Frame Analysis of Concrete Buildings

by M. Daniel Vanderbilt and W. Gene Corley

Plane frame analysis is often used in analyzing unbraced reinforced concrete buildings for horizontal and vertical static loads. Modeling the stiffness of the beam-column connections in the frames is complicated by the difficulty in correctly defining the path of moment transfer. Three models of connection behavior are discussed; the equivalent beam width, transverse torsional member, and stub beam models.

The Equivalent Frame Method, developed around the torsional member model, was introduced in the ACI 318-71 code for the analysis of single floors under vertical load. A procedure for extending the method to encompass complete buildings of either flat plate or flat slab construction (divided into parallel frames) carrying gravity and lateral loads is described. Results of studies demonstrating the effects of changes to the commentary to ACI 318-77 are given.

Keywords: beam-column frame; beams (supports); columns (supports); compressive strength; connections; edge beams; **flat concrete plates; flat concrete slabs;** flexural strength; frames; framing systems; lateral pressure; loads (forces); mathematical models; moment distribution; reinforced concrete; **static loads;** stiffness; stiffness methods; **structural analysis;** torsion.

The structural design of a reinforced concrete building typically requires that a structural analysis be made to compute actions and displacements. The actions are used in sizing the structure to have adequate strength. The displacements are used in providing for sufficient stiffness to produce a serviceable structure.

Rational analysis has often lagged behind the design and use of concrete buildings. Frame analysis of concrete buildings was not discussed in print until 1929[1,2] and was first recognized by a building code in 1933.[3] Provisions for frame analysis first appeared in the ACI Building Code in 1941.

These provisions continued in successive editions with little change until replaced by the equivalent frame method in 1971.[2] Experience with buildings designed using frame analysis shows that their strength is usually satisfactory but that problems with excessive drift occasionally occur.

Although finite-element analyses of complete buildings can be performed, their relatively high cost coupled with the generally good results obtained using frame analysis, indicate that frame analysis will continue to be the choice of many designers. The analyst performing a frame analysis faces a number of interesting problems including how to divide the structure into planar frames, how to model the beam-column connection, and how to mathematically model the idealized structure.

The objectives of this article are to (a) describe frame analysis concepts, (b) describe three models available for the beam-column connection, (c) show how the Equivalent Frame Method can be modified to encompass entire buildings (modeled as parallel planar frames), and (d) present results of studies made using the modified method. The type of structure considered is the unbraced reinforced concrete building consisting of horizontal floors and vertical columns with no other bracing members.

Frame analysis concepts

A common procedure for analyzing the building of Fig. 1a for gravity loads is to divide it into six frames by passing vertical cutting-planes through the structure midway between adjacent column planes, analyze each frame separately, and superimpose the results. Moments obtained for columns and slab sections are suitable for design but, since all gravity loads are carried twice, the column axial forces sum to twice the applied load and must be reduced for design, usually by dividing by two. The vertical deflections of a column may be different in each frame containing it when this simple expedient is used.

A basic concept of frame analysis is that the cutting planes are assumed to be planes of zero shears and twisting moments. Hence the loading on a frame

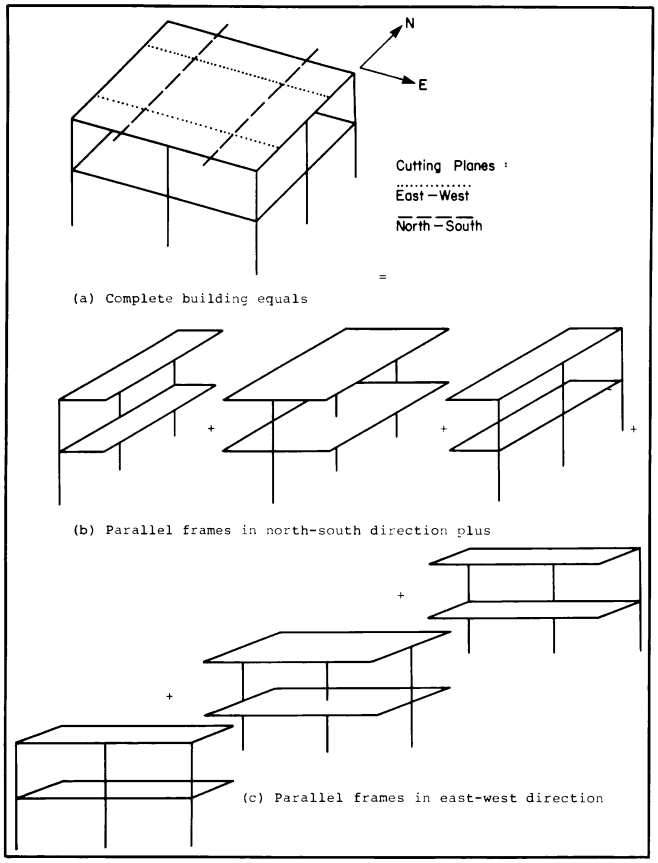

Fig. 1 — Division of building into parallel plane frames.

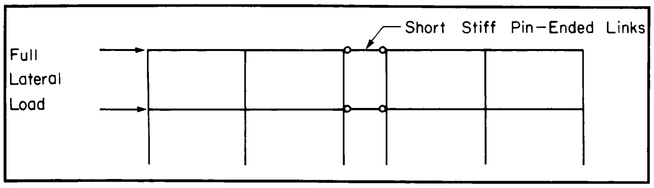

Fig. 2 — Parallel frames linked to force drift continuity.

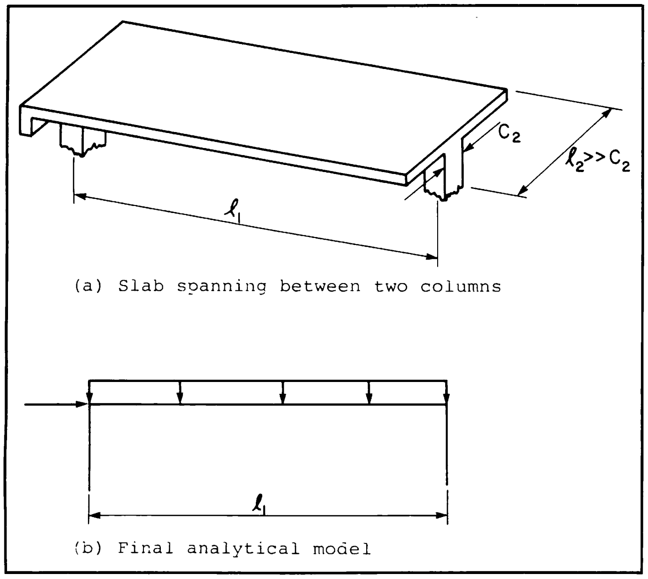

Fig. 3 — Reduction of three-dimensional structure to two-dimensional structure.

derives from the tributary area contained within the frame. The zero shear assumption is exact for planes of symmetary and is reasonable for interior panels of gravity loaded structures. For panels adjacent to a discontinuous edge (all panels in Fig. 1a), the zero shear surface is not necessarily planar and will tend to be located closer to the discontinuous edge than to the first interior support. These complications are usually ignored in practice.

