TITLE NO. 71-41

Inelastic Responses of Reinforced Concrete Structures to Earthquake Motions

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Two basic characteristics of reinforced concrete structures play an important role in determining response to strong ground motions. They are the changes in (a) stiffness and (b) energy dissipation capacity. Both can be related to the maximum displacement.

Results of dynamic tests of reinforced concrete frames are used to illustrate the effects on dynamic response of changes in stiffness and energy dissipation capacity.

It is shown that maximum inelastic response can be interpreted in terms of linearly elastic analysis by reference to a fictitious linear structure whose stiffness and damping characteristics are determined as a function of the assumed or known maximum displacement. This leads to a simplified method for estimating the design base shear taking account of inelastic response.

The object of this paper is (a) to describe basic phenomena of energy dissipation in reinforced concrete structures subjected to strong ground motion and (b) to present a simplified method for estimating the design base shear corresponding to inelastic response.

Tests of a series of reinforced concrete frames are reported in detail in Reference 1. Individual frames were subjected to steady-state dynamic base motion, simulated earthquake motion, or static lateral loading. Data from that study are used to illustrate fundamental response characteristics.

Keywords: damping capacity; dynamic tests; earthquake resistant structures; earthquakes; elastic limit; hysteresis; loads (forces); reinforced concrete; shear properties; stiffness; structural analysis; structural engineering.

TEST FRAMES

■ Basic features of dynamic response of reinforced concrete structures (proportioned so that critical strength decay does not occur with repeated displacement reversals into the inelastic range) may be illustrated using data from three test frames[1] with the dimensions shown in Fig. 1. Two frames, FDI and FEI, were tested dynamically with a weight of 4000 lb (1810 kg) attached centrally to the beam. Frame FSI was tested statically. Compressive strength of the 3/8 in. (0.95 cm) maximum aggregate concrete was 7090 psi

(498 kgf/cm²) for FDI, 5500 psi (387 kgf/cm²) for FEI and 5150 psi (362 kgf/cm²). Deformed No. 3 bars (f_y = 51 ksi or 3.6 ton/cm²) provided the longitudinal reinforcement. Plain ⅛ in. round bars (f_y = 74 ksi or 5.2 ton/cm²) were used for the closed ties.

Frame FDI was subjected for approximately 4.5 seconds to a sinusoidal base motion at a nominal frequency of 13 Hz (Fig. 2).

Frame FEI was subjected to a base motion simulating the N21E component of the 1952 Taft record. To excite the test frame into the inelastic range, the original record was scaled. Accelerations were amplified and the time axis was compressed (Fig. 3).

Frame FSI was forced, by a servoram in the plane of the frame and at midheight of beam, to go through the peak displacements measured during the test of frame FEI (Fig. 4).

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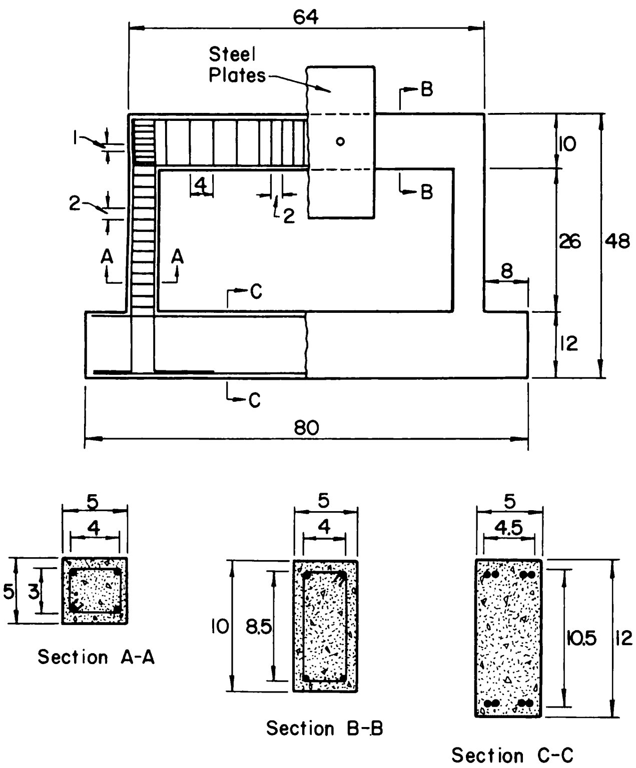
STIFFNESS CHANGE

A key property of reinforced concrete systems subjected to strong base motion is the change in effective natural frequency related to the reduction in stiffness caused by cracking and local spalling of the concrete as well as slipping and reduction in effective modulus of the steel.

Before dynamic testing, natural frequency measured at very low amplitudes (frame struck with a light hammer) was 16 Hz for both FDI and FEI. After the test runs, natural frequencies measured similarly were 10 Hz for FDI and 5 Hz for FEI.

