SP 42-34

Shear Strength Of Slabs With Shear Reinforcement

By
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Synopsis: A review is made of available data concerning the shearing strength of slabs with shear reinforcement. Results are considered for specimens both with and without a transfer of moments simultaneously with a transfer of shear through the loaded area. Data from tests on slab-column specimens for which the shear reinforcement has consisted of structural steel sections, bent bars, stirrups and prefabricated wire cages are reviewed, and the findings contrasted with the provisions of ACI Code 318-71.

Keywords: bending moments; bent-up bars; building codes; columns (supports); concrete slabs; flat concrete plates; flat concrete slabs; reinforced concrete; reinforcing steels; reviews; shear strength; shear tests; stirrups; stress transfer; structural design.

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INTRODUCTION

The strength and ductility inherent in a slab-column connection may be inadequate even though the column has an adequate strength and the slab an adequate depth. Shear reinforcement is needed if the span length of the slab is not to be reduced or the column size increased. Many types of shear reinforcement have been used successfully for slab-column connections. Those reported in the literature are illustrated in Fig. 1. They can be divided into two basic types, shearhead reinforcement, (a) through (d) in Fig. 1, and bent bar and stirrup reinforcement, (e) through (q) in Fig. 1. Although shearhead reinforcement consisting of crossing structural steel shapes was first introduced by Wheeler (1) in 1930, design provisions for that reinforcement were not incorporated in the ACI Code (10) until 1971. Bent bars and stirrups have been used in practice for a similar length of time. However, firm rules for their design were not proposed until the Committee 326 report (2) was published in 1962. Then, because the available test data were limited, Committee 326 recommended that such shear reinforcement should be considered ineffective in slabs less than 10 inches thick and only 50 percent effective in slabs greater than 10 inches thick. Since the Committee 326 report it has been clearly established that for any thickness of slab, the strength and ductility of the connection can be increased by the use of well anchored shear reinforcement. That concept is incorporated in ACI 318-71.

The object of this paper is to critically review available data concerning the shearing strength of slabs with shear reinforcement and where appropriate to document the bases for some of the design provisions of ACI 318-71. While this discussion is concerned primarily with the proportioning of the shear reinforcement for strength and ductility, practical considerations may well dictate the type of reinforcement. The use of structural sections may be undesirable if, as a consequence, additional trades are required on the job site. Bent bars may be undesirable if they cause difficulties for passage of the column bars through the connection. Stirrups may be undesirable if they cannot be readily positioned after most of the slab reinforcement is in place.

CONNECTIONS TRANSFERRING SHEAR ONLY

Shearhead Reinforcement

As indicated in Fig. 1, steel plates, collars and structural shapes have been used to increase the strength of slab-column connections. Moe (3) tested three 6-inch (15 cm.) thick slabs in which a 3/4 inch (1.9 cm.) thick steel plate was placed over the column and even with the compression surface of the slab as shown in Fig. 1(a). These plates were intended to increase the effective size of the column. With an overhang of only one inch (2.5 cm.), shears were concentrated at the corners of the plates, and the ultimate capacities differed little from the capacities for similar specimens without plates. With an overhang of 2 inches (5 cm.), the stiffness of the plate was more evenly matched with that of the slab, and the ultimate capacity was only slightly less than that for a column equal in size to the plate. Reference 4 discusses the bearing strength of concrete loaded through steel plates. Plates are characterized as flexible, semi-flexible or rigid according to whether the whole perimeter of the plate extending beyond the loaded area, only the corners of the plate, or none of the plate lifts free of the concrete before failure. For a maximum increase in the effective size of the column, the plate should be as rigid as possible without the generation of corner effects. Therefore, its characteristics should place it in the semi-flexible range close to the rigid range. It is shown in Reference 4 that the plate thickness is balanced between that for rigid and semi-flexible conditions (4) if

$$t^2 = \frac{4f'_c}{f_y} \frac{\left[c(a-c) + \frac{\pi}{8}(a-c)^2\right]}{2\pi + \frac{8c}{a-c}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad [1]$$

where t = plate thickness

f_y = yield strength of plate

a = side length of plate

Equation 1 predicts correctly the response of the plates in Moe's tests (3) if the yield strength for those plates was about 40,000 psi. (2810 kgf/cm^2).

Corley and Hawkins (5, 6) have reported tests on sixteen 5-3/4 inch (14.6 cm.) thick slabs reinforced with crossing I and Channel shapes as shown in Fig. 1(b) and 1(c). Provided the plastic moment capacity of the shearhead was adequate, these shapes increased the shear capacity in proportion to the projection of the shearhead arm. The increase was at a rate slightly less than that expected for an increase in column size equal to the projection of the shearhead arm. Therefore, when they proposed the design procedure incorporated in ACI 318-71, they specified that critical sections should be taken as shown in Fig. 2 and the shear stress on that section limited to that for a slab without a shearhead.

Strain measurements showed that the steel section and the slab behaved compositely until inclined cracking spread to the end of the shearhead arms at about half the failure load. Then all increases in shear were carried by the shearhead, and these increases were concentrated at the column face. The corresponding distribution of shear in the shearhead at failure is shown in Fig. 3. The moment, M_s, at the column face is given by

$$M_s = \frac{V_u}{8}\left[h_v + \alpha_v \left(L_v - \frac{c}{2}\right)\right] \cdots\cdots\cdots\cdots\cdots\cdots [2]$$

where c = side dimension of square column

h_v = depth of steel shape in shearhead

α_v = relative stiffness of the shearhead to that of a composite section made up of a cracked section of the slab with a width equal to that of the column plus the effective depth of the slab.

V_u = total ultimate shear = $2V_c$

L_v = length of shearhead reinforcement measured from center of column

When the moment M_s was less than M_p, the plastic moment capacity of the shearhead arm, the failure surface was forced to the end of the shearhead arm. When M_s was greater than M_p, the failure surface fell inside the end of the shearhead arm. In the latter case, the connection usually showed some ductility prior to the punching failure while in the former case the characteristics were those of the traditional shear failure.

In ACI 318-71 the M_p value required for the shearhead is the M_s value given by Eq. 2. Since the critical section used to calculate V_u does not extend to the ends of the shearhead arm, the code provisions probably result in limited ductility prior to collapse, provided M_p does not exceed the M_s value for a critical section passing through the ends of the shearhead arm. Alternatively, ductility prior to failure can be ensured by making the length of the shearhead arm greater than the length used in the initial design.

For the shearhead to respond in accordance with the concept shown in Fig. 3, the compression flanges of the steel section should be anchored within the compression zone of the slab. ACI 318-71 attempts to ensure that condition by requiring that the compression flange be within 0.3d of the compression surface of the concrete slab. Because of this restriction, it is inappropriate to use large diameter reinforcing bars, such as No. 18 bars, as shearhead reinforcement.

In Reference 6 Corley and Hawkins have noted that the shear carried by the concrete at the column perimeter under ultimate load conditions should be limited to the shear capacity for a specimen without a shearhead. Thus, V_u should not exceed the value given by Eq. 3.

$$V_u \leq 4.8 f_{ct} (c + d)d/(1 - \alpha_v) \leq 32 \sqrt{f'_c} (c + d)/(1 - \alpha_v) \cdots\cdots [3]$$

Since ACI 318-71 restricts the maximum increase in shear strength with shearhead reinforcement to 75 percent, the condition expressed by Eq. 3 governs only if α_v is less than 0.33.

Corley and Hawkins compared the strengths predicted by the code procedure (5) with their test results and those of Swedish (23) and Australian (8) tests on collars for lift slabs. Test strengths averaged 15 percent greater than the calculated strengths and the least conservative result was for a lift slab collar where the measured strength was 85 percent of the calculated strength.

In his investigation of embedded service ducts, Hanson (9) tested three 8-inch (20.3 cm.) thick, lightweight concrete slabs. Two of the specimens contained shearheads, and in one of the specimens ducts were located three inches from the end of the shearhead arm. The specimen with a shearhead and no ducts carried a load 9 percent less than that predicted by ACI 318-71. The specimen without shearhead or ducts carried a load 6 percent less than that predicted by ACI 318-71. The specimen with a shearhead and ducts had a capacity 15 percent less than that of the specimen without a shearhead.

More information is needed on the performance of shearheads in full scale structures, on the upper and lower bounds to the stiffness of the shearhead for that stiffness to be compatible with the stiffness of the slab, on the necessity for anchoring the compression flange of the shearhead in the compression zone of the slab, on appropriate methods for proportioning shearheads to ensure ductility, and on the effect of holes around columns containing shearheads.

Bent Bars and Stirrups

General considerations --There are six major considerations for detailing this type of shear reinforcement. First, the strength must be checked for every potential failure plane adjacent to the column on which the shear stress exceeds the nominal shear strength of the concrete. Second, effective anchorage must be provided for the shear reinforcement. This anchorage is difficult to achieve in a thin slab. Third, shear reinforcement must be provided to take all shears in excess of those for inclined cracking. Inclined cracking occurs within a slab at about half the ultimate load for a connection without shear reinforcement. Fourth, in order to ensure ductility under overloads the shear reinforcement should be extended at least twice the slab thickness beyond the distance at which it is no longer theoretically needed based on strength considerations. Fifth, careful attention should be given to the layout of the slab, shear and column reinforcement so that placement difficulties and possibilities for error are minimized. Because the slab section is relatively thin, small errors in the placement of the shear reinforcement may largely nullify its effectiveness. Sixth, unless ductility is mandatory, the increased labor costs resulting from the provision of shear reinforcement must be balanced against the cost of providing a similar increase in shear strength by increasing the thickness of the slab or the size of the column.

