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AUXILIARY REINFORCEMENT IN CONCRETE CONNECTIONS[a]

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INTRODUCTION

Since the late 1950's, many types of connections for precast concrete have been developed. Most of these connections involve the use of auxiliary reinforcement in the form of bearing shoes, anchoring bars, and confining hoops. Although current literature abounds with suggested types of connections, very little information is available on the detail design of these connections. It is the purpose of this paper to present an understandable, simple method in which the reinforcement concept may be verified, and all parts of the connection may be rationally sized. This method has been proved over a 10-yr span of practical design.

DESIGN PHILOSOPHY

In a precast concrete connection, forces are normally collected and funneled through a small local zone. The state of elastic stress in this local zone is difficult, if not impossible, to determine. Because of fabrication and erection tolerances, no two connections are ever identically the same, and the state of elastic stress will not be the same from one connection to the next. Furthermore, in addition to the calculated forces from external loads, connections are generally subjected to uncalculated forces arising from creep, shrinkage, settlement, and temperature changes. In light of these considerations, it is imperative that a connection possess ductility. This ductility must normally be obtained through the use of reinforcing steel, rather than through reliance on the brittle tensile or shearing strength of the concrete.

Unfortunately, many current connection designs do rely on the tensile

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strength of the concrete. A few tests on such a connection can lead to a false sense of security, for the following reasons: (1) Laboratory loading is done carefully, with even distribution of pressures and no impact, conditions notably lacking in the field; and (2) imperfections in the concrete, handling impacts, forcing during erection, and uneven fits can lead to cracks in the vital connection regions. Such cracks cannot be reliably detected and corrected after erection. The result is that the designer cannot be sure of his design, unless he provides for the "accidental" crack.

The basic approach presented herein is to assume that the concrete may crack in an unfavorable manner. Reinforcement is then provided, to prevent this hypothetical crack from having undesirable consequences. In a test of the connection, the assumed crack may not actually occur, and the connection may appear to be much stronger than the design calculations indicate. Still, the possibility of such a crack must be guarded against, since this crack will determine the minimum strength which can be depended upon in this field. Certainly, there is no one, experienced in concrete work, who has not observed an occasional crack in locations totally unpredicted by his design calculations.

A method is presented in which reinforcement may be provided to resist shearing forces that may exist along a possible crack. This method is based on the consideration of internal friction along the crack, and was originally derived from the study of interface shear in composite beams. The method predicts a safe, lower bound for the strength of the connection.

It should be noted that all equations given here are based on conditions at ultimate load, not at service load.

SHEAR-FRICTION HYPOTHESIS

Consider a cracked specimen of concrete, subjected to a normal compressive force across the crack, and a shearing force along the crack (Fig. 1). The

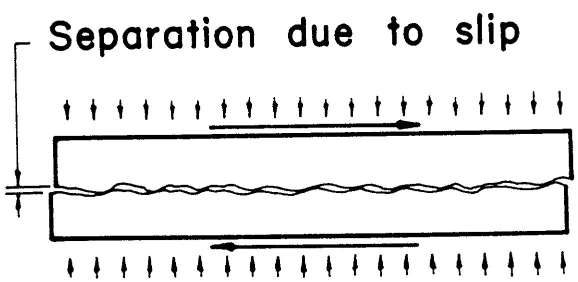

FIG. 1.—SHEAR ALONG A CRACK

shearing force may be resisted by virtue of friction along the crack. Since the crack is rough and irregular, the coefficient of friction may be quite high. Furthermore, the rough and irregular nature of the crack will cause the two pieces of concrete to separate slightly, when slip occurs along the crack.

If reinforcement is present normal to the crack, slippage and the subsequent separation along the crack will stress the steel in tension. There will then exist a balancing compressive stress along the crack, and a shearing stress

may be resisted by friction (Fig. 2). The magnitude of this shearing resistance is the compressive stress multiplied by the tangent of the angle of friction. This angle is somewhat analogous to that used in the analysis of cohesionless soils. The relation between steel area and shearing force at yield of steel (ultimate load) may be expressed either in terms of forces (Eq. 1), or in terms of stresses (Eq. 2)

$$V_u = A_s\, f_y \tan \phi \qquad (1)$$

$$v_u = pf_y \tan \phi \qquad (2)$$

The steel ratio, A_s/bd, is denoted by p, and the angle of internal friction, ϕ, is determined by tests.

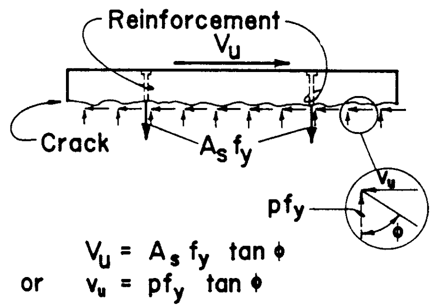

FIG. 2.—STRESSES AND FORCES AT CRACK SURFACE

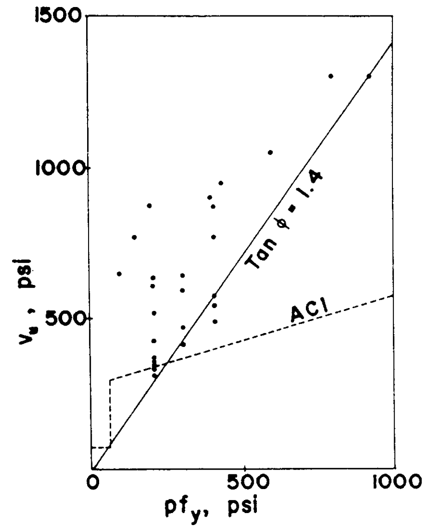

FIG. 3.—SHEAR TESTS

Push-off tests have been conducted by the Portland Cement Association (PCA) (5),[2] and by Anderson (1). The results of these tests are plotted in Fig. 3. There is considerable scatter in the data for low steel ratios. In these cases, the variable cohesive strength of the concrete may have considerable influence. Note that the shear-friction theory, which neglects the cohesive strength, forms an effective lower bound for the data. The ultimate shear strength using Section 2505 of the American Concrete Institute Building Code (ACI 318-63) is also plotted. In the development of the ACI Code, the reinforcement was regarded more as a dowel then as a tension member, and thus the effect of the reinforcement was underestimated.