The distribution of lateral load among parallel frames is far more sensitive to frame stiffness than is the gravity load distribution. For example, consider the three east-west frames of Fig. 1c. Assume that the two edge frames are supported on walls instead of columns. This reduces the rotation of an edge slab about its discontinuous edge and thus the zero shear plane moves toward the panel center. Under these conditions, the real and assumed gravity load distributions are nearly equal, but the transverse load is carried almost entirely by the edge frames. Consequently, the division of lateral loads on a tributary width basis is unsatisfactory for this case.

Linking the parallel plane frames to form one large linked plane frame is one way to enforce deflection compatibility. Thus the three east-west frames of Fig. 1c would be linked as shown in Fig. 2 and the entire transverse load applied to one frame. The linked frame modeling is recommended for all analyses. The links in Fig. 2 are symbolic. Actual linking can be accomplished through programming.

Modeling of beam-column stiffness

A single span of a plane frame is shown in Fig. 3a. The slab and its supporting columns must be reduced to the analytical model shown in Fig. 3b before the plane frame analysis can be made. It is common to assume that all members entering a joint undergo the same end rotation.

This classical assumption would be reasonable for the frame of Fig. 3a if ℓ_2 equaled c_2. Since ℓ_2 is larger than c_2, the path of moment transfer between column and beam is not easily determined and thus proper modeling of the stiffness of the beam-column connection is an important problem. Prior to 1971 the ACI Elastic Analysis Method used the classical assumption. The beam moment of inertia was computed using the width ℓ_2 and a conventional analysis was performed. Consequently computed exterior negative moments for gravity load were higher and drifts for lateral load were smaller than those for real structures.

While an exact model of connection stiffness has not been developed, several approximate models are available. The three major models are the (a) equivalent beam width model, (b) transverse torsional member model, and (c) stub-beam model. Each model is discussed here.

Equivalent beam width model

An elastic rectangular plate clamped at one end and supported on a column of width c_2 at the opposite end is shown in Fig. 4a. If the column undergoes rotation θ, the plate rotation along AA will vary as shown in Fig. 4b from θ at the column to smaller values away from the column, and can diminish to zero if ℓ_2 is sufficiently large. The equivalent width factor, α, is obtained from the requirement that the stiffness of a prismatic beam of width $\alpha\ell_2$ must equal the stiffness of the plate of width ℓ_2. This equality is obtained if the areas under the two rotation diagrams of Fig. 4b are equal.

Numerous techniques for computing α have been developed. The usual method for performing an analytical study is as follows:

1. A portion of a continuous flat-plate structure is isolated for study. The portion is usually rectangular[1,4,5,6,7,8,9,10,11] but may be round.[12] The model considered by most investigators for the analysis of laterally loaded structures is shown in Fig. 5. Other boundary conditions have been considered.

2. A round or rectangular column is located at the panel center. Two bounds on column stiffness can be considered, the rigid column and the fully flexible column. The rigid column can undergo no deformation within its boundary. The fully flexible column is obtained by designating an area at the plate center to be treated as a column. A distributed loading, whose resultant is a couple, is then applied over the area. The rigid column models an elastic structure while the flexible column does not model a realistic structural system. However, the flexible model is readily analyzed and provides a lower bound on the stiffness of a linearly elastic connection.

3. A mathematical technique is employed to compute the column rotation. The ratio of column moment to column rotation gives the connection stiffness. The stiffness of the isolated panel treated as a reference beam, with appropriate boundary conditions, is next computed. The ratio of plate to beam stiffnesses is the effective width ratio, alpha. Other information such as carry-over factors and distributions of moments, shears, and deflections is sometimes computed.

Studies show that effective width factors are a function of boundary conditions, type of loading, column stiffness, aspect ratio (ℓ_2/ℓ_1), column size (c_1/ℓ_1, c_2/c_1), Poisson's ratio and whether the reference beam contains a zero or finite-size joint. Alpha varies significantly as the aspect ratio varies from near zero to about unity but remains nearly unchanged with further increase.[10] This observation indicates that, as ℓ_2 increases, a value is eventually reached for which the plate rotation in Fig. 4b reduces to zero away from the column, and that for larger ℓ_2 the rotation diagram remains essentially unchanged.

A listing of the investigations found in the literature is given in Table 1. The first effective width study was apparently made by Tsuboi and Kawaguchi in 1965.[4,13] Several analytical techniques have been used with the Levy, finite difference, and finite element methods most popular.

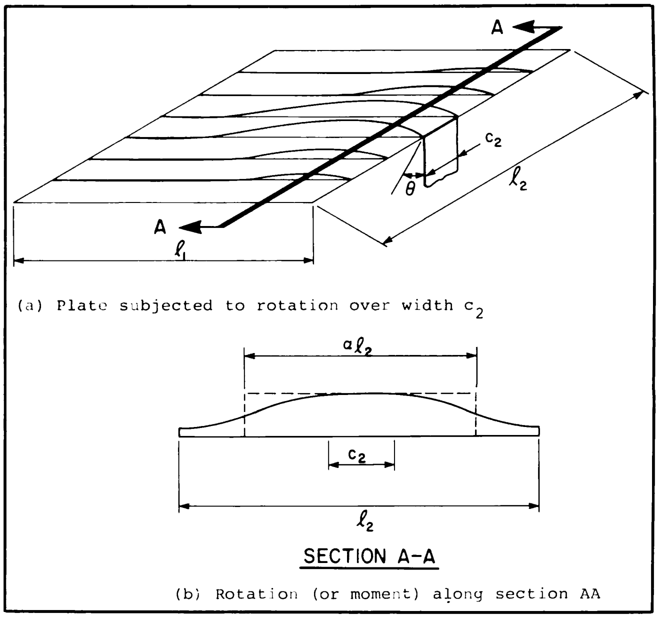

Fig. 4 — Equivalent beam width concept.

A comparison of results obtained by several investigators for a square panel containing a square column is shown in Fig. 6. The values shown are for the lateral loading case, Poisson's ratio of zero, and a support size in the reference beam of zero. Hence the α values shown are intended for use with a computer program which assumes zero size joints. The upper bound (rigid column) values are similar which shows that the different analytical techniques tend to produce similar answers.

For a rigid column and c/ℓ values greater than about 0.15, the α values exceed unity. This occurs because the effective width technique corrects for two factors at once. The first is the reduction in stiffness explained with reference to Fig. 4. The second is an increase in stiffness caused by finite rigid joint size. For large c/ℓ the increase for joint size is greater than the loss due to plate action and thus α exceeds unity.

