with varying concrete cover (Fig. 1). Both equations compared reasonably well for concrete covers up to 2.5 in. ($d_c < 2.5$ in.); however, as the cover was increased, there was a divergence in the calculated crack widths. It is interesting to note that this beam section is the same as Specimen $32R_1$ from the Kaar-Mattock study that was considered in the development of both crack width equations. The actual cover d_c for the Kaar-Mattock specimen was 1.625 in., which is approximately where the equations intersect.

Because test data for thicker covers are not available, it is not possible to determine which, if either, equation is correct in that range. Furthermore, to project an expression formulated by a statistical analysis of data outside the domain of the data is questionable. Therefore, an alternate approach for the calculation of crack widths is needed to consider thicker covers ($d_c > 2.5$ in.).

Table 1—Concrete cover ranges in Gergely-Lutz study

Investigator	Concrete cover d_c Minimum	Concrete cover d_c Maximum
Hognestad	0.81	3.31
Kaar-Mattock	1.61	2.00
Rusch-Rehm	0.75	2.20
Kaar-Hognestad	1.50	1.50
Clark	0.69	2.70
All investigators	0.69	3.31

CRACK WIDTH

To provide perspective on the calculation of crack widths, it is necessary to consider a physical model of cracking. Flexural cracking is illustrated in Fig. 2. The crack width at the level of the reinforcement can be calculated as follows

$$w_c = \varepsilon_s S_c \quad (3)$$

where

w_c = crack width;
ε_s = reinforcing steel strain = $\frac{f_s}{E_s}$;
S_c = crack spacing;
f_s = reinforcing steel stress; and
E_s = reinforcing steel modulus of elasticity.

The previous equation is based on the assumption that the reinforcing steel is uniformly strained over the crack spacing. Additionally, the tensile strain in the concrete is neglected as the tensile concrete strain relative to the steel strain is small and does not significantly affect the crack width. Furthermore, by neglecting the concrete tensile strain, the crack width is slightly overestimated and provides a conservative estimate of the crack width.

To determine the crack width at the beam surface, it is necessary to account for the strain gradient. The strain gradient is illustrated in Fig. 3, which assumes that plane sections remain plane. The crack width computed previously can be multiplied by an amplification factor β that accounts for the strain gradient. The factor *b* is computed as follows

$$\beta = \frac{\varepsilon_2}{\varepsilon_1} = \frac{h-c}{d-c} \quad (4)$$

CRACK SPACING

Crack spacing decreases with increasing load and stabilizes after the reinforcement reaches a critical stress. Further stress increases act only to widen the existing cracks. Tests indicate that the critical stress is typically in the range of 20 to 30 ksi for covers up to 3 in.; thus, under service load stresses, a stable crack pattern is typically developed.

From both analytical and experimental investigations performed by Broms,[8] it was found that the crack spacing depends primarily on the maximum concrete cover. Specifically, the minimum theoretical crack spacing will be equal to the distance from the point at which the crack spacing is considered to the center of the reinforcing bar located closest to that point. Furthermore, it was found that the maximum theoretical crack spacing is twice the minimum. Additional experiments conducted by Broms[8] support those findings for tension specimens with covers up to 6 in.

The crack spacing can be calculated as follows

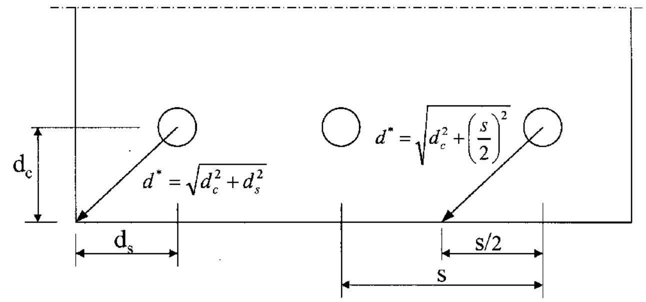

Fig. 4—Controlling cover distance.