A brief consideration of free bodies of the reinforcement will show that the

[2] Numerals in parentheses refer to corresponding items in the Appendix II.—References.

reinforcement cannot sustain the high shears by dowel action (7). Using the most optimistic assumptions of full plasticity and uniform bearing stresses, bearing stresses of approximately 70,000 psi in the concrete are obtained.

In application of the shear-friction hypothesis to design, several limitations must be observed:

1. The shear-friction hypothesis is based on the static ultimate load, after cracking of the concrete. It is not applicable to connections in which fatigue is a consideration, or where slip is highly critical.

2. If any externally applied tension may exist across the crack, reinforcement for this tension must be provided in addition to the reinforcement required by the shear-friction theory.

3. Since the reinforcement acts in tension, full tensile anchorage must be provided on both sides of the crack, sufficient to reliably develop the yield strength of the steel. The importance of this cannot be overstated. If the anchorage is by bond, a finite amount of slippage and subsequent separation of the concrete pieces on each side of the crack must take place. For #5 bars of intermediate grade, a separation of 0.01 in. is required to stress the bars to yield. It is known from tests that this separation can be developed. However, it is not known how much more can be developed. Thus, until further data become available, it is recommended that bar size and strength be limited to #6 intermediate grade.

TABLE 1.—VALUES OF TAN ϕ RECOMMENDED FOR DESIGN

Case	Description	Tan ϕ
a	Concrete to concrete, rough interface	1.4
b	Concrete to steel, composite beams	1.0
c	Concrete to steel, field-welded inserts	0.7
d	Concrete to concrete, smooth interface	0.7

4. All testing has been done on normal weight concrete. Lightweight concrete may behave quite differently, as a result of its different internal structure. The data presented herein should not be applied to lightweight concrete without further study and tests.

5. The value of the angle of internal friction, ϕ, has been assumed to be independent of concrete strength and stress level. Since this may not actually be so, extrapolation beyond the range of current test data should not be attempted without further tests. Thus, the reinforcement index, pf_y, should be limited to 15% of the concrete cylinder strength, f_c'.

6. Values of tan ϕ, determined from tests, are as follows: (1) For a crack in monolithic concrete, 1.4 to 1.7; (2) for a rough screeded interface between precast and cast-in-place concrete, 1.4; and (3) for concrete cast against steel, or against smooth concrete, 0.7 to 1.0. It is recommended that values of tan ϕ as listed in Table 1 be used in design.

Case a of Table 1 conforms to the test data presented herein. Case b corresponds to ordinary composite steel and concrete beams. The shear values for welded studs which are given in the American Institute of Steel Construction (AISC) specification may be derived from the internal friction theory, using

tan ϕ = 1.0 (2). Case c corresponds to the ordinary embedded weld plate in a concrete beam. Since the heat of welding may have deleterious effects on the concrete at the steel-concrete interface, it is recommended that tan ϕ be reduced to 0.7 (7). Case d corresponds to the ordinary construction joint. Research has been done on bolted and post-tensioned concrete connections (3), from which a minimum value of about 0.7 has been obtained for tan ϕ. Limitations 3, 4 and 5 are not absolute in the sense that they are imposed by the known nature of concrete. Rather, they are imposed in order to prevent unsafe extrapolation beyond current knowledge. Future research could result in relaxation of these limitations.

The values of tan ϕ are empirical, and seem at variance with failure theories for concrete, which generally indicate a tan ϕ of about 0.7 (14). It should be noted that the failure theories are developed from tests on uncracked specimens, generally at high stress levels. The values of tan ϕ presented herein

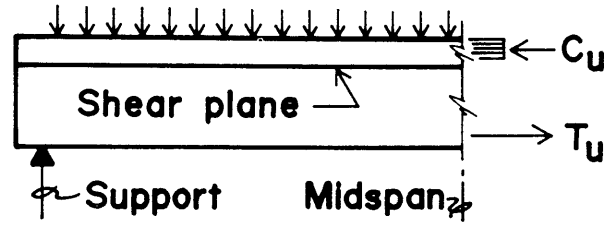

FIG. 4.—COMPOSITE BEAM AT ULTIMATE LOAD

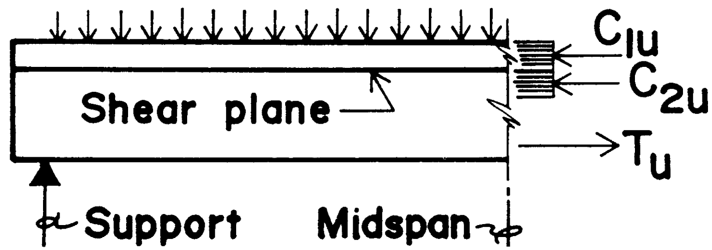

FIG. 5.—PART OF COMPRESSIVE LOAD IN STEM

are based on cracked specimens, at relatively low stress levels. It is possible to develop an apparent angle of internal friction along an irregular crack, at stress levels limited by $pf_y = 0.15f_c'$, which is greater than the true angle of friction for the uncracked material.

COMPOSITE BEAMS

The most obvious application of the shear-friction hypothesis is to the interface connection in composite beams. As an example, consider a simply supported beam with uniform load and a deck slab composite with the beam. At ultimate load, the total compressive force is normally within the top flange. A shearing force will exist along the interface between the flange and the web. From the point of zero moment (at the support of the beam), to the point of maximum moment (midspan), the total magnitude of this shearing force will be equal to the compressive force, C_u, in the flange. This is equal to T_u, the ultimate or yield tension in the flexural steel (Fig. 4). The total required area of vertical steel, from the end to the point of maximum moment, will then be $T_u/(f_y \tan \phi)$, from Eq. 1. This steel should preferably be distributed somewhat in the manner of the shear diagram for the beam. If a portion of the compressive force at ultimate load resides in the stem of the beam, then interface reinforcement need be provided only to develop the portion of the compressive force residing above the interface (Fig. 5). This simple method of design is in complete harmony with the AISC specifications for composite steel beams (13), but does not seem to have been given due consideration by concrete researchers. In most published literature on composite concrete beams, the shears at ultimate load have been computed by VQ/Ib', and these may be at considerable variance with the actual shears existing at ultimate load. Since