Test data--Test data for specimens with bent bars and stirrups are summarized in Table 1. The code for the different types of shear reinforcement as listed in Column 2 is indicated in Fig. 1.

Tests on specimens with prefabricated shearhead cages, Figs. 1(e) and 1(f), have shown that while a properly proportioned cage can increase the shear strength, several concentric cages are necessary to markedly improve the ductility of the connection. Keefe (11) tested two pairs of 5-inch (12.7 cm) thick, octagonally shaped, 37 in (94 cm) span, slabs containing 2.5 percent reinforcement and centrally loaded through a 6-inch (15 cm) diameter plate. One pair contained no shear reinforcement while the other pair contained a circular cage consisting of two rings of overlapping shear reinforcement similar to the type shown in Fig. 1(e). The diameter to the edge of the outer ring of shear reinforcement was 25 inches (63 cm). The cage increased the shear capacity by 33 percent. Moe (3) tested a 6-inch (15 cm) thick, 72 in (183 cm) span, slab with one percent reinforcement centrally loaded through a 8-inch (20.3 cm) square column. The slab contained a square cage of the type shown in Fig. 1(e) with an outside dimension of 18 inches (45.8 cm). The cage increased the shear capacity by approximately 7 percent compared to the capacity for a duplicate specimen without a cage. Tasker and Wyatt (8) tested six 4-inch (10.3 cm) thick, 5 ft. (152 cm) span, octagonally shaped slabs with 1.1 percent reinforcement. The slabs were centrally supported on a 10 in (25 cm) square lift slab collar. They contained square cages of the type shown in Fig. 1(e) with outside dimensions varying between 13 and 25 inches (33 and 63 cm) and inclinations for the reinforcement between 30 and 60 degrees. The specimens with cages failed at loads averaging 15 percent higher than those for similar specimens without cages. Wantur (12) tested eight 4.7 in (12 cm) thick, 55 in (140 cm) span slabs containing 0.98 or 1.23 percent reinforcement. The slabs were centrally loaded through a 7.1 in (18 cm) diameter column. Two of these slabs contained octagonally shaped cages of the type shown in Fig. 1(f) with a diameter of about 14 inches (135.6 cm). The shear capacities for these specimens averaged 14 percent higher than those for similar specimens without cages.

Tests on specimens with shear reinforcement consisting of bent-up bars have been reported in References 7, 13, 14, 15 and 16 and with reinforcement consisting of stirrups in References 7, 12, 14, 17 and 18.

Graf (13) tested eight thick slabs with depths of 11.8 or 19.7 inches (30 or 50 cm), spans of 59 inches (150 cm) and reinforcement ratios ranging between 0.33 and 1.01 percent. The slabs contained bent bars located in two rings around the central column and anchored as shown in Fig. 1(g). The reinforcement increased the shear strength substantially compared to that for the specimens without bent bars.

Elstner and Hognestad (14) tested nine 6-inch (15 cm) thick, 6 ft. (1.8 m) square slabs centrally loaded through a 10-inch (25.4 cm) square column stub. Reinforcement ratios ranged from 0.99 to 3.00 percent. Eight of these slabs contained differing percentages of bent bar reinforcement detailed in a manner similar to that shown in Fig. 1(h) and one contained two rows of open stirrups of the type shown in Fig. 1(m). The bend in all the bars was located directly under the column except in one specimen in which two rings of bent bars were used. Inspections of the slabs after failure indicated that collapse may have been caused by local crushing of the concrete under the bends in the bars or by insufficient anchorage length for the bars. The following equation gave the best fit to their and Graf's data:

$$v_u = 333 \text{ psi} + 0.046\, f'_c/\phi_o + (q_u - 0.050)\, f'_c \quad \cdots\cdots\cdots \quad [4]$$

where

$$q_u = \frac{A_v f_y \sin\alpha}{7/8\ bd\ f'_c} \quad \cdots\cdots\cdots\cdots\cdots\cdots \quad [5]$$

A_v = area of shear reinforcement

f_y = yield point of shear reinforcement

α = inclination of shear reinforcement to horizontal

Except for the third term, Eq. 4 is the same as their expression for the shear strength of slabs without reinforcement (24). The third term indicates that shear reinforcement was not fully effective in their slabs.

Rosenthal (15) tested four 3.9 inch (10 cm) thick slabs containing 1.05 or 1.25 percent reinforcement. Bars were bent up at two different distances from the columns and anchored as shown in Fig. 1(g). He found that the capacity increased as the amount of shear reinforcement increased.

Andersson (7) tested twenty 5.9 inch (15 cm) thick, 67.4 inch (171 cm) span slabs with reinforcement ratios varying between 0.77 and 1.09 percent. His circular slabs were centrally loaded through 5.9 or 11.8 inch (15 or 30 cm) diameter columns and they contained the four different types of shear reinforcement indicated in Fig. 1(k), (1), (m) and (n). In the slabs with the heaviest amounts of shear reinforcement, the failure cracks occurred outside the shear reinforcement. Of the four types of shear reinforcement tested, the most efficient were bent up radial bars, Fig. 1(k), and continuous vertical tangential stirrups, Fig. 1(m). The shear reinforcement increased appreciably the ultimate deflection of the slabs compared to similar specimens without shear reinforcement. The amount of increase in deflection increased in proportion to the area of the slab provided with shear reinforcement. In the same report results were also given of tests on two identical slabs by Andersson and Nylander. The 4-1/2 inch (11.4 cm) thick slabs contained bent-up radial bars and 1.8 percent reinforcement. Their behavior was similar to that of the slabs tested by Andersson.

Yitzhaki (16) tested eleven 4-inch (10 cm) thick, 67.2 in (171 cm) span circular slabs with reinforcement ratios varying between 0.39 and 1.27 percent. Shear reinforcement consisted of two rings of bent up two-way reinforcement anchored as shown in Fig. 1(a) but with the first bend located d/2 outside the column perimeter. The measured capacities equalled or exceeded V_{flex} in most cases, and in one case V_{test} was 1.4 times V_{flex}. Most of the slabs showed considerable ductility at collapse and the ultimate deflections were several times greater than those for similar specimens without bent up bars.

Franz (17, 19) tested sixteen 5.5-inch (14 cm) thick, 66-in (168 cm) diameter centrally loaded slabs with shear reinforcement consisting of bent bars detailed as shown in Fig. 1(h) for two of the slabs, stirrups detailed as shown in Fig. 1(o), mostly positioned in the radial direction, and completely enclosing the tension and compression steel for ten of the slabs, U-shaped radial stirrups, Fig. 1(p), for one of the slabs, and a combination of radial stirrups, Fig. 1(o), and column reinforcement anchored in the plane of the slab for two of the specimens. In one series of tests, the reinforcement ratio was held constant at 1.5 percent, and the amount and type of shear reinforcement varied. In a second series of tests, the shear reinforcement consisted of closed stirrups, Fig. 1(o), and the amount of slab reinforcement was varied from 0.47 to 1.15 percent. The most efficient type of shear reinforcement was the closed stirrups. The same amount of reinforcement provided by bent bars increased the shear strength only half as much while the U-shaped stirrups and bent column reinforcement were ineffective.

Wantur (12) tested four slabs, 4.7 inch (12 cm) thick, containing radial and tangential stirrups detailed as shown in Fig. 1(o) and either 0.98 or 1.23 percent slab reinforcement. Although the stirrups completely enclosed the slab reinforcement, they were located 1.1d from the column perimeter for 0.98 percent reinforcement and about 0.9d from the column for 1.23 percent reinforcement. The stirrups increased the shear strength about eight percent in the first case and about 14 percent in the second case compared to the strengths for duplicate specimens without shear reinforcement.

During an investigation of possible means for strengthening slab-column connections subjected to seismic loadings, Carpenter, et al. (18) tested a 7-1/2 inch (19 cm) thick 13 by 19 ft (3.96 by 5.79 m) slab with a central 18 inch (46 cm) square column. The slab was subjected to line loads applied along the length of the 13 ft (3.96 m) sides. In the column region the slab was reinforced with closed No. 3 stirrups at a spacing of d/2 along the arms of cross-beams extending out into the slab from each column face. The stirrups were detailed as shown in Fig. 1(q) except that overlapping stirrups were used to completely enclose four adjacent bars from the tension and compression mats. This specimen, which was well over-designed for shear strength, was intact after five cycles of reversed, moment transfer only, loadings into the general yield range. It was then subject to shear transfer only loading, and a failure occurred in flexure due to crushing across the full width of the slab when the nominal shear stress on a critical section at d/2 from the column was 4.3 $\sqrt{f'_c}$. The theoretical contribution to the shear strength possible through yielding of the stirrup reinforcement within a distance d from the columns was 3.5 $\sqrt{f'_c}$.