The lower bound values for Allen and Darvall and Mehrain and Aalami are nearly the same showing that the different analytical procedures they used produced consistent answers. Values obtained by Khan and Sbarounis were based on a fairly simplistic testing program and a coarse plane grid analytical model.

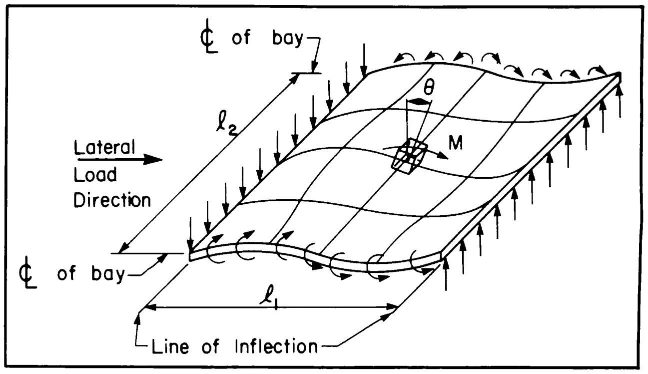

Fig. 5 — Plate-column structure analyzed in equivalent beam-width studies.

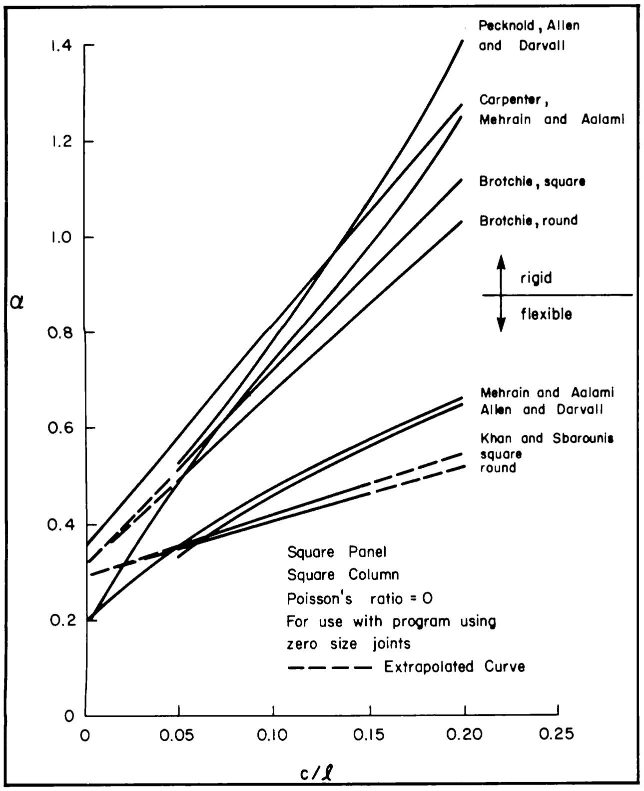

Fig. 6 — Summary of effective width factors for lateral loading, square panels.

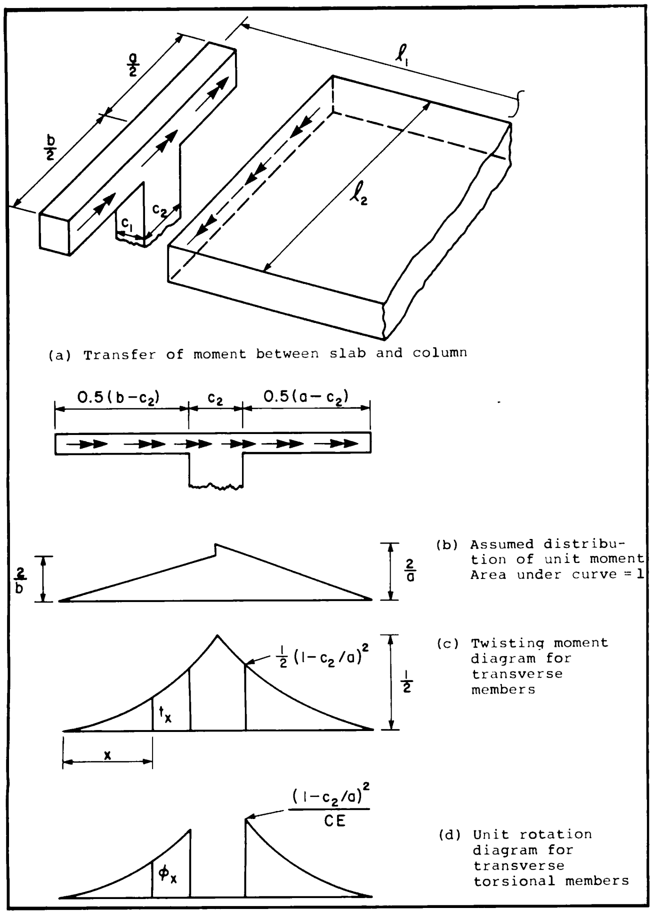

Fig. 7 — Development of torsional stiffness.

Transverse torsional member model

A part of a flat plate structure with the slab and column separated at the column face is shown in Fig. 7a. The transverse torsional member model of the slab-column connection is based on the assumption that the portions of the slab attached to the column, plus transverse beams if any, act as torsional members.

For the purpose of defining connection stiffness, the torsional members are assumed to undergo only torsional rotations but are otherwise rigid. Part of the moment transfer occurs directly between column and slab width c_2 while the remainder is transferred through the torsional members. Rotational stiffness of the joint is a function of the torsional stiffnesses of the transverse members and the flexural stiffnesses of the columns framing into the joint from above and below.

A unit moment to be transferred between column and slab is assumed distributed as shown in Fig. 7b and the resulting torsional moments are shown in Fig. 7c. The unit torsional rotation shown in Fig. 7d is obtained by dividing the torque by CG and where C is the torsional constant and G is the shear modulus of elasticity, taken as $E/2$.

Rotational stiffness is defined as moment divided by rotation where both quantities occur at the same point. In Fig. 7c and 7d the torsional moment and rotation are distributed over a length and thus the moment/rotation quotient must be integrated over the