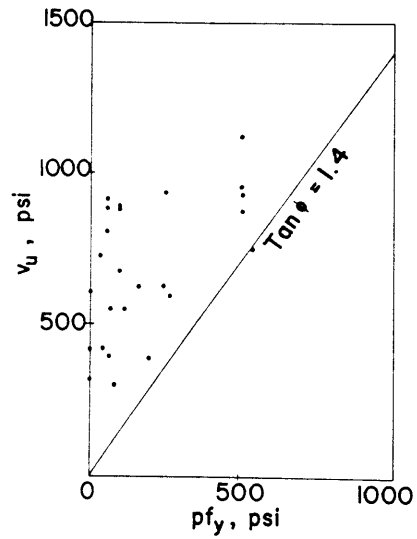

FIG. 6.—COMPOSITE BEAM TESTS

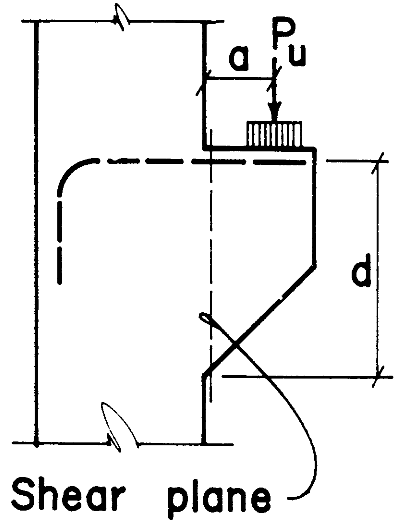

FIG. 7.—CORBEL

the assumptions behind VQ/Ib' require elastic behavior of uncracked concrete, this method of computation is obviously not correct at ultimate load. Fig. 6 shows the results of some published tests of composite concrete beams (4,9,12). Since the shears were computed by VQ/Ib', there is much scatter in the data. Nevertheless, the shear-friction hypothesis gives a safe lower bound for the data.

The design of corbels, although basically a simple problem, has been the cause of much difficulty. The shear-friction hypothesis may be applied directly to the design of corbels. If the ratio a/d > 0.7, (Fig. 7) the main horizontal steel will be controlled by flexure, as may be seen from a truss analogy. In this instance, shear is the conventional diagonal tension problem. For the more common case, with a/d < 0.7, a diagonal tension crack cannot fully develop, and the problem is one of shear rather than diagonal tension. This problem is

the one considered here. Reinforcement must be placed horizontally, to guard against the possibility of a shear failure along the plane shown in Fig. 7. The required reinforcement, A_s, is equal to $P_u/(f_y \tan \phi)$, from Eq. 2. A portion of this reinforcement, as required to resist flexural stresses, is placed in the top of the corbel. The remainder is distributed throughout the upper half of the corbel. This is necessary in order to cause the compressive forces within the corbel to "turn the corner," as shown in Fig. 8. If this is not done, the external corner of the corbel will pop off.

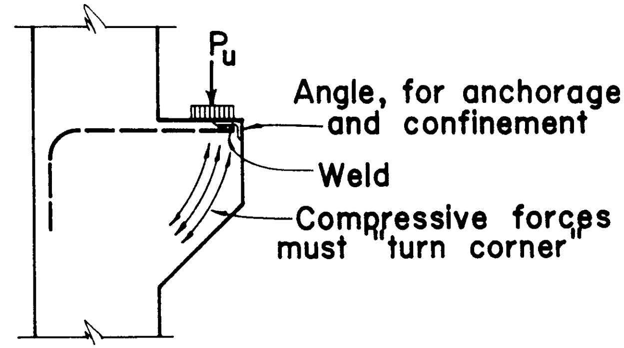

FIG. 8.—REACTION MUST "TURN CORNER"

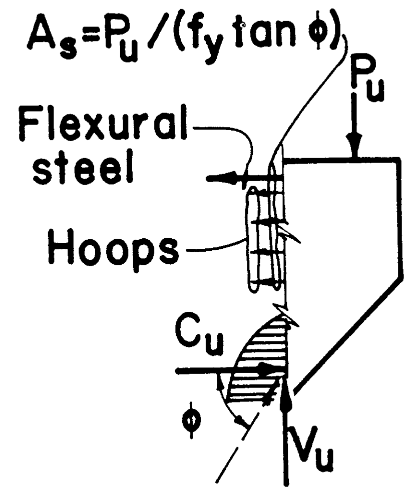

FIG. 9.—FREE BODY OF CORBEL

A crack may form just behind the loaded area. It was assumed that the steel would be able to work at yield stress at the location of this crack; thus, the steel must be fully anchored between the potential crack and the face of the corbel. This point cannot be emphasized too strongly. Since the top bars are often fairly large, a mechanical anchor is required at the end of the bar. A steel angle welded to the rebar accomplishes this nicely, and is also invaluable in preventing spalling at the outer corner. The mechanical anchor and its weld to the rebar must be able to develop the yield strength of the rebar.

Fig. 9 shows a free body of the corbel. The compressive force is equal to the tensile force, and the shear is carried through internal friction, in the region of the compressive force. Note that if the corbel were subject to lateral