Design provisions--In the 1963 ACI Code shear reinforcement was considered to be ineffective in slabs less than 10 inches (25 cm) thick, and only 50 percent effective in slabs more than 10 inches (25 cm) thick. Following the discussion by Carpenter, et al. (18), the proposed ACI 318-71 was changed so that properly anchored shear reinforcement was assumed fully effective in a slab of any thickness. When the nominal ultimate shear stress exceeds 4 $\sqrt{f'_c}$ the shear reinforcement is proportioned to carry the stress in excess of 2 $\sqrt{f'_c}$. The ultimate shear strength with shear reinforcement is limited to 6 $\sqrt{f'_c}$ and the maximum spacing between stirrups to d/2.

In the draft British Code (20) properly anchored shear reinforcement is considered effective only in slabs 8 inches (20 cm) or more thick, and this reinforcement is designed for the difference in stress between the maximum shear stress and that at inclined cracking. Shear stresses are calculated on a critical perimeter 1.5d from the loaded area, and the spacing between stirrups is limited to d.

In the CEB/FIP Recommendations (21) the ultimate load for a connection without shear reinforcement is taken as a limiting shear stress similar to that in the ACI Code times the area of the same critical section as that specified in the ACI Code. Where shear reinforcement is needed, it must be designed for at least 75 percent of the concentrated load on the slab. This reinforcement is to be placed within an area extending out about 1.5d from the column perimeter. The maximum spacing of vertical stirrups is limited to 0.75d. Bars are to be bent down within a distance 0.5d from the column and at an angle not less than 30° to the horizontal. In order to ensure adequate anchorage, the shear reinforcement must completely surround the horizontal tensile reinforcement.

Evaluation of data for bent bars and stirrups--The observations by Andersson (7) and by Carpenter, et al. (18) and the results for shearheads reported by Corley and Hawkins (5) suggest that the ultimate shear strength can be computed by assuming that the shear reinforcement carries all shear after inclined cracking. Thus, failure occurs when this reinforcement yields or the ultimate shear strength is reached in the area of the slab outside that reinforcement. Ultimate strengths predicted by this concept are compared with the available test results in Table 1.

The ratio ϕ_o, equal to V_{test} divided by V_{flex}, is listed in Column 6 of Table 1. V_{flex} is the flexural capacity calculated from yield-line theory. Results for Case 1 of Fig. 7 in Reference 24 were used for square columns and results for Case 2(a) for round columns. The theoretical shear capacity V_u, neglecting the contribution of the shear reinforcement, is listed in Column 7. V_u was taken as the greater of the values predicted by Eqs. 6 and 7. Equation 6 was developed by Moe (3) and is strictly applicable only for ϕ_o values less than unity. Equation 7 was developed by ACI-ASCE Committee 326 (2) and was intended to predict the shear strength for a ϕ_o value of unity or larger

$$V_u = [15\ (1 - 0.075\ c/d) - 5.25\ \phi_o]\ bd\ \sqrt{f'_c}\ \cdots\cdots\ [6]$$

or

$$V_u = 16d^2\ [c/d + 1]\ \sqrt{f'_c}\ \cdots\cdots\cdots\cdots\ [7]$$

V_s in Column 8 is the shear carried by the shear reinforcement if yielding occurs for all reinforcement crossing a potential inclined crack emanating at 45° from the junction of the compression surface of the slab and the perimeter of the column, or for Tasker and Wyatt's specimens, the lift slab collar. One exception is Wantur's slabs where yielding was assumed for the shear reinforcement located 1.1d from the column. Column 9 lists the theoretical shear capacity, V_{calc}, which is the greater of two quantities as indicated by

$$V_{calc} = \frac{V_u}{2} + V_s \geq V_u\ \cdots\cdots\cdots\cdots\cdots\ [8]$$

V_u is calculated from Eq. 6 or 7 as appropriate.

The measured capacity V_{test} is listed in Column 10, and the ratio of that capacity to V_{calc} is listed in Column 11. Column 12 lists the ratio of V_{test} to V'_{calc} where V'_{calc} is the capacity corresponding to a shear stress of $4\ \sqrt{f'_c}$ on a critical section located d/2 outside the outermost ring of stirrup reinforcement or the outermost bend in the bent bars. V'_{calc} is given by

$$V'_{calc} = 16d^2\ [c'/d + 1]\ \sqrt{f'_c}\ \cdots\cdots\cdots\cdots\ [9]$$

where c' = diameter or side length to outermost ring of stirrup reinforcement or the outermost bend in the bent bars.

For two slabs, I-6a and I-6b, tested by Yitzhaki there were two rings of bent bars, and the capacity corresponding to a shear failure between these two rings was less than that for a failure outside the outer ring. For these two slabs V'_{calc} is V_s for the outer ring of shear reinforcement plus half the capacity corresponding to shear stress of $4\ \sqrt{f'_c}$ on a section midway between the inner and outer rings. Where no ratios are listed in Column 12, the available data were insufficient for reliable calculations of V'_{calc}.

Examination of the values in Columns 7 through 10 shows that V_{test} exceeded V_u plus V_s only for slabs 4 and 5 tested by Franz. Radial stirrups were used in both these slabs, and a slightly flatter inclination for the inclined crack than the 45° used in these calculations results in a doubling of V_s. Therefore, it is apparent that even with adequate well-anchored shear reinforcement, V_u and V_s are not additive in their effects on the shear strength. Obviously considerable shear reinforcement is required before there is any significant increase in the shear strength of a slab. The limitation of the shear strength to the lesser of V_{calc} or V'_{calc} is a reasonable design approach. The ratios listed in Table 2 were obtained by selecting the larger of V_{test}/V_{calc} or V_{test}/V'_{calc} for each test specimen and averaging values for each investigation. The resultant grand average is 1.09. Shown in Fig. 4 is a comparison for all the test results of V_{test} to V_{pred} where V_{pred} is the lesser of the values of V_{calc} and V'_{calc}. In order to indicate the increase in shear strength due to shear reinforcement, values are normalized by dividing by V_{cr} where V_{cr} is taken as half V_u from column (7).

For Elstner and Hognestad's slabs, the average in Table 2 is 1.11, and for only one slab, B16, was the larger of the values for V_{test}/V_{calc} or V_{test}/V'_{calc} less than 1.00. For Graf's slabs the average is 1.12. For three slabs, ratios of V_{test}/V'_{calc} are not listed because a crack outside the outer ring of bent bars would have had to form over the exterior support. For Rosenthal's slabs the exact positions for the bends in the bars are not recorded, and it is likely that the number of bars effective for shear were less than that presumed in calculations of V_s. Nevertheless, the average in Table 2 is still greater than unity. For Andersson's slabs No. 62 through 65 and 75 through 81, the anchorage conditions for the bent bars were particularly poor since an inclined crack forming outside the position where the bent bar reached the compression face would parallel the anchorage for the bar. Destruction of the anchorage probably accounts for the low V_{test} values for those specimens. As explained previously, for many of Franz's slabs an inclined crack at 45° intersects only one leg of the radial stirrups. A flatter crack would significantly increase the amount of effective shear reinforcement. That behavior probably accounts for the higher average ratio for those slabs in Table 2. In contrast, for Wantur's specimens the location of the shear reinforcement at d or more from the column perimeter probably accounts for the lower average ratio in Table 2.

While the limitation of the shear strength to the lesser of the values obtained by Eqs. [8] and [9] gives good correlation with observed strengths, it does not provide any information on the potential ductilities which can be obtained with the various types of reinforcement. However, it is apparent from the observed behaviors that adequate anchorage for the shear reinforcement is essential if any ductility is to be obtained. The PCA tests (18) and the moment transfer tests conducted at the University of Washington (22) suggest that properly detailed hoops completely enclosing the tension and compression reinforcement of the slab are the best type of shear reinforcement since they also ensure adequate anchorage for the flexural reinforcement.

For slabs tested by Rosenthal and Yitzhaki, the increase in shear strength achieved through the use of shear reinforcement has exceeded three times the calculated strength for the same slab without reinforcement. In tests by other investigators strengths have frequently been doubled by the use of shear reinforcement. Therefore, provided the shear reinforcement is properly anchored, it is reasonable to permit in slabs increases in shear strength due to shear reinforcement of the same magnitude as those permitted for beams. That value is double the increase currently permitted by ACI 318-71. The spacing of stirrups should be limited to 0.5d.

CONNECTIONS TRANSFERRING SHEAR AND MOMENT

Data on the strength of connections with shear reinforcement transferring moment and shear are meager and are limited to information reported in References 6, 18 and 22. The work of Carpenter, et al. (18) in examining the strengthening of connections for seismic loadings has been discussed in the previous section. In addition to testing the specimen with stirrups in the column region they tested two specimens of similar proportions containing shearheads detailed in accordance with ACI 318-71 requirements. Moment-deflection relationships for slab 1 without any shear reinforcement, slab 3 with a shearhead and slab 5 with stirrups are shown in Fig. 5. While the shearhead permitted development of the flexural capacity of the specimen, it did not insure ductility. The first loading reversal after the initial loading into the inelastic range resulted in failure. In contrast, properly anchored stirrups insured both strength and ductility.