Table 2—Bottom face crack width test data

Investigator	Reinforcement stress levels, ksi	No. of specimens	No. of observations	
			Average crack width	Maximum crack width
Clark	20, 25, 30, 35, 40, 45	54	287	287
Hognestad	20, 30, 40, 50	8	32	32
Kaar-Hognestad	40	8	0	7
Kaar-Mattock	40	13	13	13
	Total	83	332	339

$$S_c = \Psi_s d^* \quad (5)$$

where

S_c = crack spacing;

d^* = controlling cover distance; and

Ψ_s = crack spacing factor: 1.0 for minimum crack spacing; 1.5 for average crack spacing; and 2.0 for maximum crack spacing

Based on these studies, it is evident that the point on the beam face located furthest from the reinforcing steel controls the crack spacing and the resulting crack width. Therefore, two cases can control the crack spacing: the distance determined by the spacing of the reinforcement, or the distance determined by the side cover, as illustrated in Fig. 4. By using the appropriate crack spacing factor Ψ_s, the theoretical minimum, average, or maximum crack widths can be calculated. Typically, the maximum crack width is of interest.

The crack spacing discussed up to this point applies only to the use of uncoated reinforcement. Since epoxy coating affects bond, it is reasonable to expect that the coating will affect the crack spacing. From tests conducted by Treece and Jirsa,[9] it was found that epoxy coating significantly increased the width and spacing of cracks with the average width of cracks increasing up to twice the width of cracks in specimens with uncoated bars. Based on those tests, the crack spacings calculated previously should be doubled to account for the effect of epoxy coating.

ANALYSIS OF MEASURED CRACK WIDTHS

Crack widths calculated based on the physical model presented previously were compared to test data from Hognestad,[3] Kaar and Mattock,[4] Kaar and Hognestad,[5] Clark,[6] and Sozen-Gamble.[10] Generally, this is the same data used in the Gergely-Lutz[2] study, except that the Rüsch-Rehm[7] data were not included since the reinforcing steel does not conform to U.S. reinforcement standards, and the goal of the current study is to compare with bars used in U.S. practice. Tests conducted by Sozen and Gamble were included since that test series considered cracking of beams with large size bars (No. 14 and 18) that were not considered in previous crack width analyses.

As in the Gergely-Lutz study, two separate cases were considered. Analyses were conducted for crack widths measured on the side face at the level of the steel and for crack widths measured on the bottom tensile face.

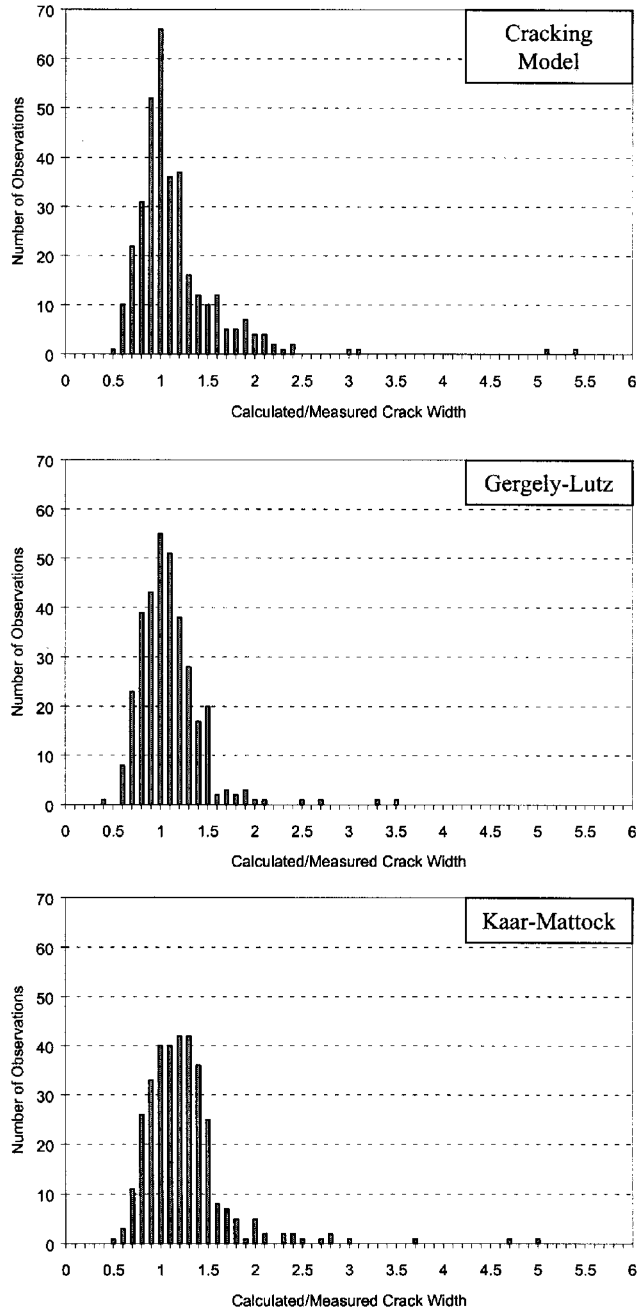

Fig. 5—Maximum bottom face crack width comparison.