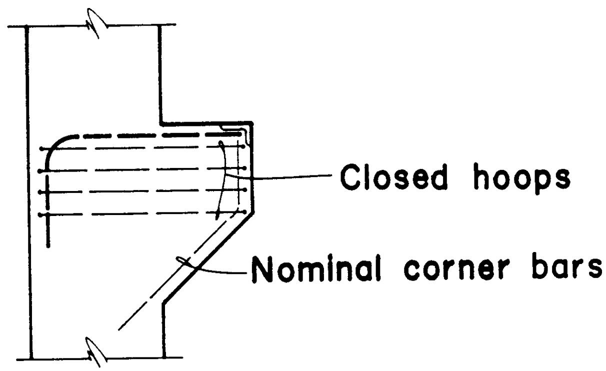

FIG. 10.—CORBEL DETAIL

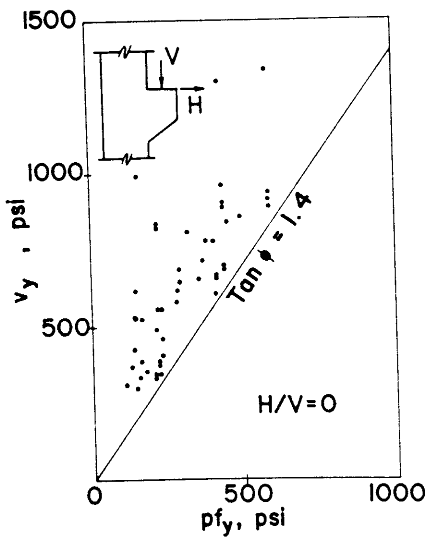

FIG. 11.—CORBEL TESTS WITHOUT HORIZONTAL TENSION

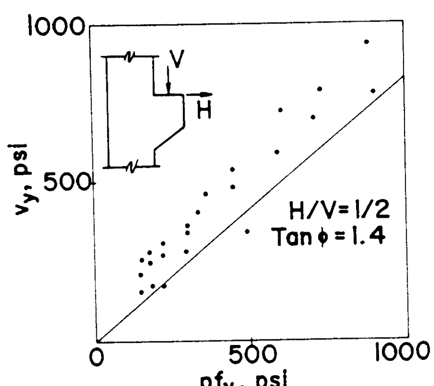

FIG. 12.—CORBEL TESTS WITH HORIZONTAL TENSION

tension, the compressive force in the concrete would not be equal to the tensile force in the steel. In this case, extra tensile steel would be required to resist the external tension, in addition to the steel required by shear-friction. The

effect of an external tensile force is considered in detail later. Fig. 10 shows the completed corbel detail.

The Portland Cement Association has made extensive tests on corbels (8). Fig. 11 shows the results of those PCA tests which had a vertical reaction but no horizontal tension. The shear-friction hypothesis gives a good lower bound for the data. Similar results are obtained for corbels with horizontal tension. When horizontal tension is present, the shear-friction hypothesis gives

$$V_u = (A_s f_y - H_u) \tan \phi \quad (3)$$

$$v_u = \left(pf_y - \frac{H_u}{bd} \right) \tan \phi \quad (4)$$

in which H_u = horizontal tensile force at ultimate load. Fig. 12 shows the results for the PCA tests, in which the horizontal tension was one half of the vertical load.

The test results plotted in Figs. 11 and 12 are for those tests in which yielding of the steel was obtained. In some tests, yielding was not obtained, due to a variety of other failures, such as bearing splitting of corbel end, and over reinforcing. Proper detailing will prevent these failures (4).

BEAM BEARING SHOES

Precast beams are often supported on narrow bearings at their ends. A crack may form, as shown in Fig. 13, separating the bearing from the beam. To guard against this, reinforcing sized by Eq. 1 must be placed across the crack. Since this reinforcing must be capable of working at tensile yield at the crack, an angle shoe is welded to the end of the bars, for anchorage. A crack may also form as shown in Fig. 14, separating the anchoring steel from the main tensile steel of the beam. To guard against this, vertical steel hoops are placed around the anchor bars. The area, A_v, of this steel will be $T_u/(f_y \tan \phi)$, in which T_u = the tension in the anchoring bars. Often, steel required for stirrups or end block reinforcing can also be used for this purpose.

The commonly used details shown in Fig. 15 are poor ones. A crack may form as shown on the left, missing all the steel. The detail shown on the right is very bad, because welding to cold bent reinforcing bars can cause catastrophic embrittlement in the region of the bend. Never weld to bent rebars.

The previous description of beam bearing design assumed that no horizontal tension was present. This ideal condition usually occurs only if positive steps are taken to prevent horizontal tension. If horizontal restraint is present (stiff walls, piers, columns, etc.), tension will develop due to such causes as temperature and shrinkage. If moment restraint is present, tension will develop if the beam cambers after erection due to creep, or if positive moment occurs due to settlement. In either event, the tension which may develop is unmanageable.

In precast construction, there are two approaches possible: (1) Completely flexible construction, using neoprene bearing pads and providing no connections other than gravity between members; and (2) construction using rigid or "hard" connections with the structure well tied together. The writer's opinion is that these two approaches are mutually exclusive (i.e., they cannot be combined), and that the second is far preferable. See Ref. (2) for amplification.

The question then becomes how to tie the structure together, while providing for unwanted tension. This can be done by: (1) Making columns or walls flexible, so that tension cannot develop due to temperature, etc.; (2) designing beams so that long-term creep is downward, producing negative moment at the support and therefore horizontal compression at the bearing (this comes automatically, except for certain highly prestressed elements with low superimposed dead load); and (3) if items 1 and 2 are not possible, separating the

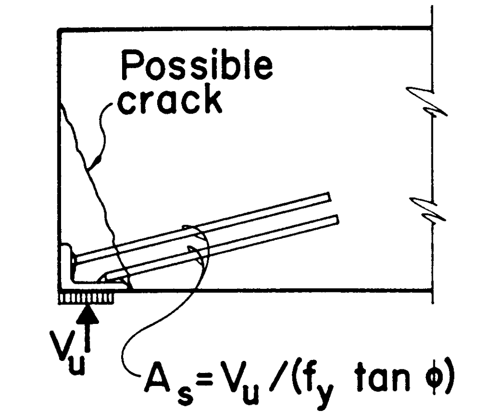

FIG. 13.—BEAM BEARING, VERTICAL CRACK

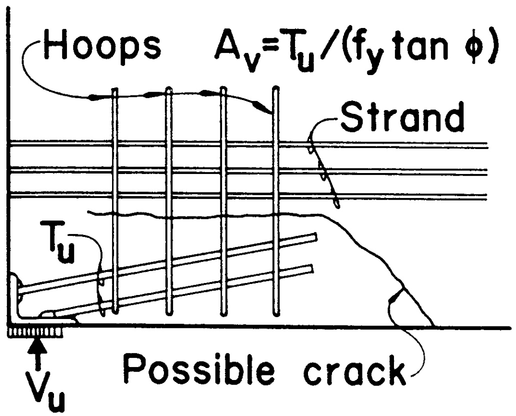

FIG. 14.—BEAM BEARING, HORIZONTAL CRACK

continuity connection from the bearing connection. Thus, the horizontal continuity steel can yield without destroying the bearing connection. Because of possible unforeseen conditions, item 3 is highly desirable even if conditions 1 and 2 are satisfied. These requirements are met by the detail shown in Fig. 16. The negative rebar is sized either for full or partial live-load negative moment. This rebar must be anchored into the cast-in-place concrete by the