Hawkins and Corley (6) tested fourteen lightweight concrete specimens containing shearhead reinforcement and simulating conditions at an edge column in a flat plate without spandrel beams. The slab was 6-in (15 cm) thick and 4 ft by 9 ft (122 by 274 cm) with the column centered at the edge of the long side. Principal variables were the length and strength of the shearhead arms and the width of the column face transverse to the direction of moment transfer. The shearhead was effective in increasing the shear capacity at the transverse face in a manner similar to that for a shearhead in a connection transferring shear only. However, at the torsion faces of the column the shearhead was relatively ineffective. It altered the torsional capacity only in proportion to the effect of the shear strength on the torsion strength as indicated by the interaction equations of ACI 318-71. Consequently, the capacities predicted by combining the procedures in ACI 318-71 for moment transfer and shearheads resulted in non-conservative estimates of the measured shear strengths for long shearheads. Knowledge of these results was the principal reason for restricting the shearhead provisions of ACI 318-71 to shear transfer situations only. Hawkins and Corley have shown that their results can be accurately predicted using their beam analogy (25) or predicted conservatively to a degree which is independent of the shearhead length by using ACI 318-71 procedures and differing critical sections for shear and moment transfer. The nominal shear stress is limited to the customary value specified in ACI 318-71. However, that stress is calculated as the sum of the stress due to shear on the critical section for a shearhead plus the shear stress due to moment transfer on a critical section d/2 from the column perimeter.

Hawkins (22) tested six specimens simulating conditions at an interior column in a flat plate structure and containing stirrup reinforcement of the type shown in Fig. 1(q). The slab was 6-in (15 cm) thick and seven foot (213 cm) square and it was supported on 12-in square (30 cm) central column. Loadings resulted in initial M/V_c ratios of 0.5 or 2.0. Variations were made in the amount and spacing of the stirrup reinforcement and the percentage of reinforcement in the slab. With an adequate amount of properly anchored stirrups, the strength and ductility of the connection were substantially greater than that of a similar connection without stirrup reinforcement. As observed for connections transferring shear only, the stirrups carried all the shear after applied loads exceeded about half the failure load for a connection without stirrups. To insure effectiveness, the spacing between stirrups in the "integral beams" had to be limited to d/2, and the stirrups had to be detailed as shown in Fig. 1(o) or (q). Stirrup reinforcement in excess of that necessary to carry the ultimate loads had negligible effect on the performance of the connections. For slabs with adequate stirrup reinforcement plastic deformations resulted in large rigid body rotations of the connection. Cycling of the load between zero and that for plastic deformations resulted in a punching failure on the compression surface of the slab at the column junction in the second cycle. However, that failure did not have any marked effect on the subsequent stiffness and strength of the connection. This investigation did not define the distance that the "integral beams" had to extend into the slab in order to ensure ductile behavior. The minimum extension in those tests was 18-in (45.7 cm) beyond the column perimeter.

CONCLUSIONS

(1) Even for thin slabs, properly detailed shear reinforcement consisting of shearheads, bent bars, or stirrups can be fully effective in increasing the shear capacity. For bent bars or stirrups, considerable amounts of shear reinforcement are needed before the capacity is appreciably greater than that for a connection without shear reinforcement.

(2) For slabs with properly detailed bent bars or stirrups and connections transferring shear only, the shear capacity equals the lesser of the following strengths: (a) the strength for a slab without shear reinforcement for a critical section located d/2 beyond the end of the stirrups or the bend in the bent-up bars, or (b) half the capacity for a slab without shear reinforcement plus the vertical component of the yield strength of the shear reinforcement intersected

by a crack inclined at 45 degrees to the column-compression surface interface.

(3) The increases in shear stress possible with bent bars or stirrups need not be limited to 50 percent of the capacity of the connection without shear reinforcement as is currently required by ACI 318-71 provisions. A more realistic limit would be the 10 $\sqrt{f'_c}$ value currently used for beams.

(4) Shear reinforcement is fully effective only when it is properly detailed. Where shear reinforcement is needed it must extend to a distance of at least 1.5d from the column perimeter. Bars must be bent down within a distance 0.5d of the column at an angle not less than 30° to the horizontal. The maximum spacing between vertical stirrups should be 0.5d and every 45° line extending from the junction of the column and the compression surface of the slab to the mid-depth of the slab should be crossed by at least one stirrup for every 90 degree quadrant of the slab surrounding the connection. Stirrup reinforcement should completely enclose adjacent longitudinal tensile and compression bars of the slab and the ends of the stirrups should be terminated by bending them around the longitudinal bars through at least 145 degrees. Longitudinal compression reinforcement for the slab should be continuous through the region of the connection.

(5) For shearheads designed in accordance with ACI 318-71, failure can occur due to exhaustion of the strength of the concrete at the column face. For shearheads with α_v values less than 0.33, the increase in strength with shearhead reinforcement may be less than the 75 percent maximum permitted by ACI 318-71. Maximum increases should be limited by the provisions of Eq. 3.

(6) In connections transferring moment and shear, shearhead reinforcement is effective only for shear stresses caused by shear transfer. The sum of the stress caused by shear transfer on the critical section for the shearhead plus the maximum shear stress caused by moment transfer on a critical section d/2 from the column perimeter should be limited to 4 $\sqrt{f'_c}$ or the corresponding value for lightweight aggregate concrete.

REFERENCES

1. Wheeler, W. H., "Thin Flat-Slab Floors Prove Rigid Under Test," Engineering News-Record, Vol. 116, No. 2, January 9, 1936, pp. 49-50.

2. ACI-ASCE Committee 326, "Shear and Diagonal Tension," Proc. ACI, Vol. 59, January, February and March, 1962, pp. 1-30, 277-334 and 353-396.

3. Moe, J., "Shearing Strength of Reinforced Concrete Slabs and Footings Under Concentrated Loads," Development Department Bulletin D47, Portland Cement Association, April, 1961, 130 pp.

4. Hawkins, N. M., "The Bearing Strength of Concrete Loaded Through Flexible Plates," Magazine of Concrete Research, Vol. 20, No. 63, June, 1968, pp. 95-102.

5. Corley, W. G. and Hawkins, N. M., "Shearhead Reinforcement for Slabs," Proceedings, American Concrete Institute, Vol. 65, October, 1968, pp. 811-824.

6. Hawkins, N. M. and Corley, W. G., "Moment Transfer to Columns in Slabs with Shearhead Reinforcement," Shear Symposium Volume, American Concrete Institute, Detroit, Michigan, 1973.

7. Andersson, J. L., "Punching of Concrete Slabs with Shear Reinforcement," Transactions No. 212, 1963, Royal Institute of Technology, Stockholm, Sweden.

8. Tasker, H. E. and Wyatt, R. J., "Shear in Flat-Plate Construction Under Uniform Loading," Special Report No. 23, October, 1963, Australian Commonwealth Experimental Building Station, Sydney, Australia.

9. Hanson, J. M., "Influence of Embedded Service Ducts on the Strength of Flat Plate Structures," Portland Cement Association Research and Development Bulletin, RD 005.01D, 1970.

10. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, 1971.

11. Keefe, R. A., "An Investigation on the Effectiveness of Diagonal Tension Reinforcement in Flat Slabs," Thesis, Massachusetts Institute of Technology, June, 1954, 43 pp.

12. Wantur, H., "Dimensioning of Flat Slabs in the Column Region with Simultaneous Consideration of the Bending and Vertical Reinforcement," Ph.D. Dissertation, University of Ghent, 1969, 184 pp.

13. Graf, O., "Strength Tests of Thick Reinforced Concrete Slabs Supported on All Sides Under Concentrated Loads," ("Versuche über die Widerstandsfëhigkeit von allseitigen aufliegenden dicken Eisenbetonplatten unter Einzellastan"), Deutscher Ausschuss fur Eisenbeton (Berlin), No. 88, 1938, 22 pp. (In German)

14. Elstner, R. C. and Hognestad, E., "Shearing Strength of Reinforced Concrete Slabs," ACI Journal, Proceedings, Vol. 53, No. 1, July, 1956, pp. 29-58.

15. Rosenthal, I., "Experimental Investigation of Flat Plate Floors," ACI Journal, Proceedings, Vol. 56, August, 1959, pp. 153-166.

16. Yitzhaki, D., "Punching Strength of Reinforced Concrete Slabs," Proceedings, American Concrete Institute, Vol. 63, May, 1966, pp. 527-542.

17. Comité Européen du Béton, Bulletin No. 57, Dalles, Structures Planes, Theme II: Poinconnement, I, September, 1966.

18. Carpenter, J. E., Kaar, P. H. and Hanson, N. W., Discussion of "Proposed Revision of ACI 318-63: Building Code Requirements for Reinforced Concrete," ACI Journal, Vol. 68, No. 3, September, 1970, pp. 696-697.