Bottom face cracks

Crack widths on the bottom face were calculated according to the cracking model as well as the Gergely-Lutz and Kaar-Mattock equations. For the calculation of the crack spacing, the controlling cover dimension d^* was determined by the larger of either the side cover d_s or the bar spacing $s/2$, as previously illustrated in Fig. 4. The factor β was calculated based on the elastic, cracked sectional properties.

The test data included in the evaluation of bottom face cracking are listed in Table 2. The reinforcement stress levels, as well as the number of observations for both average and maximum crack widths from each investigator, are provided.

The calculated crack widths were divided by the measured widths to evaluate the accuracy of the various calculation methods. The results are presented in Fig. 5, in which the results of

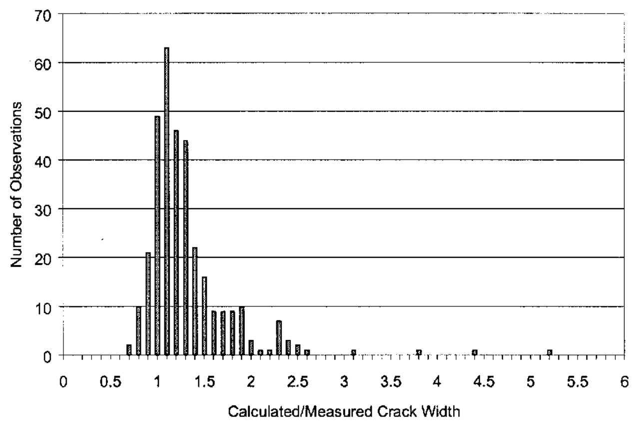

Fig. 6—Average bottom face crack width (cracking model).

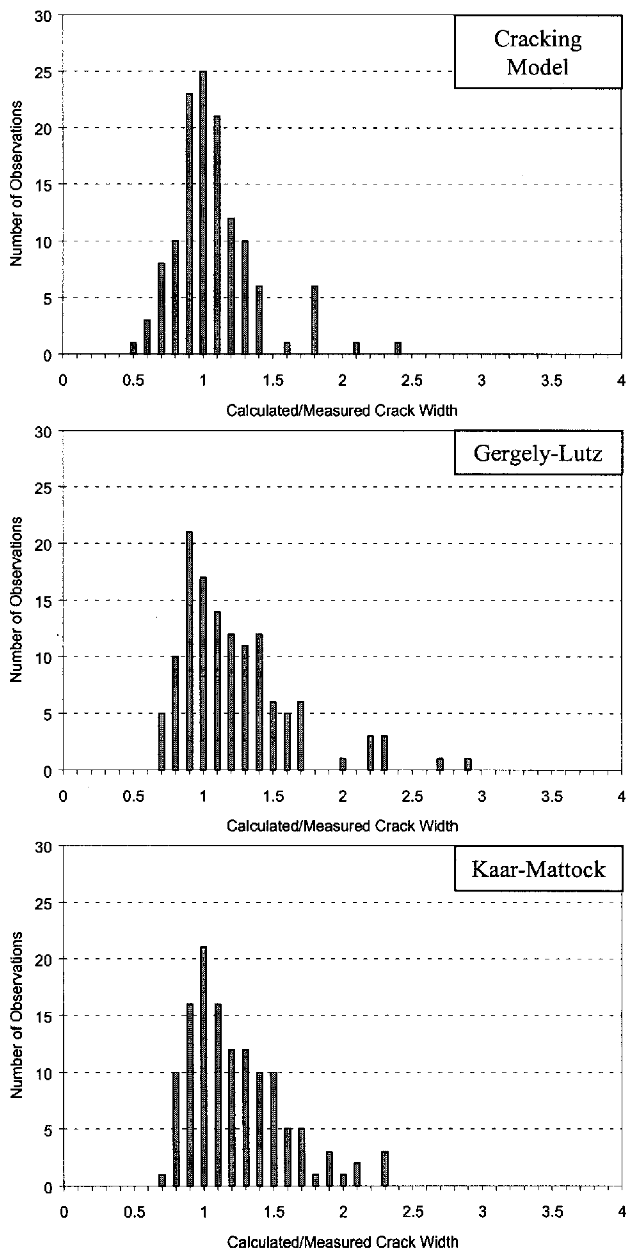

Fig. 7—Maximum side face crack width comparison.

the three different methods are compared for maximum crack widths. It can be seen that all three equations provide reasonable results, with no one method providing more accurate results than the others. The crack width model provides a fairly accurate procedure, considering the scatter inherent in crack widths.