hooped stirrups which project up from the beam. These stirrups are sized by $A_v = T_u/f_y \tan \phi$, in which A_v = the total stirrup area, and T_u = yield tension of the negative rebar. The beam bearing shoe is not welded to the corbel, to avoid needless exposure to horizontal tension. If tension is sure to be absent, H_u = zero. If otherwise, the maximum possible H_u is then the vertical reaction times the static coefficient of friction, μ, between the corbel and the beam. Therefore, H_u may be either zero, in which case the auxiliary rebar is designed by Eq. 1, or μV_u. For the latter, the anchor shoe rebar (Fig. 13) is sized by Eq. 3, in which $A_s = (H_u + V_u/\tan \phi)/f_y$, as the hoops (Fig. 14) are sized to match. Also, the corbel steel must be designed for the same H_u. Note that if

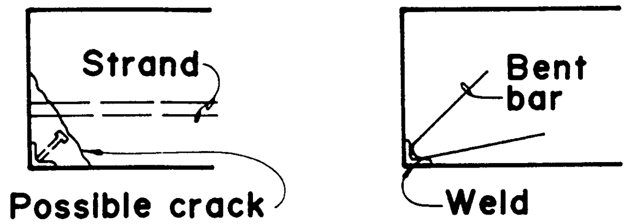

FIG. 15.—WEAK BEAM DETAILS

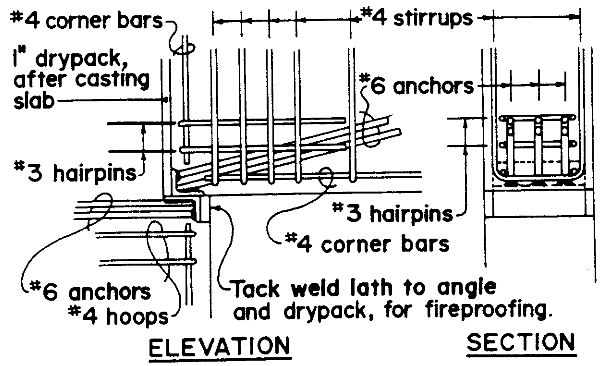

FIG. 16.—BEAM BEARING DETAIL

the coefficient of friction, μ, is 0.7, the amount of shear-friction steel is doubled.

This description is undoubtedly conservative, since the writer's firm has designed, using Eq. 1, many precast and prestressed buildings generally fulfilling conditions 1 and 2, but with beams welded to corbels (details otherwise similar to Fig. 16), with no known instances of connection distress.

LAPPED SPLICES

Lapped splices near points of maximum tensile or compressive stress should generally be avoided. However, they are sometimes necessary. In such cases, there is a possibility of splitting of the concrete between the bars. To protect against this, hooping should be placed around the bars, as shown in Fig. 17. This hooping may be computed by $A_s = T_u/f_y \tan \phi$, exactly analogous to the hoop design in the preceding example.

The Comite European du Beton (CEB) Code (10) gives a treatment of this problem, based on a truss analogy. If $\tan \phi$ is assumed as 1.0, the shear-friction hypothesis gives the same results as the CEB Code.

An important effect of stirrups in ordinary reinforced beams is their action in preventing the tension reinforcement from separating from the bottom of the

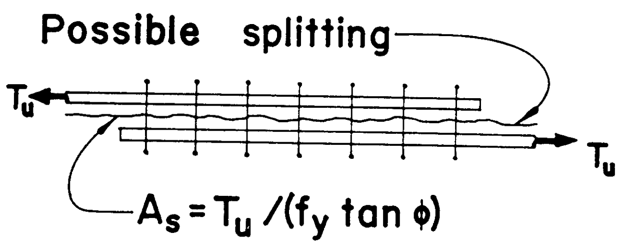

FIG. 17.—LAPPED SPLICE

beam (11). This action is easily understood, using the principles discussed in the preceding paragraph.

CONFINEMENT REINFORCING

High compressive stresses often exist in local zones of connections, and the elastic stresses may be decidedly nonuniform, due to uneven bearing caused by construction tolerances. The complex state of elastic stress may result in cracks, whose location and orientation are unpredictable. Confinement reinforcing may be used, to prevent such cracks from causing a failure. The required reinforcement may be determined from the shear-friction hypothesis. Consider a cracked specimen, loaded with the compressive stress f_1 (Fig. 18). To prevent slippage along the crack, a confining pressure, f_2, is required. The crack may be at any angle, α. The critical angle, α, will be $45° + \phi/2$, and the required confining pressure may be found from a Mohr diagram (Fig. 18). The relation between f_1 and f_2 is

$$f_1 = f_2 \tan^2 \left(45° + \frac{\phi}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

In the previous applications of shear friction, the angle of internal friction, ϕ, was taken as 54.5° ($\tan \phi = 1.4$). This was determined from tests in which the stress was in the range of 10% to 20% of f'_c. However, confinement reinforcing is normally used in cases where the stress at ultimate load will be in the neighborhood of f'_c. Therefore, when sizing confinement steel, a smaller angle of internal friction should be used. If ϕ is taken as 37° ($\tan \phi = 0.75$), Eq. 5 gives $f_2 = 0.25 f_1$. This is in good agreement with tests conducted by Krahl, et al. (6). Numerous tests of spiral columns have shown that spiral reinforcement is about 2.0 times as effective as the same volume of reinforce-

ment placed vertically. This is equivalent to saying that the additional vertical pressure which may be sustained by virtue of the spirals is four times the lateral confining pressure induced by the spirals, corresponding to $f_2 = 0.25 f_1$.