Natural-frequency changes noted above are consistent with stiffness changes for FSI. The initially measured low amplitude frequency corresponds to the frame with virtually intact concrete. Excursions into the inelastic range reduce the effective stiffness of the frame (Fig. 4), hence the effective natural frequency.

Change in apparent natural frequency related to changes in force displacement response is a property of all reinforced concrete systems and can be generally understood from the moment curvature relationship.



All Dimensions In Inches (1.0 in. = 2.54 cm)
Longitudinal Reinforcement, Def. No. 3 Bars
Transverse Reinforcement, Plain 1/8 in. Round Bars

Fig. 1—Dimensions of test frames

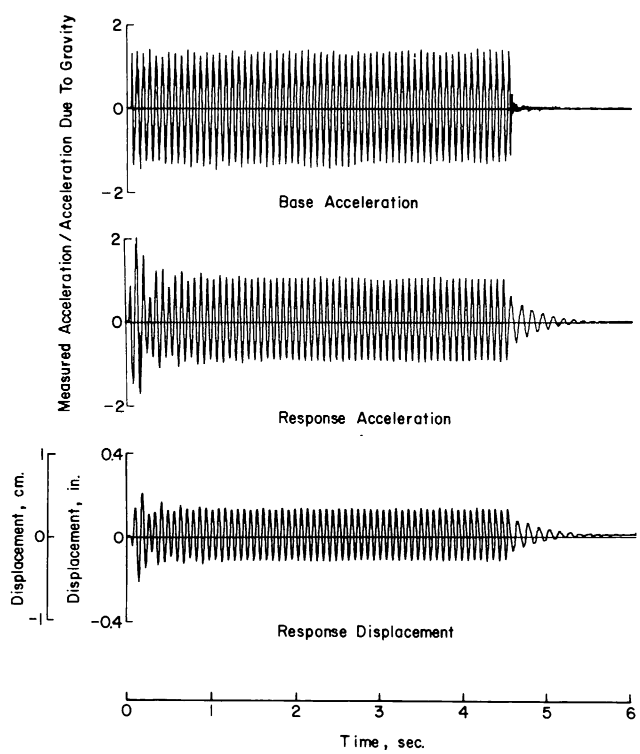


Fig. 2—Measured base acceleration, response acceleration and response displacement for Frame FDI

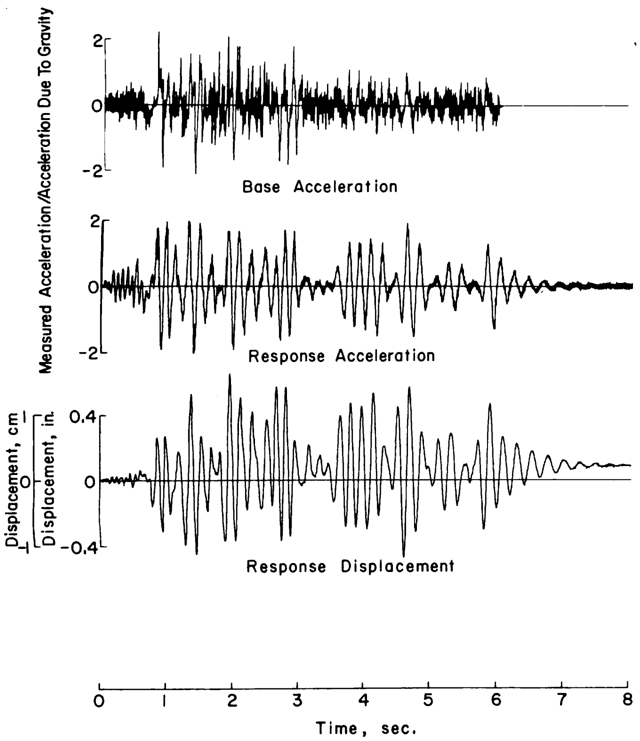


Fig. 3—Measured base acceleration, response acceleration and response displacement for Frame FEI

CHANGES IN HYSTERETIC RESPONSE

The area within a cycle of the force displacement curve is a critical parameter for dynamic response because it is a measure of energy dissipated by the vibrating system in that cycle.

Force displacement curves shown for Frame FSI illustrate two significant and typical trends for structures with moderately reinforced sections, with increase in displacement into inelastic range of response: (1) the stiffness, measured by the slope of a line drawn through the points on the hysteretic curve corresponding to maximum and minimum displacements, decreases and (2) the area within the hysteresis loop increases. Also, for a given set of displacement limits, the hysteresis loop area is larger if either displacement for that cycle exceeded previously attained values.

INELASTIC DYNAMIC RESPONSE

Fig. 2 records the response acceleration (as a ratio of the acceleration of gravity) and displacement at center of attached mass as well as the base acceleration for frame FDI.