Fig. 6 presents a histogram for the average crack width data to illustrate the flexibility provided by the physical model. Comparisons with the other crack width equations were not possible since both equations were developed specifically for maximum crack widths. It can be seen that the model also provides an effective method for the computation of average crack widths.

Side face cracks

Crack widths on the side face at the level of the reinforcement were calculated according to the cracking model as well as the Gergely-Lutz and Kaar-Mattock equations. For the calculation of the crack spacing, the controlling cover dimension d^* was determined by side cover d_s, as previously illustrated in Fig. 4. A value of 1.0 was used for the strain gradient factor β, since the crack widths were measured at the level of the reinforcement.

The test data included in the evaluation of side face cracking are listed in Table 3. The reinforcement stress levels, as well as the number of observations for both average and maximum crack widths from each investigator, are provided.

Table 3—Bottom face crack width test data

Investigator	Reinforcement stress levels, ksi	No. of specimens	No. of observations	
			Average crack width	Maximum crack width
Hognestad	20, 30, 40, 50	29	109	109
Kaar-Mattock	40	13	13	13
Sozen-Gamble	40	6	6	6
	Total	48	128	128

The calculated crack widths were divided by the measured widths to evaluate the accuracy of the various calculation methods. The results, presented in Fig. 7, include a comparison of the different calculation methods for maximum crack widths. While again the three methods reasonably determine the crack width, it appears that the crack width model provides the best estimate of the crack width.

Fig. 8 presents a histogram for the average crack width data. As mentioned previously, comparisons with the other crack width equations were not possible, since both equations were developed specifically for maximum crack widths. It can be seen that the cracking model can also be used effectively to compute average crack widths for side face cracking.

CRACK CONTROL

The crack width model clearly illustrates that the crack spacing and width are functions of the distance between the reinforcing steel. Therefore, crack control can be achieved by limiting the spacing of the reinforcing steel. Maximum bar spacings can be determined by limiting the crack widths to acceptable limits.

Based on the physical model presented, the equation for the calculation of maximum crack width for uncoated reinforcement is as follows

$$w_c = 2\frac{f_s}{E_s}\beta\sqrt{d_c^2 + \left(\frac{s}{2}\right)^2} \qquad (6)$$

For epoxy coated reinforcement, the equation should be multiplied by a factor of 2. The equation can be rearranged to solve for the permissible bar spacing, s

$$s = 2\sqrt{\left(\frac{w_c E_s}{2 f_s \beta}\right)^2 - d_c^2} \qquad (7)$$

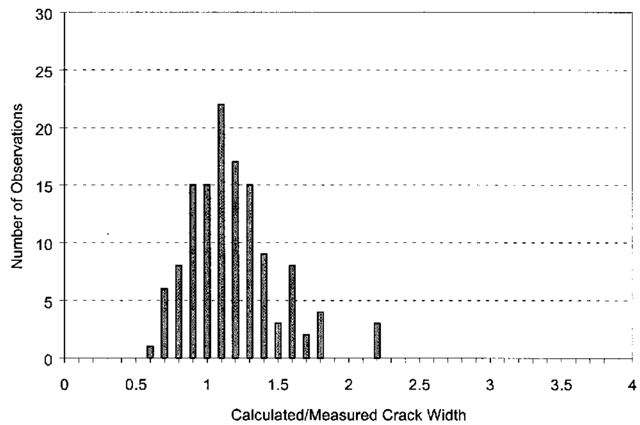

Fig. 8—Average side face crack width (cracking model).