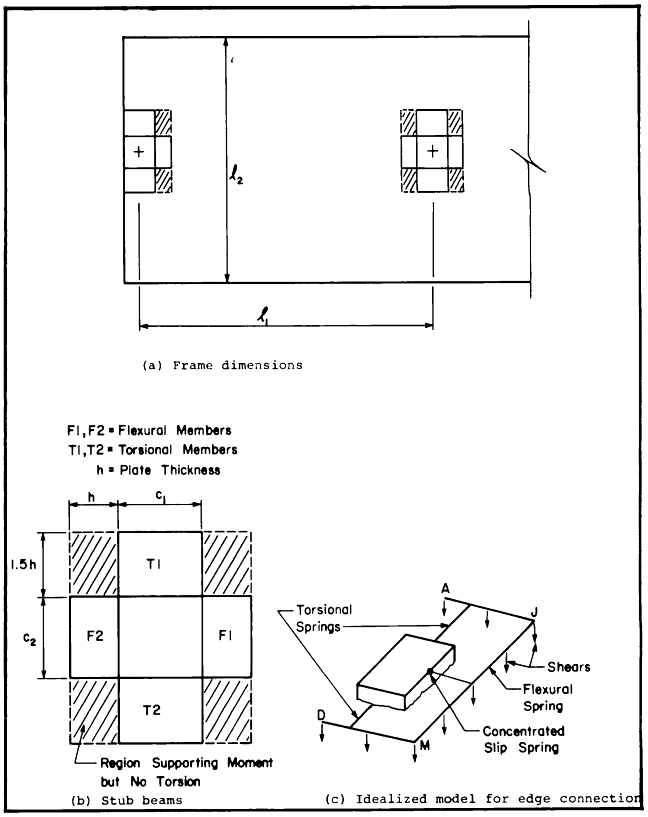

Fig. 8 — Beam analogy for moment transfer by Hawkins.

torsional member lengths. Rather than performing this integration, an approximate procedure is followed.[14,15]

The rotation, θ_t, of one torsional member is defined as one-third the area of its unit rotation diagram and the stiffness is computed as the moment transferred over the member length. 0.5, divided by θ_t. The two transverse torsional member stiffnesses are summed to give the total torsional stiffness as

$$K_t = 9EC/a\,(1 - c_2/a)^3 + 9EC/b(1 - c_2/b)^3 \qquad \text{Eq. (1)}$$

where a and b are the ℓ_2 values on either side of the column. The value of K_t given by Eq. (1) is not correct for an elastic analysis. Rather, it is an approximate equation for modeling joint stiffness that has been calibrated through comparisons of computed and measured moments for the University of Illinois and other test slabs.[15,16]

The torsional members and columns act as springs in series. Their flexibilities are summed to give

$$\frac{1}{K_{ec}} + \frac{1}{K_t} + \frac{1}{\Sigma K_c} \qquad \text{Eq. (2)}$$

where K_{ec} = rotational stiffness of an equivalent column which replaces the real columns, each of stiffness K_c, plus torsional members.

The lateral torsional member model of connection behavior was developed at the University of Illinois as part of an investigation that culminated in the development of the Equivalent Frame Method (EFM) of analysis. In this development the unit moment was assumed to be uniformly distributed over the member lengths. Jirsa[16] modified the distribution to that shown in Fig. 7b as part of a study on pattern loadings. Corley and Jirsa described the procedure in a paper[15] on the EFM.

Stub beam model

A portion of a frame including one edge and one interior column is shown in Fig. 8a. The stub beam model is based on the assumption that the slab can be treated as a beam of width ℓ_2 and span ℓ_1, connected to the columns through the stub beams shown in Fig. 8. Part of the moment in the ℓ_1 direction transfers through the flexural members and the remainder transfers through the torsional members.

Strength and stiffness properties for the stub members are computed for all stages of behavior and thus the moment-rotation curve for the connection is defined through failure. The stub beam model was described in a 1971 paper by Hawkins and Corley.[18] Research that forms the basis for the stub beam model has been in progress at the University of Washington since 1968.[19]

An idealized edge column connection is shown in Fig. 8c. Tests have shown that the torsional rotations per unit length decrease to near zero at 1.5h from the column length. This observation fixes the torsional member length. Tests show that the rotation, θ_t, of the free end of a torsional member with

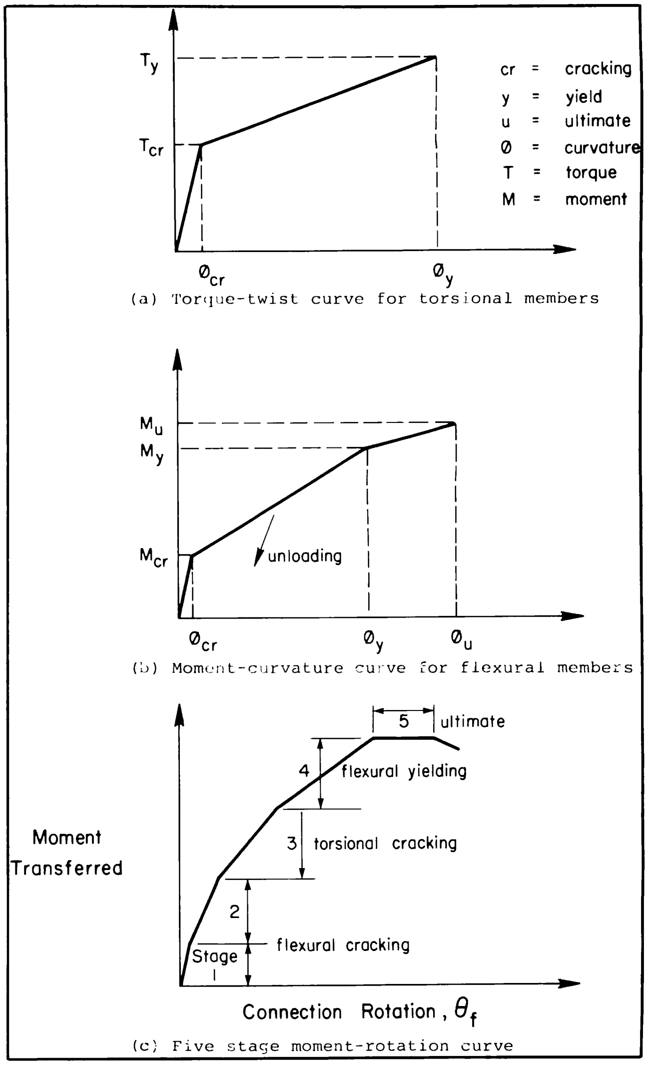

Fig. 9 — Behavior of components and complete edge connection.

respect to its fixed end is centered on the column face.

Modeling of stiffness loss due to plate cracking that occurs at column corners at low load levels is obtained by assuming the hatched areas in Fig. 8 can support flexure but not torsion. Bar AJMD in the idealized connection is rigid to enforce deflection compatibility. Torsional springs attached to the column sides transfer moment through torsion and also support shear. The flexural member attached to the front face transfers moment through flexure and carries the remaining shear. Tests show that significant rotations of the column-slab connection arise from bond slip that occurs over the face of the column supporting the flexural member. This bond slip can be concentrated at a point as shown.

The moment-rotation behaviors are assumed as shown in Fig. 9a,b. Procedures for computing the points on these curves are based on code provisions except that the ultimate point on the torque-rotation curve is computed using an interaction curve. After these two curves have been computed for a given connection, its complete behavior can be defined.