19. Franz, G., "Versuche an Stalbetonkröpern der Flachdecke im Stützenbereich," Bericht über die Versuchsreihe I (II) für die Deutsche Forschungsgemeinschaft, Bad Godesberg, Institut für Beton and Stahlbeton, Technische Hochschule Karlsruhe, 1963 (1964).

20. Unified British Code for Structural Use of Reinforced Concrete in Buildings, British Standards Institute 1972.

21. Comité Européen du Béton-Federation Internationale de la Precontrainte, "International Recommendations for the Design and Construction of Concrete Structures," Paris, June, 1970.

22. Hawkins, N. M., Progress Report on NSF Grant No. GK-16375, "Shear and Moment Transfer Between Concrete Flat Plates and Columns," Department of Civil Engineering, University of Washington, December, 1971.

23. Andersson, J. L., "Genomstansning av Lift Slabs" (Punching of Lift Slabs), Bulletins of the Division of Building Statics and Structural Engineering No. 49 (in Swedish), 1964, Royal Institute of Technology, Stockholm, Sweden; also Nordisk Betong No. 3, 1964.

24. Criswell, M. E. and Hawkins, N. M., "Shear Strength of Slabs-Basic Principles and Their Relation to Current Methods of Analysis," ACI Shear Symposium Volume, American Concrete Institute, Detroit, Michigan, 1973.

25. Hawkins, N. M. and Corley, W. G., "Transfer of Unbalanced Moment and Shear from Flat Plates to Columns," SP-30, Cracking, Deflection, and Ultimate Load of Concrete Slab Systems, American Concrete Institute, Detroit, Michigan, 1971, pp. 147-176.

NOTATION

a = side of square plate used as shearhead reinforcement for a square column

A_v = area of shear reinforcement, Eq. [5]

b = perimeter of loaded area or column

c = side length of square column or diameter of circular column

c' = side length or diameter for outermost ring of stirrup reinforcement or outermost bend in bent bars

d = effective depth

f'_c - concrete cylinder compressive strength

f_{ct} = splitting tensile strength of concrete

f_y = yield strength of reinforcement

h_v = depth of steel shape in shearhead

L_v = length of steel shape in shearhead measured from center of column

M_p = plastic moment capacity of steel shape in shearhead

M_s = moment in shearhead reinforcement at column face

q_u = value given by Eq. 5

t = thickness of steel plate, Eq. 1

V_{cr} = shear force at inclined cracking

V_{calc} = shear force calculated from Eq. 8

V'_{calc} = shear force calculated from Eq. 9

V_{flex} = ultimate flexural capacity determined from yield line analysis

V_{pred} = lesser of values V_{calc} and V'_{calc}

V_s = contribution of shear reinforcement to V_u

V_{test} = measured shear force for failure

V_u = ultimate shear capacity

v_u = V_u/bd, Eq. 4

α = inclination of shear reinforcement to horizontal

α_v = relative stiffness of shearhead to that of composite section made up of a cracked section of the slab with a width equal to $(c + d)$

ϕ_o = V_{test}/V_{flex}

Table 1

Strength of Slabs with Shear Reinforcement

Slab No. (1)	RC† (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	ϕ_o (6)	V_u kips ($kgf \times 10^{-3}$) (7)	V_s kips ($kgf \times 10^{-3}$) (8)	V_{calc} kips ($kgf \times 10^{-3}$) (9)	V_{test} kips ($kgf \times 10^{-3}$) (10)	$\frac{V_{test}}{V_{calc}}$* (11)	$\frac{V_{test}}{V'_{calc}}$** (12)
ELSTNER-HOGNESTAD (14)											
B-3	B	10.00 (25.4)	4.50 (11.2)	43.9 (3.08)	1.04	55.6 (25.2)	29.1 (13.2)	56.9 (25.8)	64.5 (29.3)	1.13	0.87
B-5	B	10.00 (25.4)	4.50 (11.2)	45.6 (3.20)	0.61	76.2 (34.6)	27.1 (12.3)	76.2 (34.6)	85.0 (38.6)	1.12	1.10
B-6	B	10.00 (25.4)	4.50 (11.2)	49.6 (3.48)	0.77	75.5 (34.2)	54.3 (24.6)	92.1 (41.8)	105.3 (47.8)	1.14	1.25
B-10	B	10.00 (25.4)	4.50 (11.2)	82.0 (5.76)	0.83	120.0 (54.5)	54.3 (24.6)	120.0	120.0	1.00	0.86
B-12	VS	10.00 (25.4)	4.50 (11.2)	81.5 (5.72)	0.86	117.4 (53.4)	103.0 (46.8)	161.7	177.0	1.09	1.07
B-15	B	10.00 (25.4)	4.50 (11.2)	84.2 (5.92)	0.74	130.0 (59.0)	81.6 (37.1)	146.6	155.0	1.06	1.09
B-16	B	10.00 (25.4)	4.50 (11.2)	81.0 (5.79)	0.80	121.0 (54.9)	108.8 (49.4)	169.3	168.0	0.99	0.90
B-17	B	10.00 (25.4)	4.50 (11.2)	45.8 (3.22)	0.89	64.6 (29.3)	29.2 (13.3)	64.6 (29.3)	80.3 (36.4)	1.24	1.03

Table 1 (continued)

Slab No. (1)	RC† (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	ϕ_o (6)	V_u kips (kgfx10^{-3}) (7)	V_{calc} kips (kgfx10^{-3}) (8)	V_{test} kips (kgfx10^{-3}) (9)	V_{test} kips (kgfx10^{-3}) (10)	$\frac{V_{test}}{V_{calc}}$* (11)	$\frac{V_{test}}{V'_{calc}}$** (12)
GRAF (13)											
1355	B	7.90 (20.0)	10.70 (27.2)	47.0 (3.30)	0.98	143.8 (65.3)	159.0 (72.2)	230.9 (104.8)	271.2 (123.2)	1.17	0.67
1356	B	7.90 (20.0)	10.80 (27.4)	47.0 (3.30)	1.04	150.0 (68.1)	158.0 (71.2)	223.0 (101.2)	288.8 (131.0)	1.24	0.71
1376	B	7.90 (20.0)	18.70 (47.5)	48.7 (3.42)	0.99	268.0 (121.8)	350.0 (159.0)	484.0 (220.0)	507.2 (230.0)	1.05	a
1377	B	7.90 (20.0)	18.70 (47.5)	47.0 (3.30)	0.97	262.0 (119.0)	350.0 (159.0)	481.0 (218.0)	496.1 (225.0)	1.03	a
1361	B	11.80 (30.0)	10.70 (27.2)	48.7 (3.42)	0.97	214.0 (97.2)	218.0 (99.0)	325.0 (147.6)	388.0 (176.0)	1.19	0.92
1363	B	11.80 (30.0)	18.50 (47.0)	48.7 (3.42)	0.85	418.0 (190.0)	459.0 (208.0)	668.0 (303.0)	679.1 (308.0)	1.02	a
MOE (3)											
S8-60	SH	8.00 (20.3)	4.50 (11.2)	57.8 (4.06)	0.98	65.4 (29.7)	34.0 (15.4)	66.7 (30.3)	82.5 (37.4)	1.23	1.12

Table 1 (continued)

Slab No. (1)	RC (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	ϕ_o (6)	V_u kips (kgfx10^{-3}) (7)	V_s kips (kgfx10^{-3}) (8)	V_{calc} kips (kgfx10^{-3}) (9)	V_{test} kips (kgfx10^{-3}) (10)	$\frac{V_{test}}{V_{calc}}$* (11)	$\frac{V_{test}}{V'_{calc}}$** (12)
ROSENTHAL (15)											
I/1	B	9.00ϕ≠ (22.9)	3.13 (7.95)	52.8 (3.71)	0.86	31.7 (14.4)	42.9 (19.5)	58.8 (26.7)	54.0 (24.5)	0.92	0.87
I/2	B	9.00ϕ≠ (22.9)	3.13 (7.95)	47.6 (3.34)	1.01	25.9 (11.8)	71.5 (32.5)	84.4 (38.3)	73.6 (33.4)	0.87	1.03
I/3	B	9.00ϕ≠ (22.9)	3.13 (7.95)	48.6 (3.41)	0.96	27.2 (12.3)	71.5 (32.5)	85.1 (38.6)	71.5 (32.4)	0.84	0.98
I/4	B	9.00ϕ≠ (22.9)	3.13 (7.95)	68.5 (4.81)	0.95	38.4 (17.4)	42.9 (19.5)	62.1 (28.2)	70.5 (32.0)	1.13	0.69

* $V_{calc} = V_s + V_{u/2} \geq V_u$ .... Eq. [8]

** V'_{calc} = shear for stress of $4\,\overline{f'_c}$ on critical perimeter d/2 outside the outermost ring of shear reinforcement .... Eq. [9]

† RC = reinforcement code as shown in Fig. 1

≠ϕ = circular

Table 1 (continued)