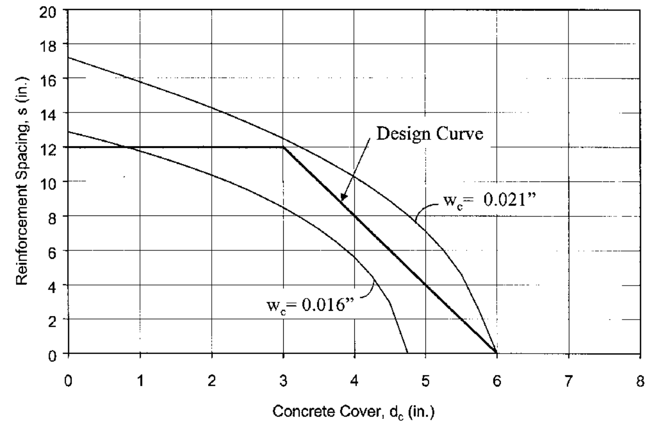

Fig. 9—Grade 60 reinforcement spacing.

where

s = maximum permissible bar spacing, in.;
w_c = limiting crack width, in.;
E_s = 29,000 ksi;
f_s = 0.6 f_y, ksi;
β = 1.0 + 0.08d_c; and
d_c = bottom cover measured from center of lowest bar, in.

A reinforcement stress of 60 percent of yield was selected to account for the service stress and corresponds to that recommended by ACI 318-95.[11] The factor β varies as the cover increases. Therefore, based on a review of sections with varying cover, the previous equation was developed.

According to Gergely,[12] "the only real reason to limit surface cracks in most structures is appearance." Furthermore, "investigations have concluded that the total amount of corrosion is influenced very little by the crack width and even whether transverse cracks are present or not. Thus the limitation of crack width for corrosion protection is unnecessary and can even be counterproductive if it is achieved by a decrease of cover." Other discussions[13,14] have supported the viewpoint that a correlation between corrosion and surface crack width does not exist. Therefore, the limiting crack width was selected as 0.016 in., which is based on ACI 318-95[11] design recommendations for interior exposure conditions. A 1/3 increase in crack widths (0.021 in.) was considered acceptable, considering the large scatter that is inherent in crack widths and that crack control is primarily an aesthetic consideration. Based upon these values, the graph shown in Fig. 9 was developed for Grade 60 reinforcement stressed at 36 ksi (0.6 f_y). Both limiting crack widths are presented.

For design purposes, a simplified design curve is illustrated. As shown in the graph, for Grade 60 reinforcement, a maximum bar spacing of 12 in. should be used for concrete covers up to 3 in. thick. As the thickness of the cover is increased beyond 3 in., there is a decrease in the permissible bar spacing. This approach simplifies design since, for most typical cases (d_c < 3 in.), crack control is accomplished by limiting the bar spacing to 12 in. Only in cases where the covers are thicker would it be necessary to consider a reduction in the maximum bar spacing.

A similar curve is presented in Fig. 10 for Grade 75 reinforcement. In that case, the maximum bar spacing is decreased to 9.6 in. due to the increased working stress of the steel at service levels.

CONCLUSIONS

To control unsightly cracking in elements with a cover of 2.5 in. or more, the currently used expressions are at a disadvantage because they are based strictly on statistical reasoning and limited beyond 2.5 in. This paper presents a new formulation of the equation for calculating crack width that is based on the physical phenomenon. Use of this equation is supported by an evaluation of existing test data. The equation is used to develop a pragmatic solution for the control of cracking in reinforced concrete structures.

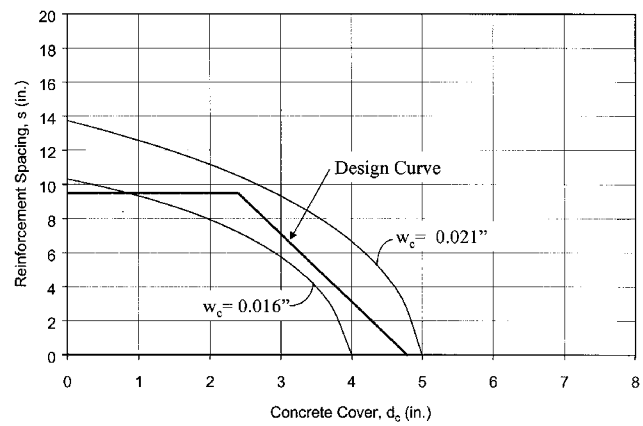

Fig. 10—Grade 75 reinforcement spacing.