Part of the deflection along edge JM arises from torsional rotations, part from flexure, and part from

TABLE 1 — Effective beam width studies

Investigator	Date	Type of study and comment
Tsuboi and Kawaguchi[13]	1960	analytical study; experimental study of 9 microconcrete specimens; experimental study and agreed within a few percent
Brotchie[7]	1960	analytical study; beam to column and column to beam transfer studied
Khan and Sbarounis[17]	1964	analytical study = plane grid analysis, zero support size; experimental study of aluminum plates to investigate support size
Carpenter[12]	1965	PhD thesis, analytical study; experimental study of multipanel plexiglas plates; extensive study made, good agreement between analytical, test
Aalami[1]	1972	finite difference study
Mehrain and Aalami[9]	1974	finite element, several boundary conditions
Pecknold[10]	1975	Levy analysis
Allen[4]	1976	Levy analysis, MS thesis
Allen and Darvall[5]	1977	Levy analysis
Wong and Coull[11]	1978	Finite element, Levy; results essentially same as Mehrain and Aalami

concentrated bond slip. Procedures for estimating bond slip are available.[19] The five-stage behavior for a connection in a slab that is under-reinforced in flexure is given in Fig. 9c. The process for computing this curve is nonlinear because the curves of Fig. 9a,b are nonlinear and is iterative since the distribution of the total shear and moment among the stub beams is not initially known. The moment rotation curve for an interior connection can exhibit seven stages of behavior.

Comparison of three models

The equivalent beam width model is a child of analysis. Predictive techniques are all based on elastic analyses. The few verification tests made were on elastic specimens. Verification tests of real structures have not been made. The method is appealingly simple since it is only necessary to multiply ℓ_2 by α and proceed to make the analysis using any software package such as STRUDL, SAP IV, etc.

Some analysts obtain an effective width by arbitrarily selecting some portion of the column strip as effective while others select from one of the studies represented in Table 1 and Fig. 6. The analyst who elects to use the effective width model should use a lower bound value since the assumptions made for the lower bound model indirectly and imprecisely account for the loss in stiffness that occurs due to cracking and bond slip. The equivalent beam width model should not be used with gravity load analyses since its use will result in overestimating exterior negative moments.

The transverse torsional member model was calibrated using the results of the most comprehensive studies of reinforced concrete floor slabs made.[20] Test series included only gravity loaded structures. Both the equivalent width and stub beam models were developed only for flat plates. Test structures used in calibrating the transverse torsional model included two flat plates, two flat slabs, and two two-way slabs. Consequently, the transverse torsional member model is more generally applicable.

The stub beam model has been developed based largely on test results. Comparisons between computed and measured moment-rotation curves for many edge and interior test specimens for both monotonic and cyclic loading show excellent agreement. While the stub beam model is a superior research tool for predicting the complete strength-stiffness behavior of a flat-plate to column connection, it is not in its present form a useful design tool. Many observations made during its development are of interest and some have immediate application in design[18,19]

Equivalent frame for gravity load analysis

The transverse torsional member model of moment transfer is a key component of the Equivalent Frame Method. The parts of the method defined in ACI 318-77 are:

1. Frames to be analyzed, and the loads acting on each, are defined as described with reference to Fig. 1. For gravity loads a single floor of a frame may be isolated and the far ends of the columns assumed fixed.
2. Transverse torsional members are defined as described above and the torsional stiffness, K_t, computed using Eq. (1).
3. Beam and column rotational end stiffness, and beam fixed end moments are computed considering the nonprismatic nature of the members.
4. The equivalent column stiffnesses, K_{ec}, are computed using Eq. (2).

The method of analysis is not prescribed in ACI 318-77. However the history of the method, the language of ACI 318-77, and the form of design aids[14] indicate that moment distribution is the expected method. While ACI 318-77 permits the analysis of complete frames for any loading, the method as now described was developed and calibrated only for single floor structures under gravity loading.[3,15] Extension of the method to accommodate the analysis of complete frames for static gravity and lateral loads is described next.

Equivalent frame for complete frame analysis

Two floors from a multistory single-bay frame, and their attached columns, are shown in Fig. 10a. Rotational stiffnesses are computed treating each member as nonprismatic. Beams and columns are shown connected through transverse torsional members of stiffness K_t. These members can deform in torsion but are otherwise rigid. The Equivalent Frame Method in ACI 318-77 requires that the two columns and one torsional spring at each joint be replaced by a single equivalent column of stiffness K_{ec}. This procedure cannot be applied to the frame of Fig. 10a. Hence the procedure must be modified before complete frames can be analyzed.

Extension of the Equivalent Frame Method to encompass complete frames consisted of three steps: (a) developing a procedure for dividing torsional stiffnesses among either the beams or the columns to produce substructures all having the same general configuration, (b) deriving the general form of the stiffness matrix in local coordinates for this general substructure, and (c) writing program EFRAME (Equivalent FRAME) to implement the first two steps in analyzing complete buildings.

The first step was accomplished as follows. Consider a torsional member of stiffness K_t and two columns at a joint. If the torsional spring undergoes a rotation causing a moment K_t in the torsional member, this moment transfers to the columns in proportion to their relative flexural stiffnesses. On this basis it was assumed that the torsional stiffnesses could be divided among the columns in proportion to their relative flexural stiffnesses, as shown in Fig. 10b. The torsional springs can be assigned to the slab-beam members instead of the columns giving a K_{es}, rather than a K_{ec} model. This modeling is discussed later.

Fig. 11a shows the general form of the substructure obtained through the K_t distribution. The substructure consists of flexural member BC in series with two torsional links, AB and CD. Its stiffness matrix is defined by the degrees of freedom shown in Fig. 11b. The substructure of Fig. 11b can be analyzed as a plane grid using familiar stiffness techniques[21] to obtain its stiffness matrix. Details of these computations and the general form of the substructure stiffness matrix are given elsewhere.[22,23]

Program EFRAME was written to implement steps (a) and (b).[22] This program may be used to analyze single frames or linked parallel frames using either the extended EFM or a conventional plane frame analysis. In either case all variations in member geometry such as column capitals, drop panels, brackets, transverse, and longitudinal beams are considered.