Thus, confining reinforcement may be sized to provide a lateral confining stress of 25% of the main stress. This is a conservative approach, since it completely neglects the strength the concrete would have without confining reinforcement. Also, the ratio of major stress to minor stress is quite sensitive to the exact value of ϕ, which will be larger for smaller stresses. Nevertheless, the shear-friction theory gives insight into the demonstrated value of confinement reinforcing. It should be noted that this description on confinement reinforcing is not based on test data or directly applicable field experience.

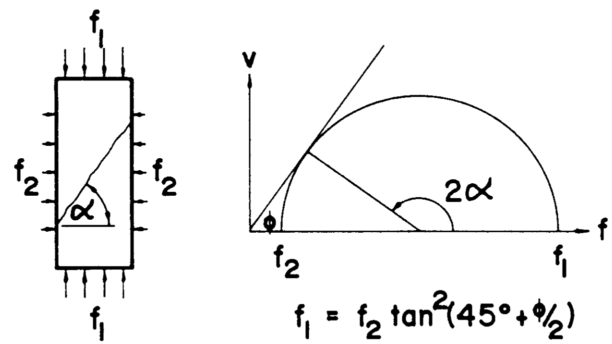

FIG. 18.—CONFINEMENT

Therefore, it should not be used for designs with the major principal stress greater than f'_c unless ad hoc tests are made to verify the design.

FUTURE RESEARCH

A considerable amount of test data exists, showing that the shear-friction model will give good results for various applications. Research does not exist, however, to prove that the model represents the true behavior of concrete. The key point is the action of the steel in tension, rather than as a dowel. This could readily be proved (or disproved) by testing specimens in which the reinforcement is external to the concrete, and is anchored only in tension. Having performed these tests, attention could then be directed to the factors which influence the magnitude of $\tan \phi$.

Much of the current research in concrete connections seems to be directed toward finding complex empirical relations to fit a specific set of test data. The designing engineer is often faced with configurations which do not fit the geometry range of any possible set of a priori tests. Therefore, physical models which permit reasonable extrapolation are sorely needed. The models need not be exceedingly accurate, provided they will predict a safe, lower bound of strength. Tests should be used to verify rational approaches (based on physical models of behavior), rather than to create irrational approaches (based on statistical curve fitting to large numbers of empirical data points). The shear-friction hypothesis provides the basis for a rational approach to design of connections.

CONCLUSIONS

An understandable, simple method has been presented for the design of auxiliary reinforcement in concrete connections. This method is based on a physical model, which may be judiciously used to extrapolate designs into regions not covered by previous tests. Rather than attempting to predict the nature of cracking in a concrete connection, cracking in an unfavorable manner is assumed. Reinforcement is then provided, in accordance with the shear-friction hypothesis, to assure that this cracking will not lead to premature failure. The method has a wide range of applications, as described in this paper. Because the method is based on a physical model rather than on a curve fitted to empirical data, it may be circumspectly extended to other applications as well.

ACKNOWLEDGMENTS

The origin of the shear-friction idea is unknown. The idea was first introduced into the writer's office by Ernest Basler. The idea has been further developed by the writer's partners, particularly by Halvard Birkeland. Several European papers contain the idea in implicit form, and a recent Russian paper (Ref. 7) states the concept explicitly. The concept has been in use in designs by the writer's office since 1958.

APPENDIX I.—DESIGN EXAMPLE

This example goes through the design of a prestressed beam and its connections, using the shear-friction hypothesis. Those parts of the design which use the hypothesis are given in detail. Other parts are summarized, and follow ACI 318-63. Where nomenclature is not given, see ACI 318-63.

Given information is:

1. Multiple span precast, pretensioned beams on 15 ft centers, interior bay clear span = 48.5 ft, assumed cross section 12 in. × 30 in., f'_c = 6.0 ksi, f'_{ci} = 4.5 ksi.
2. Beams continuous for live load moment only, composite with 5-in. cast-in-place slab. Slab f'_c = 3.0 ksi. Negative moment steel at supports in slab, f_y = 40.0 ksi. Other rebar, f_y = 40.0 ksi. Live load = 100 psf.
3. Space limitations require beam bearing and column cap details similar to Fig. 16.
4. Walls and columns are flexible, offering only small restraint. Foundation conditions are good, with very small differential settlements expected.

Required information for an interior span is: (1) Design of interface shear connection of slab to beam; and (2) design of beam and column bearings.

Loads on Beam.—The beam acts as a simply supported member for dead loads, and as a continuous member for live loads. Loads are as follows.

Distributed Loads:
At service load, due to dead load,

$$w_D = w_{\text{beam}} + w_{\text{slab}}$$
$$= \frac{12 \times 30}{144} \times 0.160 + \frac{5}{12} \times 0.150$$
$$= 0.400 + 0.938 = 1.338 \text{ kips per lin ft}$$

Live load,

$$w_L = 15 \times 0.100 = 1.500 \text{ kips per lin ft}$$

At ultimate load (ACI 318-63, Eq. 15-1),

$$w_u = 1.5w_D + 1.8w_L = 1.5 \times 1.338 + 1.8 \times 1.500 = 4.71 \text{ kips per lin ft}$$

Shear (Reaction) at Support:
At service load,

$$V_D = w_D \frac{L}{2} = 1.338 \times \frac{48.5}{2} = 32.4 \text{ kips}$$

$$V_L = w_L \frac{L}{2} = 1.500 \times \frac{48.5}{2} = 36.3 \text{ kips}$$

At ultimate load (ACI 318-63, Eq. 15-1),

$$V_u = 1.5V_D + 1.8V_L = 1.5 \times 32.4 + 1.8 \times 36.3 = 48.6$$
$$+ 65.4 = 114.0 \text{ kips}$$

Negative Moment at Support:
At service load,

$$M_D = 0$$

$$M_L = w_L \frac{L^2}{11} = 1.500 \times 48.5^2 \times \frac{12}{11} = 3{,}850 \text{ kip-in.}$$