Design recommendation

Based on the physical model, the following design recommendation is presented that addresses the use of both coated and uncoated reinforcement. The design curves are illustrated for Grade 60 and 75 reinforcement in Fig. 9 and 10, respectively.

The maximum spacing of reinforcement shall be given by

$$s = 12\alpha \left[2 - \frac{d_c}{3\alpha_s}\right] \le 12\alpha_s \qquad (8)$$

where

$$\alpha_s = \frac{36}{f_s}\gamma_c$$

d_c = thickness of concrete cover measured from extreme tension fiber to center of bar or wire located closest thereto, in.;
s = maximum spacing of reinforcement, in.;
α_s = reinforcement factor; and
γ_c = reinforcement coating factor: 1.0 for uncoated reinforcement; 0.5 for epoxy-coated reinforcement, unless test data can justify a higher value.

Calculated stress in reinforcement at service load f_s (kips/in.^{2}) shall be computed as the moment divided by the product of steel area and internal moment arm. It shall be permitted to take f_s as 60 percent of specified yield strength f_y.

CONVERSION FACTORS

1 in. = 25.4 mm
1 kip = 4.448 kN
1 ksi = 6.895 MPa

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Farmington Hills, Mich., 1971, 78 pp.

2. Gergely, P., and Lutz, L. A., "Maximum Crack Width in Reinforced Concrete Flexural Members," *Causes, Mechanism, and Control of Cracking in Concrete*, SP-20, American Concrete Institute, Farmington Hills, Mich., 1968, pp. 87-117.

3. Hognestad, E., "High-Strength Bars as Concrete Reinforcement—Part 2: Control of Flexural Cracking," *Journal*, PCA Research and Development Laboratories, V. 4, No. 1, Jan. 1962, pp. 46-63.

4. Kaar, P. H., and Mattock, A. H., "High-Strength Bars as Concrete Reinforcement—Part 4: Control of Cracking," *Journal*, PCA Research and Development Laboratories, V. 5, No. 1, Jan. 1963, pp. 15-38.

5. Kaar, P. H., and Hognestad, E., "High-Strength Bars as Concrete Reinforcement—Part 7: Control of Cracking in T-Beam Flanges," *Journal*, PCA Research and Development Laboratories, V. 7, No. 1, Jan. 1965, pp. 42-53.

6. Clark, A. P., "Cracking in Reinforced Concrete Flexural Members," ACI JOURNAL, *Proceedings* V. 27, No. 8, Apr. 1956, pp. 851-862.

7. Rüsch, H., and Rehm, G., "Versuche mit Betonformstahlen," Deutscher Ausschuss für Stahlbeton, *Bulletins*, No. 140, Part 1, 1963; No. 160, Part 2, 1963; and No. 165, Part 3, 1964. (in German)

8. Broms, B. B., "Crack Width and Crack Spacing in Reinforced Concrete Members," ACI JOURNAL, *Proceedings* V. 62, No. 10, Oct. 1965, pp. 1237-1255.

9. Treece, R. A., and Jirsa, J. O., "Bond Strength of Epoxy-Coated Reinforcing Bars," PMFSEL *Report* No. 87-1, University of Texas at Austin, Jan. 1987, 85 pp. See also *ACI Materials Journal*, V. 86, No. 2, Mar.-Apr. 1989, pp. 167-174.

10. Sozen, M. A., and Gamble, W. L., "Strength and Cracking Characteristics of Beams with No. 14 and No. 18 Bars Spliced with Mechanical Splices," ACI JOURNAL, *Proceedings* V. 66, No. 12, Dec. 1969, pp. 949-956.

11. ACI Committee 318, "Building Code Requirements for Structural Concrete (ACI 318-95) and Commentary (ACI 318R-95)," American Concrete Institute, Farmington Hills, Mich., 1995, 369 pp.

12. Gergely, P., "Role of Cover and Bar Spacing in Reinforced Concrete," *Significant Developments in Engineering Practice and Research: A Tribute to Chester P. Siess*, SP-75, American Concrete Institute, Farmington Hills, Mich., 1981, pp. 133-147.

13. Darwin, D. et al., "Debate: Crack Width, Cover, and Corrosion," *Concrete International*, V. 8, No. 5, May 1985, pp. 20-35.

14. Oesterle, R. G., "The Role of Concrete Cover in Crack Control Criteria and Corrosion Protection," PCA R&D Serial No. 2054, Portland Cement Association, 1997.