Response data, in the form of measured deflections and rotations for gravity and lateral loads are not available for multistory elastic buildings. Thus it is not possible to verify either the extended equivalent frame or the effective beam width models. The EFRAME program has been satisfactorily tested against results from other elastic analyses. Test results for one reinforced mortar model structure are

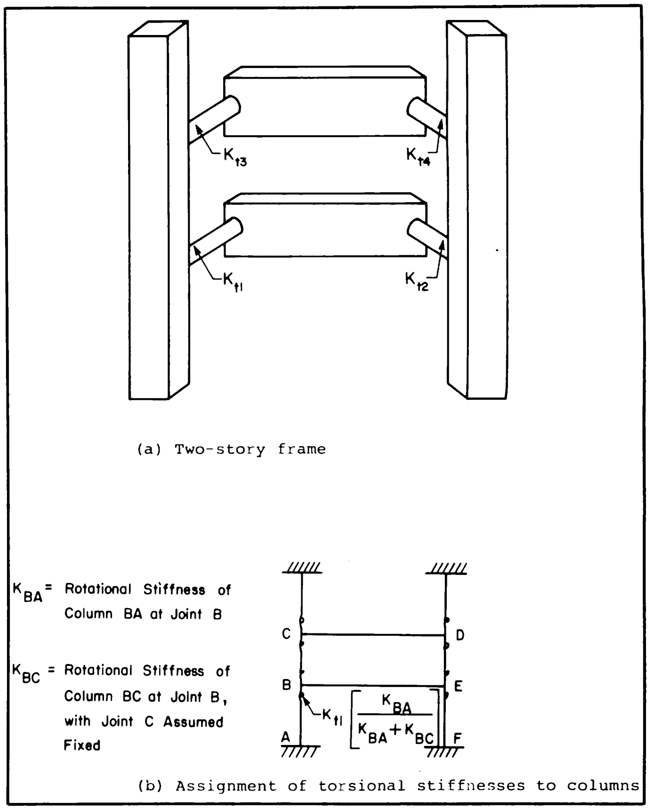

Fig. 10 — Torsional springs in multistory frame.

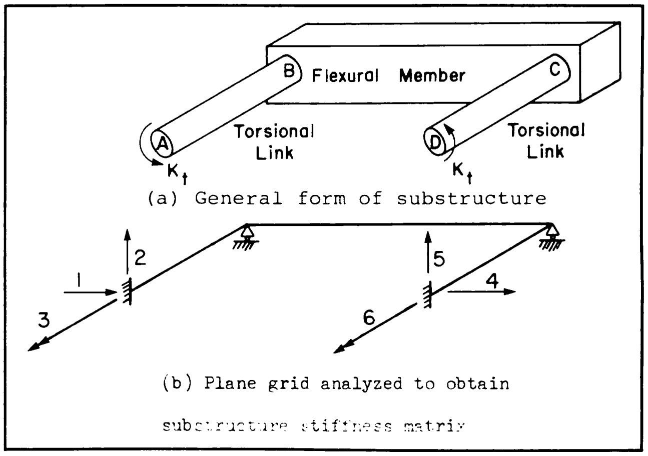

Fig. 11 — Substructure consisting of flexural member in series with two torsional links.

available for comparison with EFRAME calculations.

Eight flat plate structures having the layout shown in Fig. 12 were built in a study of the static and dynamic behavior of reinforced concrete buildings. Information on construction of these models using wire reinforcing and mortar are given by Zelman.[24] Test results for one structure are reported by Hartley.[25] Results for the other seven do not exist. The structure was loaded with forces and torques applied one at a time to floor centers to obtain a flexibility matrix. Gravity load other then self weight was not applied. Hence, the slabs away from the columns experienced little, if any, cracking.

Computed and measured deflections are shown in Fig. 13 for load applied in the transverse direction.

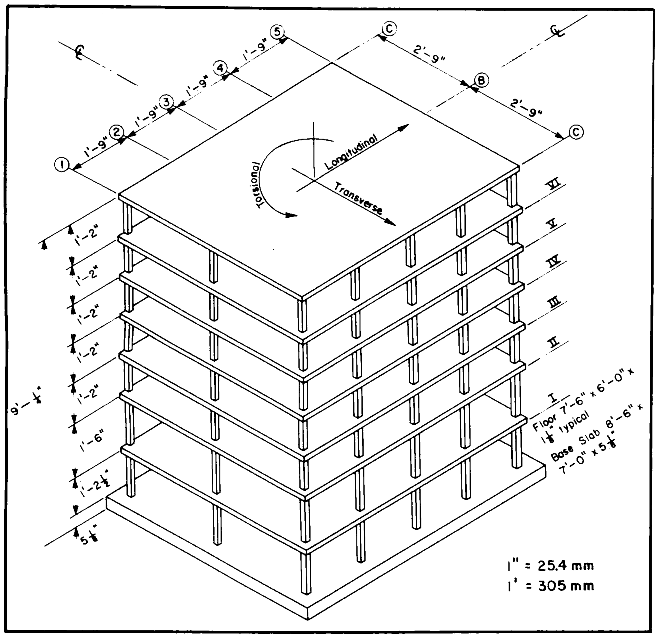

Fig. 12 — Isometric view of Canadian slab model.

Computed drifts for the K_{ec} and K_{es} models were virtually identical. Computed values are about 15 percent higher than measured values. The computed values were obtained using EFRAME and dividing the building of Fig. 13 into three interior and two edge frames linked as shown in Fig. 2.

Computed and measured drifts are shown in Fig. 14 for load applied in the longitudinal direction. Use of the full ℓ_2 value results in computed drifts considerably greater than measured values. Agreement about as good as is shown in Fig. 13 was obtained by reducing the effective ℓ_2 to ℓ_1. Elastic studies made in developing alpha[22] showed it to be insensitive to increases in ℓ_2 for ℓ_2/ℓ_1 greater than unity. Based on these observations it is recommended that the upper limit on the value of ℓ_2 for use in computing K_t be taken as ℓ_1.

Sensitivity studies

Studies were made using the extended Equivalent Frame Method to determine the effects of varying some key parameters and of using different modeling options.[22,26] Pertinent results of these studies follow.

K_{ec} versus K_{es} modeling

Comparisons of computed drifts for laterally loaded buildings show that the K_{es} values are always smaller than the K_{ec} values but are seldom more than 5 percent smaller. Since drift represents the integrated curvatures over all the structure, it is relatively insensitive to whether the columns or the beams are softened by being placed in series with the torsional members. Drifts computed using a conventional plane frame analysis with the equivalent beam width factor alpha = 1, are typically about half those computed using either the K_{ec} or K_{es} model.

Comparisons of wind moments show column moments to generally be larger, and beam moments smaller, for the K_{es} model. The comparisons given in Fig. 13 and 14 indicate that the extended equivalent frame model gives conservative drift estimates. For gravity loads the beam moments are very sensitive to the option chosen (K_{ec} vs K_{es}) especially at interior supports. The negative moments at interior supports for the K_{es} modeling are typically only about half those for the K_{ec} modeling. Hence, only the K_{ec} model should be used for gravity load analysis. Either model may be used for lateral load analysis.

