Prestressed Concrete Structures

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Step 8: Check the reserve of strength after cracking.

The ACI Code requires that ϕM_n exceeds $1.2M_{cr}$ where M_{cr}, is calculated on the basis of the modulus of rupture

$$\begin{aligned} f_r &= 7.5\sqrt{f'_c} \\ &= 7.5\sqrt{4500} \\ &= 503 \text{ psi (3.47 MPa)} \end{aligned}$$

The moment to cause a tensile stress of 503 psi (3.47 MPa) on the top surface at B can be calculated from the stresses given in Table 10-3 as

$$503 = -479 - 1258 + \frac{M_{cr}}{12 \times 20^2/6}$$

Hence

$$M_{cr} = 1792 \text{ in.-kips/ft} = 149 \text{ ft-kips/ft (664 kNm/m)}$$

Thus, at B,

$$\frac{\phi M_n}{M_{cr}} = \frac{247}{149} = 1.66 > 1.20$$

Hence post-cracking capacity is adequate.

Similarly, at D,

$$\frac{\phi M_n}{M_{cr}} = \frac{232}{146} = 1.59 > 1.20$$

Step 9: Check the deflections at service loads.

Although some cracking is predicted to occur at support B under full service loading, we will first estimate the deflections by neglecting the influence of this cracking. As there are no partitions in this assembly hall, only the immediate deflection due to live load needs to be checked.

For a two-span continuous beam, the maximum deflection due to loading one span is

$$\Delta = \frac{1.6w\ell^4}{185EI} = \frac{1.6 \times 0.100 \times 64^4 \times 12^3}{185 \times 3824 \times 12 \times 20^3/12} = 0.82 \text{ in. (21 mm)}$$

As the maximum permissible deflection for live load is $\ell/360 = 65 \times 12/360 = 2.17$ in. (55 mm) the deflection due to live load will not be critical. Cracking over the support will cause a local reduction in stiffness, but as the midspan deflections are influenced mainly by the stiffness in the span, cracking only over the support will cause no more than a 15% increase in midspan deflections.

10.15 INFLUENCE OF RESTRAINT MOMENTS ON LOAD DISTRIBUTION AT ULTIMATE

A number of engineers have argued that while it is appropriate to include the restraint moments due to prestress when checking service load conditions, these "secondary" moments should be ignored when checking overload capacity ("ultimate loads"). It is contended that once the member has cracked, the calculated restraint moments have no relation to reality.

Figure 10-53 illustrates the manner in which the support moment for the two-span, continuous slab, designed in Section 10.14, increases as the uniform load on the slab increases. Three predicted relationships are shown in the figure: an elastic analysis including the prestress restraint moments, an elastic analysis neglecting these restraint moments, and a nonlinear analysis which accounts for the influence of concrete cracking and material nonlinearities. This nonlinear analysis was performed using the computer program TEMPEST, developed by Vecchio (Ref. 10-7).

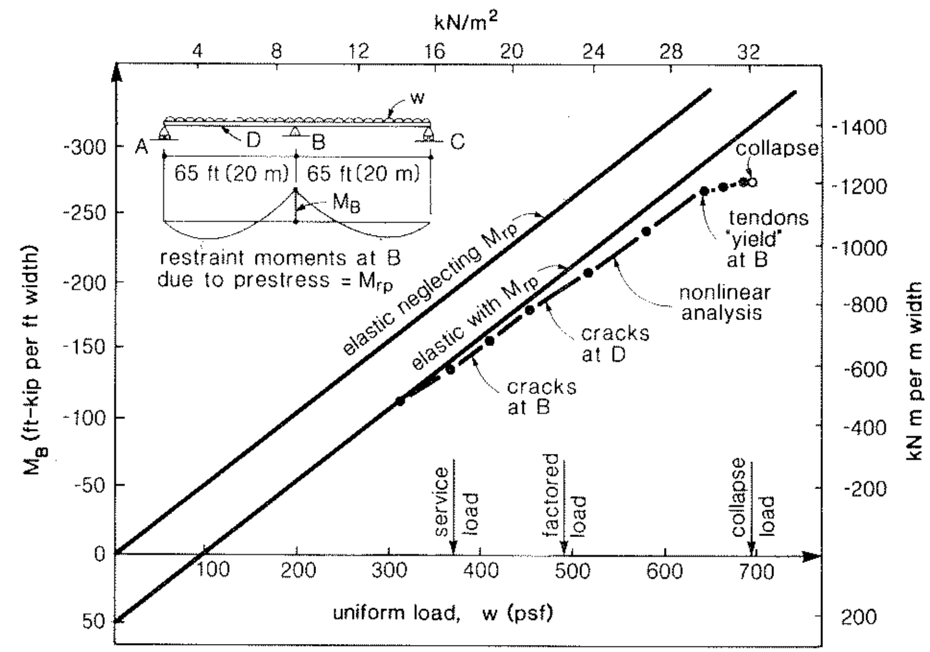

Figure 10-53 Predicted magnitude for the moment at support B.