Slab No. (1)	RC (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	ϕ_o (6)	V_u kips ($kgf x 10^{-3}$) (7)	V_s kips ($kgf x 10^{-3}$) (8)	V_{calc} kips ($kgf x 10^{-3}$) (9)	V_{test} kips ($kgf x 10^{-3}$) (10)	$\frac{V_{test}}{V_{calc}}$ (11)	$\frac{V_{test}}{V'_{calc}}$ (12)
						ANDERSSON (7)					
62	BR	5.90ϕ (15.0)	4.72 (12.0)	63.5 (4.46)	0.91	49.0 (22.2)	66.9 (30.4)	91.4 (41.5)	78.0 (35.4)	0.85	0.69
63	BR	5.90ϕ (15.0)	4.72 (12.0)	63.6 (4.47)	0.93	49.0 (22.2)	66.2 (30.0)	90.7 (41.2)	79.6 (36.2)	0.87	0.70
64	BR	5.90ϕ (15.0)	4.72 (12.0)	63.6 (4.47)	0.90	49.4 (22.4)	110.4 (50.1)	135.1 (62.3)	83.5 (37.9)	0.62	0.75
65	BR	5.90ϕ (15.0)	4.76 (12.1)	63.5 (4.46)	0.90	49.8 (22.6)	110.8 (50.3)	135.7 (62.5)	83.9 (38.1)	0.62	0.75
66	VS	5.90ϕ (15.0)	4.69 (11.9)	64.2 (4.50)	0.92	49.2 (22.3)	22.4 (10.2)	49.2 (22.3)	65.8 (29.9)	1.38	0.67
67	VS	5.90ϕ (15.0)	4.76 (12.1)	64.8 (4.55)	0.93	50.4 (22.9)	22.4 (10.2)	50.4 (22.9)	66.3 (30.1)	1.31	0.67
68	IS	5.90ϕ (15.0)	4.72 (12.0)	62.9 (4.42)	0.86	50.0 (22.7)	31.8 (14.4)	56.8 (25.8)	61.8 (28.1)	1.09	0.66
69	IS	5.90ϕ (15.0)	4.76 (12.1)	61.6 (4.33)	0.78	51.6 (23.4)	31.8 (14.4)	57.6 (26.2)	56.1 (25.5)	0.97	0.60
70	RS	5.90ϕ (15.0)	4.76 (12.1)	60.4 (4.24)	0.84	49.9 (22.7)	33.8 (15.3)	58.3 (26.5)	61.4 (27.8)	1.05	0.39
71	RS	5.90ϕ (15.0)	4.84 (12.3)	62.1 4.36)	0.89	49.6 (22.5)	33.8 (15.3)	58.6 (26.6)	65.8 (29.9)	1.12	0.40
76	BR	11.80ϕ (15.0)	4.81 (12.2)	64.4 4.52)	0.93	84.4 (38.3)	69.2 (31.9)	111.4 (50.6)	120.3 (54.6)	1.08	0.87
77	BR	11.80ϕ (30.0)	4.92 (12.5)	65.6 (4.61)	0.96	86.8 (39.4)	68.6 (31.2)	112.0 (50.9)	123.8 (56.1)	1.10	0.86

Table 1 (continued)

Slab No. (1)	RC (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	ϕ_o (6)	V_u kips ($kgf \times 10^{-3}$) (7)	V_s kips ($kgf \times 10^{-3}$) (8)	V_{calc} kips ($kgf \times 10^{-3}$) (9)	V_{test} kips ($kgf \times 10^{-3}$) (10)	$\frac{V_{test}}{V_{calc}}$ (11)	$\frac{V_{test}}{V'_{calc}}$ (12)
ANDERSSON (7) (continued)											
78	BR	11.80φ (30.0)	4.72 (12.0)	66.0 (4.64)	0.95	83.4 (37.8)	140.0 (63.6)	181.7 (82.4)	136.6 (62.0)	0.75	0.99
79	BR	11.80φ (30.0)	4.69 (11.9)	66.2 (4.65)	0.96	83.4 (37.8)	142.6 (64.7)	184.3 (83.7)	138.0 (62.6)	0.74	0.99
80	BR	11.80φ (30.0)	4.76 (12.1)	65.7 (4.62)	0.97	82.2 (37.3)	88.8 (40.3)	129.9 (58.9)	102.0 (46.3)	0.78	0.72
81	BR	11.80φ (30.0)	4.72 (12.0)	63.1 (4.44)	0.97	78.4 (35.6)	89.5 (40.6)	128.7 (58.4)	106.2 (48.2)	0.82	0.80
82	VS	11.80φ (30.0)	4.72 (12.0)	64.6 (4.54)	0.85	87.6 (39.8)	39.2 (17.8)	87.6 (39.8)	103.6 (47.0)	1.18	1.07
83	VS	11.80φ (30.0)	4.69 (11.9)	60.0 (4.21)	0.81	82.8 (37.6)	39.2 (17.8)	82.8 (37.6)	103.6 (47.0)	1.25	1.16
84	IS	11.80φ (30.0)	4.84 (12.3)	61.8 (4.34)	0.96	79.8 (36.2)	42.3 (19.2)	82.2 (37.4)	92.8 (42.1)	1.12	0.84
85	IS	11.80φ (30.0)	4.81 (12.2)	62.0 (4.36)	1.00	77.5 (35.2)	42.3 (19.2)	81.2 (37.4)	88.4 (40.2)	1.08	0.80
NYLANDER-ANDERSON (7)											
a	B	10.16 (25.8)	4.65 (11.8)	64.3 (4.52)	0.67	85.8 (39.0)	77.5 (35.2)	120.4	152.2	1.27	--

Table 1 (continued)

Slab No. (1)	RC (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	o (6)	V_u kips (kgfx10^{-3}) (7)	V_s kips (kgfx10^{-3}) (8)	V_{calc} kips (kgfx10^{-3}) (9)	V_{test} kips (kgfx10^{-3}) (10)	$\frac{V_{test}}{V_{calc}}$ (11)	$\frac{V_{test}}{V'_{calc}}$ (12)
					YITZHAKI (16)						
I-1a	B	8.67ϕ (22.0)	3.24 (8.24)	45.8 (32.2)	0.98	27.8 (12.6)	43.5 (19.8)	57.4 (26.1)	54.0 (24.5)	0.94	1.02
I-1b	B	8.67ϕ (22.0)	3.24 (8.24)	37.5 (26.4)	1.02	22.0 (10.0)	38.5 (17.5)	49.5 (22.5)	48.2 (21.9)	0.98	1.08
I-2a	B	8.67ϕ (22.0)	3.24 (8.24)	44.6 (31.3)	1.20	22.4 (10.2)	72.2 (32.8)	83.2 (37.8)	73.8 (33.5)	0.89	1.17
I-2b	B	8.67ϕ (22.0)	3.24 (8.24)	46.1 (32.4)	1.12	24.8 (11.3)	73.0 (33.2)	85.4 (38.8)	71.6 (32.5)	0.84	1.10
I-4	B	8.67ϕ (22.0)	3.24 (8.24)	58.5 (41.1)	1.02	34.4 (15.6)	77.7 (35.3)	94.9 (43.1)	70.5 (32.0)	0.74	0.85
I-5	B	8.67ϕ (22.0)	3.24 (8.24)	49.5 (34.8)	1.22	24.4 (11.1)	29.8 (13.5)	42.0 (19.1)	34.7 (15.8)	0.82	0.80
I-6a	B	8.67ϕ (22.0)	3.24 (8.24)	52.5 (36.9)	1.29	25.4 (11.5)	59.6 (27.1)	72.3 (32.8)	57.5 (26.1)	0.80	0.98
I-6b	B	7.88ϕ (20.0)	3.24 (8.24)	51.3 (36.1)	1.29	29.7 (13.5)	59.6 (27.1)	74.5 (33.8)	57.5 (26.1)	0.77	0.96
I-7	B	8.67ϕ (22.0)	3.24 (8.24)	41.2 (28.9)	1.20	20.6 (9.4)	44.9 (20.4)	55.2 (25.1)	43.8 (19.9)	0.79	1.10
I-8	B	8.67ϕ (22.0)	3.24 (8.24)	43.5 (30.6)	1.40	21.1 (9.6)	29.8 (13.5)	40.4 (18.3)	64.1 (29.1)	1.59	1.05
I-9	B	8.67ϕ (22.0)	3.24 (8.24)	45.8 (32.2)	1.33	22.2 (10.1)	59.6 (27.1)	70.7 (32.1)	75.0 (34.0)	1.06	1.41

Table 1 (continued)