Beta studies

In the following discussion the parameter beta is used to describe stiffness reduction due to cracking where beta is defined as:

$$\beta = \text{effective } I/\text{gross } I \qquad \text{Eq. (3)}$$

and I is the moment of inertia of the slab-beam member. Thus beta = 1 refers to an uncracked beam

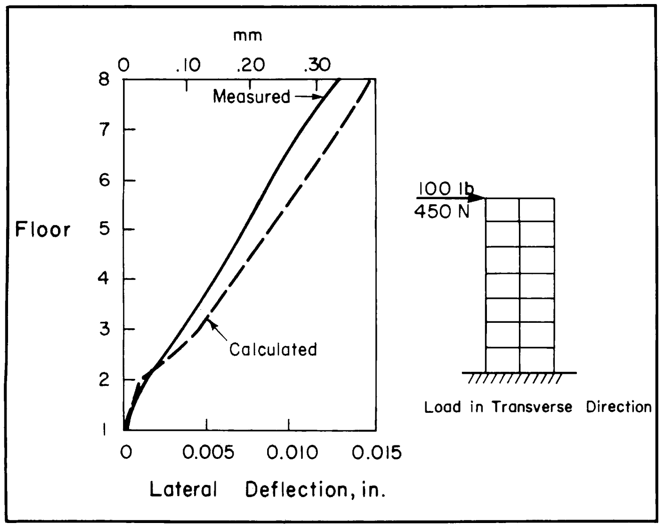

Fig. 13 — Deflections of Canadian slab model.

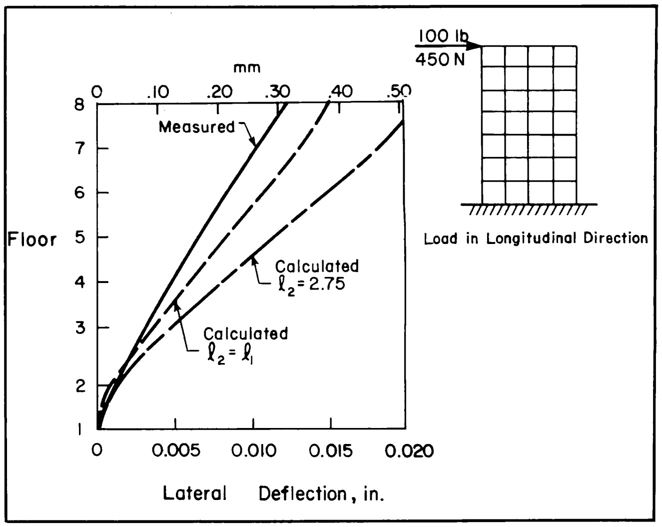

Fig. 14 — Effects on longitudinal deflections of varying transverse span for Canadian slab model.

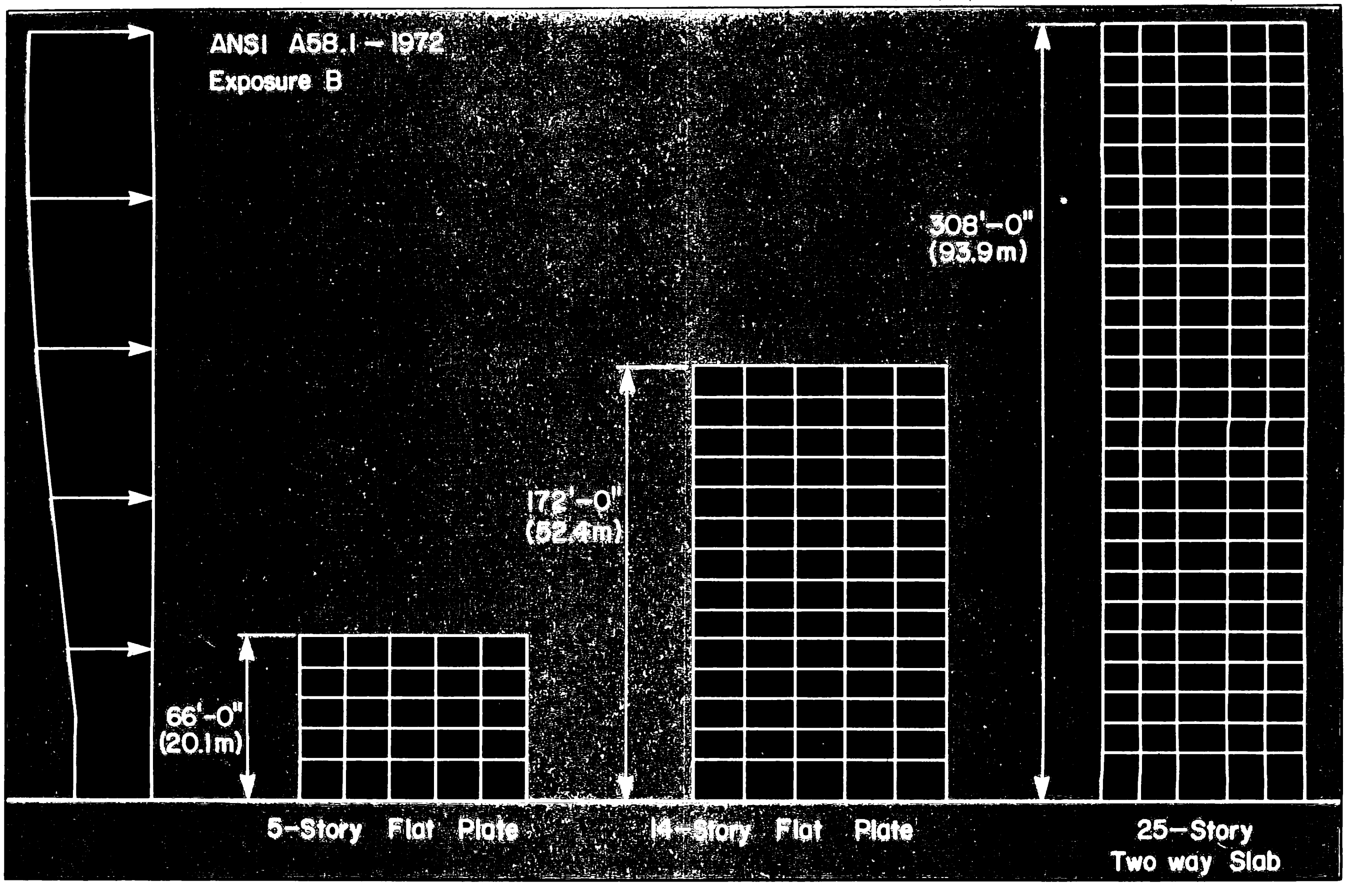

Fig. 15 — Structure and wind profile for beta studies.

while beta = ⅓ describes a beam whose stiffness has been reduced by cracking to only one-third its uncracked value.

Three representative structures were analyzed for selected wind loads to examine the effects on drift of varying beta. Elevations of the structures analyzed are shown in Fig. 15. Lateral loads were computed for an ANSI exposure B profile.[27]

Drifts computed for the three structures are shown in Fig. 16. The equivalent frame values are for the K_{ec} model. The K_{cs} values were about 5 percent smaller. As expected, softening the structure to represent cracking by reducing beta increases drift significantly.