From Fig. 10-53 it can be seen that including the influence of restraint moments resulted in an elastic analysis which gave values closer to those of the more sophisticated, nonlinear analysis. Figure 10-53 also shows that considerable redistribution can occur prior to failure. That is, the moment over the support may increase at a slower rate, while the moment in the span increases at a faster rate. The ACI Code permits allowance to be made for such redistribution. Thus the one-way slab considered could be designed for a support moment which is reduced by

$$20\left[1-\frac{\omega_p+\frac{d}{d_p}(\omega-\omega')}{0.36\beta_1}\right]\text{ percent} = 6.8\%$$

provided that an appropriate increase is made in the corresponding moments in the span. Note that a redistribution of considerably more than 6.8% would be required to reduce the elastic moments, calculated ignoring restraint, to the "nonlinear" values.

It is true that an elastic analysis based on uncracked stiffness values can lead to unrealistically high values of restraint moments, particularly for those cases involving the restraint of axial deformations (see Section 10.10). However, this deficiency in the elastic analysis does not justify neglecting the presence of restraint actions. Such a neglect would violate the code limitations on allowable amounts of redistribution. To obtain more realistic estimates of the restraint moments, either a nonlinear analysis can be performed, or the flexural stiffnesses of the members can be reduced to allow for cracking.

References

10-1 Leonhardt, F., *Prestressed Concrete – Design and Construction*, English translation 2nd ed., Wilhelm Ernst und Sohn, Berlin, 1964, 677 pp.

10-2 Standing Committee on Structural Engineering of the Federal Construction Council, Building Research Advisory Board, Division of Engineering, "Expansion Joints in Buildings," Technical Report No. 65, National Research Council, National Academy of Science, Washington, D.C., 1974.

10-3 Prestressed Concrete Institute, *PCI Design Handbook: Precast and Prestressed Concrete*, 3rd ed., PCI, Chicago, 1985.

10-4 Cross, Hardy, "Analysis of Continuous Frames by Distributing Fixed-End Moments," *Transactions of the American Society of Civil Engineers*, Paper No. 1793, May 1930, 10 pp.

10-5 Vecchio, F.J., and Sato, J.A., "Thermal Gradient Effects in Reinforced Concrete Frame Structures," *ACI Structural Journal*, Vol. 87, No. 3, May-June 1990, pp. 262–275.

10-6 ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-89) and Commentary – ACI 318 R-89," American Concrete Institute, Detroit, 1989, 353 pp.

10-7 Vecchio, F.J., "Nonlinear Analysis of RC Frames Subjected to Thermal and Mechanical Loads," *ACI Structural Journal*, Vol. 84, No. 6, Nov.-Dec. 1987, pp. 492–501.

Demonstration Problems

10-1 The cast-in-place concrete frame shown in Fig. 10-54 is composed of rectangular members 10 × 20 in. (254 × 508 mm) in cross section. When the three frictionless tendons are post-tensioned to a force of 200 kips (890 kN), what will be the values of the restraint actions induced at the supports? What will be the horizontal deflection of point A due to stressing of the tendons? Assume that E_c = 4000 ksi (27 600 MPa).

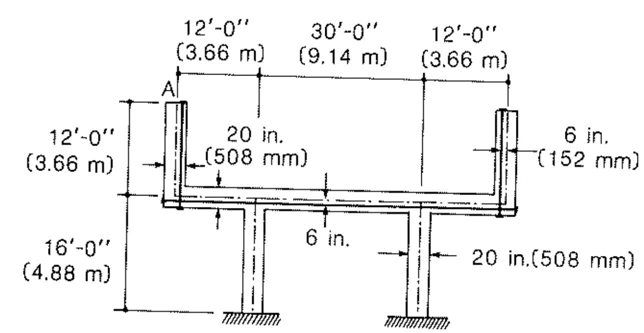

Figure 10-54 Cast-in-place, post-tensioned frame.

10-2 Determine the fixed-end moments caused by post-tensioning the four members shown in Fig. 10-55.