Slab No. (1)	RC (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	ϕ_o (6)	V_u kips (kgfx10^{-3}) (7)	V_s kips (kgfx10^{-3}) (8)	V_{calc} kips (kgfx10^{-3}) (9)	V_{test} kips (kgfx10^{-3}) (10)	$\frac{V_{test}}{V_{calc}}$ (11)	$\frac{V_{test}}{V'_{calc}}$ (12)
FRANZ (17, 19)											
2	B	8.27φ (21.0)	4.44 (11.3)	59.6 (4.19)	0.73	62.5 (28.4)	75.7 (34.4)	107.0 (48.6)	77.4 (35.1)	0.80	1.07
3	B	8.27φ (21.0)	4.44 (11.3)	59.6 (4.19)	0.71	63.2 (28.7)	75.7 (34.4)	107.3 (48.8)	86.6 (39.4)	0.82	1.10
4	CSR	8.27φ (21.0)	4.44 (11.3)	59.6 (4.19)	0.80	59.8 (27.2)	17.9 (8.1)	59.8 (27.2	88.5 (40.2)	1.48	d
5	CSR	8.27φ (21.0)	4.44 (11.3)	59.6 (4.19)	0.89	56.5 (25.7)	35.8 (16.2)	64.1 (29.2)	97.3 (44.2)	1.51	d
6	CSR	8.27φ (21.0)	4.44 (11.3)	59.6 (4.19)	0.88	56.8 (25.8)	47.8 (21.7)	76.2 (34.6)	96.2 (43.7)	1.26	d
7	VS	8.27φ (21.0)	4.44 (11.3)	59.6 (4.19)	0.73	62.5 (28.4)	40.8 (18.5)	72.1 (32.8)	79.7 (36.2)	1.12	d
8	CSR&T	8.27φ (21.0)	4.44 (11.3)	62.0 (4.36)	0.89	58.8 (26.7)	57.1 (26.0)	86.5 (39.3)	97.4 (44.2)	1.13	d
9	CSR&T	8.27φ (21.0)	4.44 (11.3)	56.4 (3.96)	0.90	53.2 (24.2)	57.1 (26.0)	83.7 (38.0)	97.4 (44.2)	1.16	d
10	CSR&T	8.27φ (21.0)	4.44 (11.3)	57.9 (4.07)	1.03	50.0 (22.7)	42.9 (19.5)	67.9 (30.8)	81.9 (37.2)	1.21	d
11	CSR&T	8.27φ (21.0)	4.44 (11.3)	60.6 (4.26)	1.17	47.2 (21.4)	42.9 (19.5)	66.5 (30.2)	77.4 (35.1)	1.16	d
12	CSR&T	8.27φ (21.0)	4.44 (11.3)	58.6 (4.12)	1.34	41.6 (18.9)	42.9 (19.5)	63.7 (29.0)	73.0 (33.2)	1.14	d
13	CSR&T	8.27φ (21.0)	4.44 (11.3)	61.2 (4.30)	1.47	43.4 (19.7)	28.6 (13.0)	50.3 (22.8)	63.0 (28.6)	1.25	d

Table 1 (continued)

Slab No. (1)	RC (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	ϕ_o (6)	V_u kips (kgfx10^{-3}) (7)	V_s kips (kgfx10^{-3}) (8)	V_{calc} kips (kgfx10^{-3}) (9)	V_{test} kips (kgfx10^{-3}) (10)	$\frac{V_{test}}{V_{calc}}$ (11)	$\frac{V_{test}}{V'_{calc}}$ (12)
FRANZ (17, 19) (continued)											
14	CSR&T	8.27ϕ (21.0)	4.44 (11.3)	55.8 (3.92)	0.84	54.6 (24.8)	66.6 (30.2)	93.9 (42.6)	92.9 (42.2)	0.99	d
15	CSR&T	8.27ϕ (21.0)	4.44 (11.3)	57.9 (4.07)	0.87	58.1 (26.4)	56.5 (25.7)	85.6 (38.9)	88.5 (40.2)	1.03	d
16	CSR&T	8.27ϕ (21.0)	4.44 (11.3)	57.6 (4.05)	1.30	40.8 (18.5)	37.7 (17.1)	58.1 (26.4)	70.8 (32.2)	1.22	d
WANTUR (12)											
2	CST	7.09ϕ (18.0)	4.00 (10.2)	63.9 (4.49)	1.08	41.8 (19.0)	26.7 (12.1)	47.6 (21.6)	52.2 (23.7)	1.10	d
3	CST	7.09ϕ (18.0)	4.00 (10.2)	66.6 (4.68)	1.09	43.2 (19.6)	40.0 (18.2)	61.6 (28.0)	52.8 (24.0)	0.85	d
5	CST	7.09ϕ (18.0)	4.00 (10.2)	65.7 (4.62)	0.99	45.7 (20.8)	40.0 (18.2)	62.8 (28.5)	59.9 (27.2)	0.96	d
6	CST	7.09ϕ (18.0)	4.00 (10.2)	67.7 (4.76)	1.01	46.4 (21.1)	33.4 (15.2)	56.6 (25.7)	60.8 (27.6)	1.07	d
7	SH	7.09ϕ (18.0)	4.00 (10.2)	67.1 (4.72)	1.12	42.6 (19.3)	39.6 (18.0)	61.4 (27.9)	54.4 (24.7)	0.89	d
8	SH	7.09ϕ (18.0)	4.00 (10.2)	66.8 (4.70)	1.02	45.6 (20.7)	39.6 (18.0)	62.4 (28.3)	60.8 (27.6)	0.98	d

Table 1 (continued)

Slab No. (1)	RC (2)	c in. (cm) (3)	d in. (cm) (4)	$\sqrt{f'_c}$ psi (kgf/cm^2) (5)	ϕ_o (6)	V_u kips ($kgf x10^{-3}$) (7)	V_s kips ($kgf x10^{-3}$) (8)	V_{calc} kips ($kgf x10^{-3}$) (9)	V_{test} kips ($kgf x10^{-3}$) (10)	$\frac{V_{test}}{V_{calc}}$ (11)	$\frac{V_{test}}{V'_{calc}}$ (12)
TASKER-WYATT (8)											
B1, 2,3	SH	10.00 (25.4)	2.63 (6.7)	61.5 (4.32)	1.10	32.8 (14.8)	36.6 (16.6)	53.0 (24.1)	40.0 (18.2)	0.76	1.03
B4	SH	10.00 (25.4	2.63 (6.7)	61.5 (4.32)	1.18	32.8 (14.8)	28.4 (12.9)	44.8 (20.4)	43.0 (19.5)	0.96	0.89
B5	SH	10.00 (25.4)	2.63 (6.7)	56.5 (3.97)	1.17	30.1 (13.7)	23.2 (10.5)	38.3 (17.4)	41.9 (19.0)	1.09	d
G1	SH	10.00 (25.4)	2.63 (6.7)	57.3 (4.03)	1.10	30.5 (13.8)	23.2 (10.5)	38.5 (17.5)	44.6 (20.2)	1.16	0.89

a: Not calculated since cracking would be required over support

b: Insufficient information for calculations. However, V'_{calc} is unlikely to govern.

d: V'_{calc} does not control.

c: With bent column reinforcement.

Table 2

Average Ratios of Measured to Calculated Strengths for Slabs with Shear Reinforcements

Investigation	No. Specimens	Average Strength Meas./Calc.
Elstner-Hognestad (14)	8	1.11
Graf (13)	6	1.12
Rosenthal (15)	4	1.02
Andersson (7)	20	1.04
Yitzhaki (16)	11	1.10
Franz (19)	15	1.19
Wantur (12)	6	0.98
Tasker-Wyatt (8)	4	1.06
	Average	1.09