The lateral deflections of a structure are computed to assess its serviceability. After computing drifts, they must be compared with acceptance criteria to determine the structure's adequacy. There is currently no consensus on what constitutes an acceptable drift criterion nor whether a criterion should be based on static or dynamic behavior. The static criterion of $h/500$ where h = structure height, is shown plotted in Fig. 16 since a literature review showed this to be the value most often quoted.[22]

A beta value of one-third is judged to represent a realistic lower bound for slab stiffness and is recommended as the default value. A two-way structure is more sensitive to reducing beta than is a flat plate since more of the total system stiffness is in the beam members of a two-way structure. The designer of a two-way structure would probably elect to compute the cracked moment of inertia rather than use the minimum value of beta = ⅓. It is clear that reducing beta produces a more conservative estimate of drifts. However, the comparisons indicate that this increase is not unduly conservative. Note that beta is intended to account for loss in stiffness from all causes including cracking of beams, columns, and torsional members.

An alternate way of analyzing the structure of Fig. 10a is to write a plane frame program which considers four degrees of freedom (DOF) at each joint: two orthogonal displacements and two rotations. The beam rotations at a joint would then be associated with one rotational DOF, the column rotations with the other, and the torsional member used to link the two. A version of EFRAME has been developed to implement this direct modeling.

Limited studies show the direct modeling produces computed drifts and gravity load moments nearly equal to the indirect K_{ec} modeling. The direct model offers the advantage that there is only one analytical model to consider (Fig. 10a) rather than the two indirect models (K_{ec}, K_{cs}) discussed above. A disadvantage is that far more degrees of freedom and hence a larger stiffness matrix are involved. The indirect models offer the advantage of fewer degrees of freedom. Also a method devised to allow the K_{ec} model to be implemented using conventional plane frame analysis programs shows promise.[22]

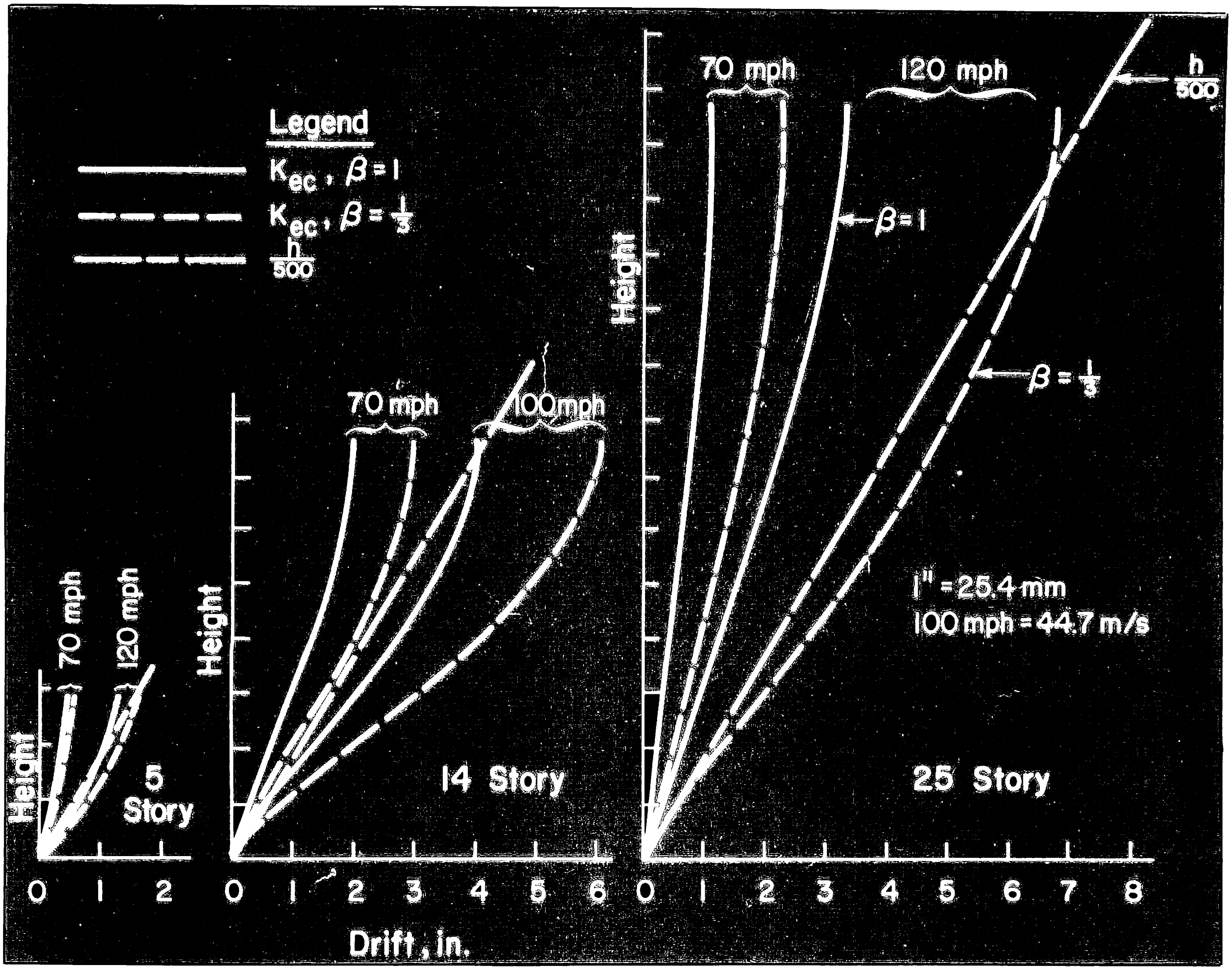

Fig. 16 — Influence of beta on drift.

Conclusions

A difficult part of analyzing a reinforced concrete building is modeling the transfer of moment between slab and columns. The three major transfer models are the equivalent beam width, transverse torsional member, and stub beam models. The equivalent beam width model is based on elastic analyses of laterally-loaded flat plates and thus its validity is limited. The transverse torsional member model is based on studies of analyses of elastic structures and tests of reinforced concrete structures. The method incorporates the effects of spandrel beams, longitudinal beams, drop panels, column capitals, and cracking since all of these were present in the calibration structures. The stub-beam model provides the most accurate model at all stages of stiffness and strength of flat plates but is not yet suitable as a routine design tool.

The Equivalent Frame Method of analysis, which employs the transverse torsional member modeling, has been extended to be applicable to complete buildings supporting both gravity and lateral loading. A list of recommendations for using the EFM follows:

1. Link parallel frames into one "super-frame" for all analyses but especially lateral load analyses.
2. Use only the K_{ec} model with beta = 1 for gravity load analysis.
3. Use either model for lateral load analysis with beta less than one to provide conservative drift estimates. A beta value of one-third is recommended unless a more detailed analysis of cracking is made.
4. Use an effective ℓ_2 in computing K_t equal to the smaller of the real ℓ_1 and ℓ_2 values.
5. Ignore cantilever spans in computing K_t.

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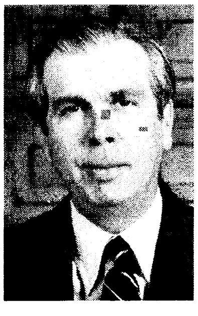

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