(ACI 318-63, Sect. 904 (c), considered realistic for this case)
At ultimate load (ACI 318-63, Eq. 15-1),

$$M_u = 1.5M_D + 1.8M_L = 0 + 1.8 \times 3{,}850 = 6{,}930 \text{ kip-in.}$$

Positive Moment at Midspan:
At service load,

$$M_D = w_D \frac{L^2}{8} = 1.338 \times 48.5^2 \times \frac{12}{8} = 4{,}720 \text{ kip-in.}$$

$$M_L = w_L \frac{L^2}{16} = 1.500 \times 48.5^2 \times \frac{12}{16} = 2{,}650 \text{ kip-in.}$$

(ACI 318-63, Sect. 904 (c), considered realistic for this case)

At ultimate load (ACI 318-63, Eq. 15-1),

$$M_u = 1.5M_D + 1.8M_L = 1.5 \times 4{,}720 + 1.8 \times 2{,}650 = 7{,}090$$
$$+ 4{,}760 = 11{,}850 \text{ kip-in.}$$

The beam is analyzed. Pretension is selected as 354 kips total (10-1/2-in. diam 270K strands straight, and 5-1/2-in. diam 270K strands harped at mid-span) with eccentricities of 10.0 in. at midspan and 3.6 in. at the supports. With full service load, the composite beam just reaches cracking stress at midspan. The stress blocks for the beam are such that it will creep downward under long-term load. Shear is investigated by Eqs. 26-12 and 26-13 of ACI 318-63. Only nominal stirrups are required, #4 at 22 in. o.c., by Eq. 26-11 of ACI 318-63. As will be shown later, more stirrups will be required due to horizontal shear between the slab and the beam. To avoid problems of welding anchor bars to shoes and of field-bending stirrups, all rebars are selected as ASTM A15 Intermediate Grade (f_y = 40 ksi, with anchor rebars limited to 0.35% carbon).

Interface Shear Connection.

Find Moment Couple at Support:

Effective couple arm,

$$jd = d - \frac{a}{2} \cong 30 \text{ in.}$$

$$C_u = T_u = \frac{M_u}{jd} = \frac{6{,}930}{30} = 231 \text{ kips}$$

Check jd (ACI 318-63, Sect. 1601 (a)):

$$d \cong 30 + 5 - 3 = 32.0 \text{ in.}$$

$$a = \frac{C_u}{0.85\, f'_c b} = \frac{231}{0.85 \times 6.0 \times 12} = 3.8 \text{ in.}$$

$$d - \frac{a}{2} = 32.0 - 1.9 = 30.1 > 30, \text{ therefore ok}$$

Find Moment Couple at Midspan:

Effective couple arm,

$$jd = d - \frac{a}{2} \cong 29 \text{ in.}$$

$$C_u = T_u = \frac{M_u}{jd} = \frac{11{,}850}{29} = 409 \text{ kips}$$

Check jd (ACI 318-63, Sect. 1601 (a)):

$$d \cong 30 + 5 - 5 = 30 \text{ in.}$$

$$a = \frac{C_u}{0.85\, f'_c b} = \frac{409}{0.85 \times 3.0 \times 92} = 1.7 \text{ in.}$$

$$d - \frac{a}{2} = 30 - 0.9 = 29.1 > 29, \text{ therefore ok}$$

Thus C_u is entirely within the slab.

Find Total Shear Along Interface Between Slab and Beam (Fig. 19):

$$V_u = C_{u\,\text{midspan}} + T_{u\,\text{support}} = 409 + 231 = 640 \text{ kips}$$

Find Stirrups Required at Interface, For Half-span:

Use Eq. 1. For rough screeded "popcorn" surface of beam cast with no-slump concrete, assume tan ϕ = 1.4. Then, with a capacity reduction factor of 0.85 ACI 318-63, Sect. 1504 (b), the total stirrup area required is $A_v = V_u/0.85 f_y \tan\phi = 640/0.85 \times 40 \times 1.4 = 13.5$ sq in. This would be 13.5/2 × 0.20 = 33.7 sq in. say 34 #4 stirrups. Previously, shear investigation of the beam itself showed that #4 stirrups at 22 in. o.c. (24.25 × 12/22 = 13.2 sq in. say 14 #4 stirrups for half span) were required. Therefore, interface shear governs. This situation is usual for prestressed beams. The stirrups are distributed roughly according to the beam shear diagram (Fig. 20). To

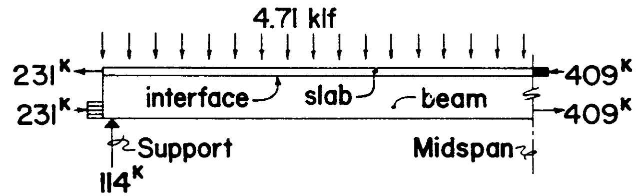

FIG. 19.—FREE BODY, LEFT HALF SPAN

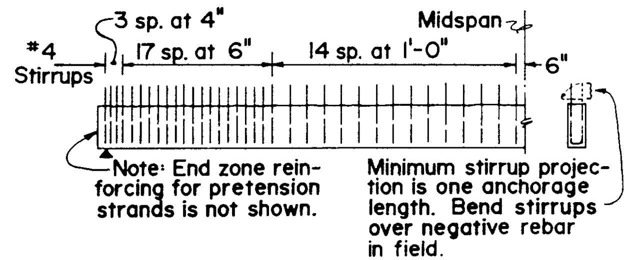

FIG. 20.—STIRRUP ARRANGEMENT

obtain full tensile anchorage in the slab, the stirrups are extended one anchorage length and are bent down (Fig. 20) in the field. Fig. 20 does not show the end-zone reinforcing required at the support for pretension strand bond transfer.

Find Stirrups Required to Anchor Negative Moment Steel to Beam at Support:

The yield tension of the moment steel is the tensile part of the moment couple divided by the capacity reduction factor, or T_u = 231/0.9 = 257 kips. Using a capacity reduction factor of 0.85, the required stirrup area is $A_v = T_u/0.85 f_y \tan\phi = 257/0.85 \times 40 \times 1.4 = 5.4$ sq in. or 5.4/2 × 0.20 = 13.5 sq in. say 14 #4 stirrups. The live-load moment diagram shows that the negative moment steel must extend about 12 ft in from the support. Referring to Fig. 20, about 20 #4 stirrups are provided, which exceeds the 14 required.