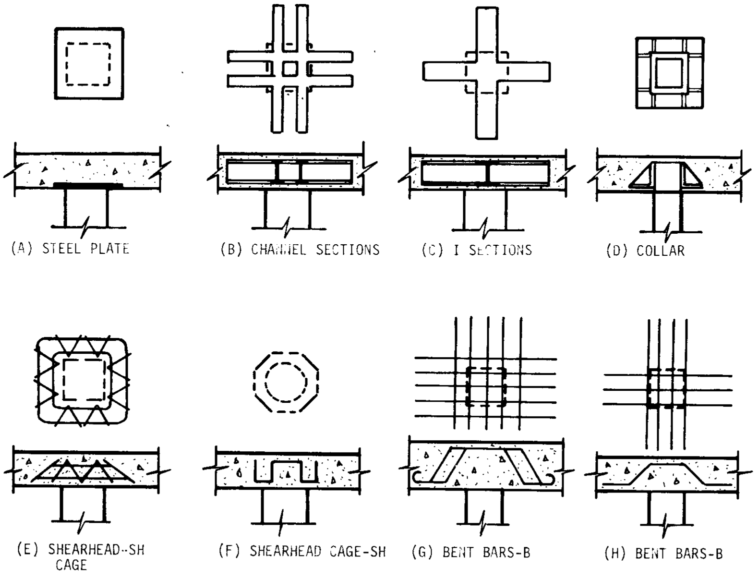

Fig. 1a--Shear reinforcements

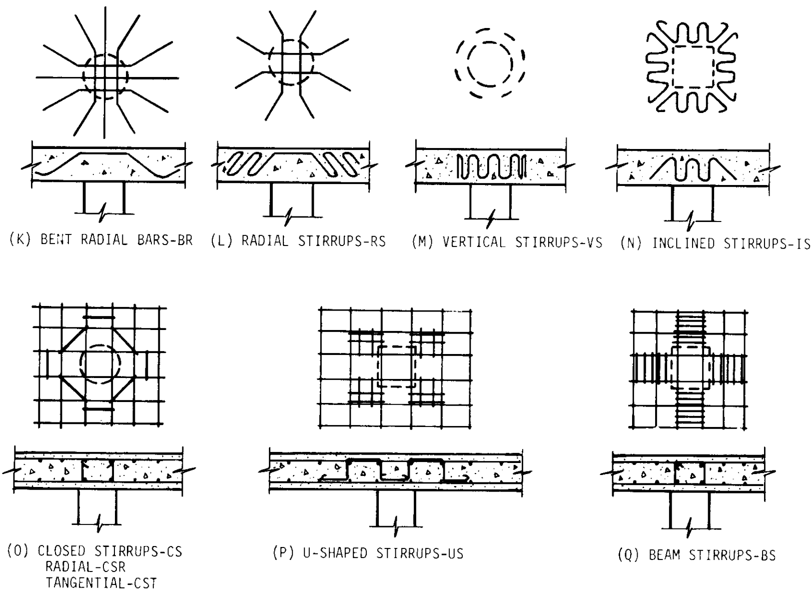

Fig. 1b--Shear reinforcements

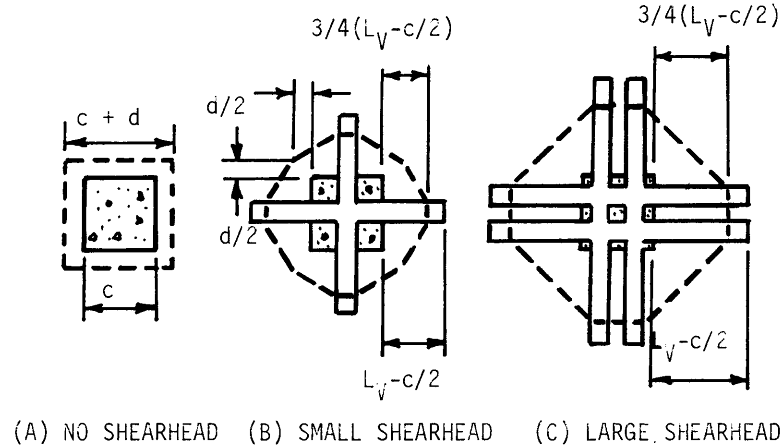

Fig. 2 --Critical sections for slabs with shearheads

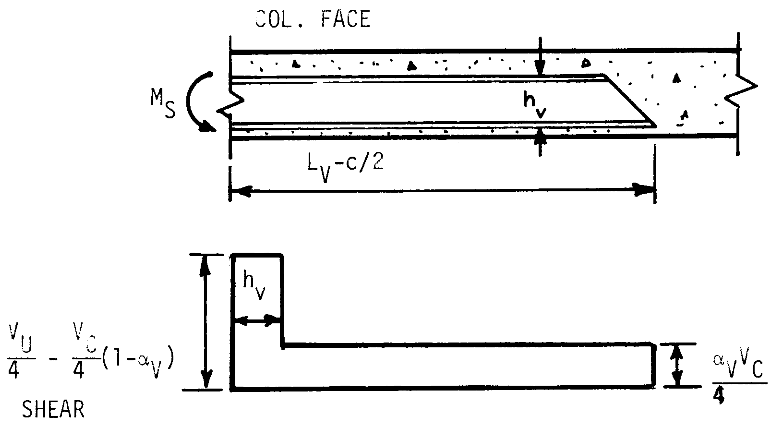

Fig. 3 --Idealized distribution for shear in shearhead at ultimate load

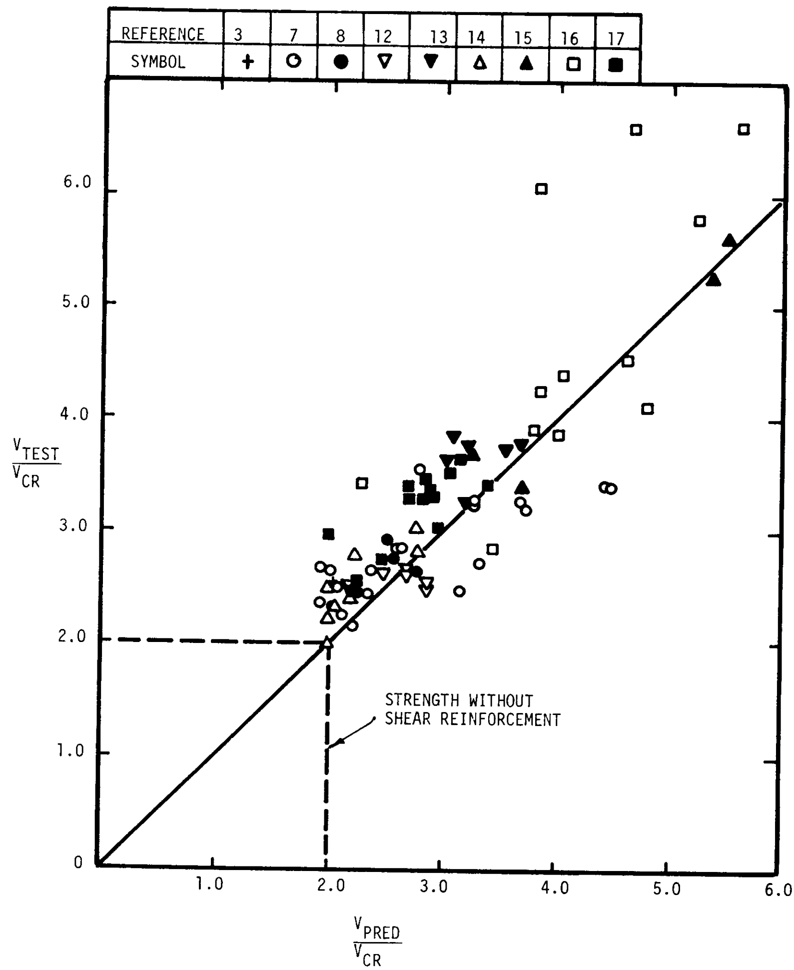

Fig. 4 --Comparison of V_{test} and V_{pred} for slabs containing bent bars and stirrups

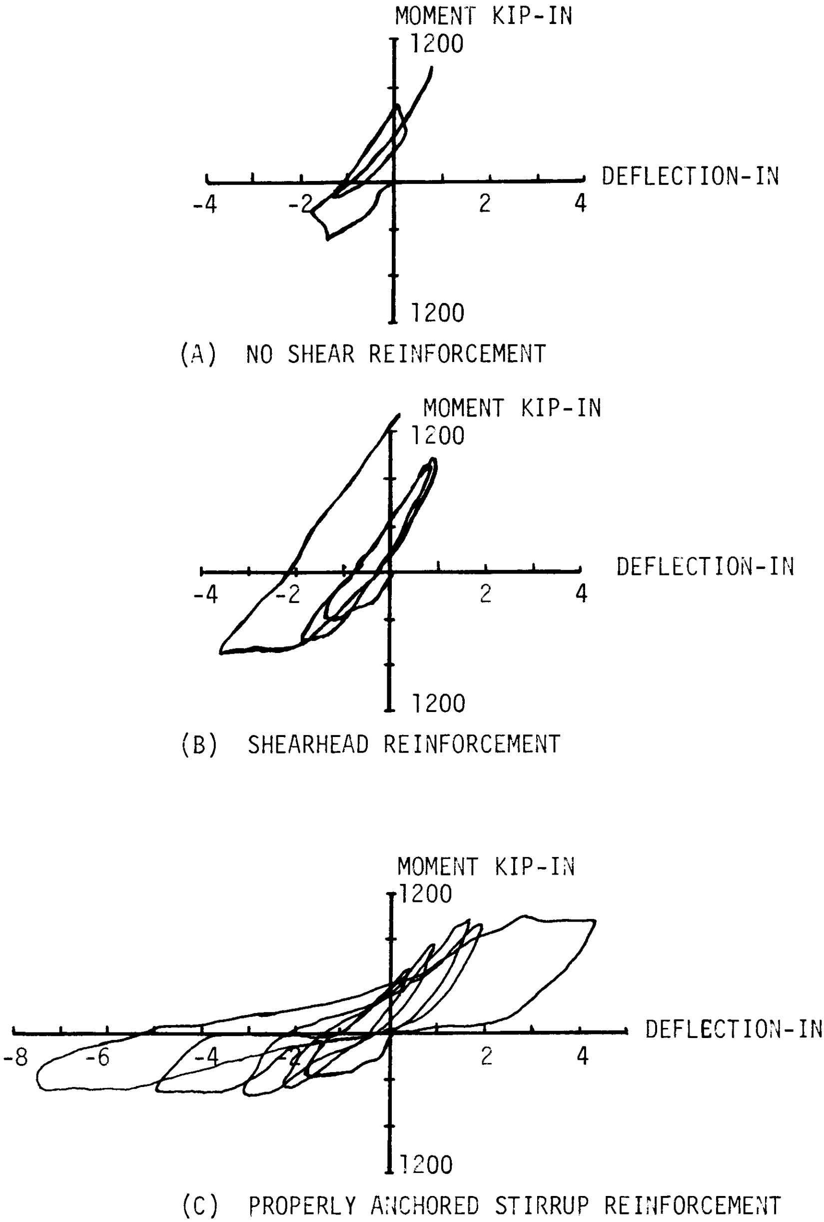

Fig. 5 --Moment-deflection curves for PCA reversed loading tests