Beam Bearing Shoe.

Find Design Reactions:

Referring to the previous description of bearing shoes, conditions 1, 2, and 3 are satisfied, and the horizontal tension, H_u, may be assumed as zero. In reality, this force would be compressive, making the assumption of $H_u = 0$ conservative. The vertical reaction is increased 10% (ACI 512-64, Par. 202.2), and a capacity reduction factor of 0.85 is used, giving $V_u = 1.1 \times 114.0/0.85 = 147.5$ kips.

Size Anchor Steel:

Using Eq. 1 and Fig. 13, $A_s = V_u/f_y \tan\phi = 147.5/40 \times 1.4 = 2.63$ sq in. Use six #6 ($A_s = 6 \times .44 = 2.64$ sq in.). The length beyond the potential crack

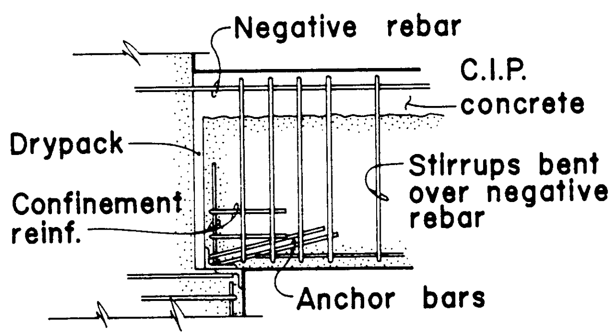

FIG. 21.—BEARING DETAIL

must be the anchorage length of the pretension strand (approximately $50 \times 0.5 = 25$ in.) or of the anchor bars (about 16 in.), whichever is greater. Use six #6 × 2 ft 4 in. (Fig. 21).

Size Stirrups to Transfer Anchor Tension to Strands:

By Eq. 1 and Fig. 14, $A_v = T_u/f_y \tan\phi$. The yield tension of the anchor bars is $T_u = f_y A_s = 40 \times 2.64 = 105.6$ kips. Therefore, $A_v = 105.6/40 \times 1.4 = 1.88$ sq in., or five #4 stirrups ($A_v = 5 \times 2 \times 0.2 = 2.0$ sq in.). Referring to Fig. 20, five stirrups are provided (Fig. 21).

Size Confinement Steel:

To prevent splitting in the plane of the beam due to the support reaction, provide steel to carry 25% of the vertical reaction, $A_s = 0.25\ V_u f y = 0.25 \times 147.5/40 = 0.92$ sq in. Use two #3 horizontal hairpins ($A_s = 4 \times 0.11 = 0.44$ sq in.) to carry half the confinement force. The other half is carried by the adjacent two #4 vertical stirrups already provided (Fig. 21).

Size Anchor Shoe Angle:

For a 12-in. wide beam, with 1 in. at each end for fireproofing, use an angle 10 in. long. The upstanding leg of the well-anchored angle provides excellent confinement to the concrete at the corner. Therefore, the bearing stress may be taken as $1.0 f_c'$ at ultimate load, and $A_{brg} = V_u/f_c' = 147.5/6.0 = 24.6$ sq in. The bearing width must be $24.6/10 = 2.5$ in. Allowing 1/2 in. for clearance, and 1/2 in. for erection tolerance, use a 3-1/2-in. lower leg. To provide room for welding the upper three anchor bars, use a 3-in. upper leg. Referring to Fig. 2, the concrete will exert a compressive pressure on the upstanding leg. Because of the location of the anchor bars, flexure due to this pressure will not limit the thickness of the angle. To permit good welding (one-half the rebar diameter = 3/8-in. fillet all around), use an angle 3 in. × 3-1/2 in. × 3/8 in. (Fig. 21).

Corbel Bearing Shoe.—The corbel is designed in exactly the same way as the beam bearing, with the exception that the concrete area along the shear plane (Fig. 7) must be sufficient to keep the steel reinforcement index, pf_y, below $0.15 f_c'$. Note also that the ratio, a/d, must be below 0.7 (Fig. 7). Otherwise, the corbel steel must be sized by truss analogy instead of by shear-friction (Fig. 21).

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APPENDIX III.—NOTATION

The following symbols are used in this paper:

A_s = area of steel crossing "crack" normal to shear interface; also beam moment steel;
A_v = area of beam stirrup steel;
a = corbel moment arm, distance from column face to load;
b = width of concrete along "crack" at shear interface;
b' = width of area of contact between precast and cast-in-place concrete;
C_u = compression part of moment couple in beam at ultimate load;
C_{1u} = that part of C_u above interface;
C_{2u} = that part of C_u below interface;
d = length or depth of concrete along "crack" at shear interface;
f'_c = cylinder compressive strength of concrete;
f_y = yield strength of reinforcing steel;
f_1 = principal compressive stress on confined concrete, at failure;
f_2 = confining pressure on concrete, at failure due to f_1;
H/V = ratio of horizontal force to vertical force (see H_u and V_u);
H_u = horizontal tensile reaction on corbel or beam bearing at ultimate load;
I = moment of inertia of transformed composite section;
p = steel ratio, A_s/bd;
P_u = vertical reaction on corbel at ultimate load;
Q = statical moment of the transformed area outside of the contact surface about the neutral axis of the composite section;
$\tan \phi$ = tangent of angle of internal friction, ϕ;
T_u = tension in rebar at yield, or tensile part of moment couple in beam at ultimate load, or tension in beam bearing anchor bars at ultimate load;
V = total shear from VQ/Ib';
v_u = average shear stress on area of "crack" at shear interface at ultimate load, V_u/bd, not a measure of diagonal tension;
V_u = shear force along shear interface at ultimate load;
v_y = average shear stress on concrete interface at yield of reinforcing steel (similar to but not the same as v_u);
α = angle of potential failure plane;
μ = static coefficient of friction; and
ϕ = angle of internal friction in concrete, not a capacity reduction factor.