Response acceleration reached initially a maximum of 2.0 g but was reduced rapidly to a nearly steady-state average of 1.1 g. At time of maximum acceleration, peak-to-peak displacement was 0.45 in. (1.143 cm) and was reduced rapidly to 0.26 in. (0.6604 cm). The overall impression is that of a

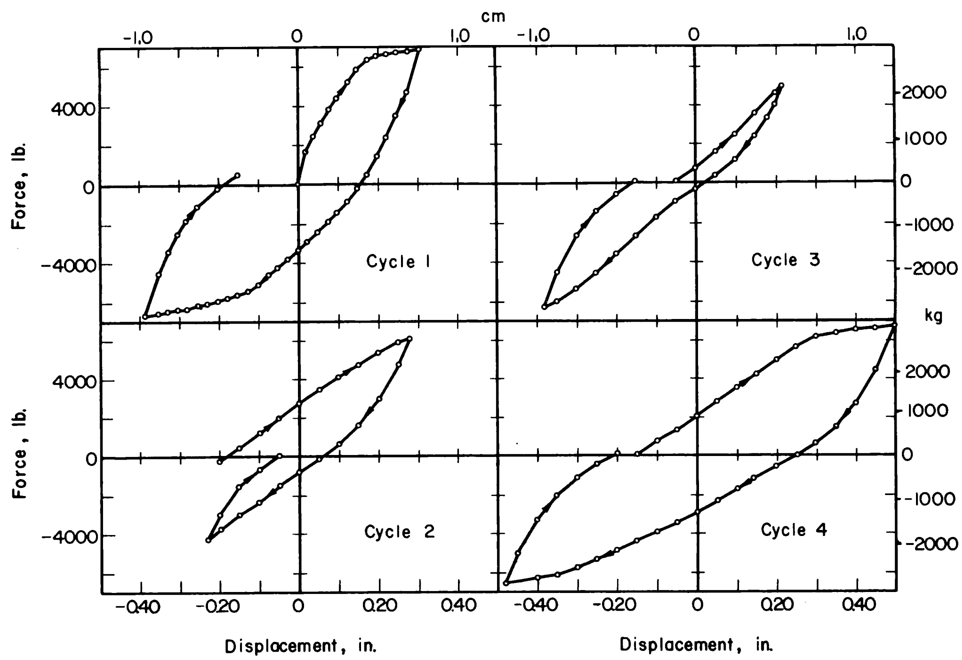


Fig. 4—Measured force-displacement relationship for Frame FSI. (First four cycles. "Static" loadings.)

system going momentarily through resonance, an impression that is consistent with the expected changes in hysteretic response. Initial natural frequency of 16 Hz is reduced drastically by the maximum excursion which also increases the hysteresis loop area. Thus the frame becomes less sensitive to the base motion.

Because of the base motion, the response of Frame FEI is more complex (Fig. 3), but its response mechanism is quite similar to that of FDI. The frame was "softened" by the first displacement beyond cracking, and responded to the remainder of the motion as a system with a lower apparent natural frequency and greater capability to dissipate energy.

INTERPRETATION OF INELASTIC RESPONSE IN TERMS OF LINEAR RESPONSE ANALYSIS

Response history of reinforced concrete test structures can be simulated by analytical models[1-3] which recognize in detail the changes in hysteretic response. Such models require considerable computational expense. Therefore, interpretation of the earthquake response of reinforced concrete structures in terms of simpler models is desirable.

Acceleration and displacement responses of a series of linearly elastic single-degree-of-freedom oscillators[4,5] for base motions of frames FDI and FEI are plotted in Fig. 5 for an arbitrarily selected damping ratio of 0.05.[1]

Consider Fig. 5a plotted for the base motion of FDI. As natural frequency decreases from 16 Hz, both calculated acceleration and displacement responses reach a peak and then attenuate. Given the information on the change in effective stiffness (and therefore the natural frequency), the measured response (Fig. 2) can be understood qualitatively in terms of the spectral-response curves.

The spectral-response curves in Fig. 5b indicate that as the natural frequency decreases from 16 Hz, the acceleration response stays almost constant at approximately 5 g and then starts decreasing while the displacement response increases continually. Given the information on hysteretic behavior, these trends also help interpret qualitatively the responses of FEI (Fig. 3). During strong motion, the effective natural frequency of FEI is rapidly reduced. Its "position" on the x-axis of Fig. 3b shifts to the right, reducing the maximum acceleration response. Simultaneously with the change in stiffness, there occurs an increase in energy-dissipation capacity so that the maximum acceleration response is actually reduced more than that indicated by the curve plotted for a constant damping factor in Fig. 3b.

To obtain a quantitative relationship between linear-response analysis and inelastic response, it is necessary to consider the effect of the hysteresis loop as well as changes in stiffness, as discussed in the next section.

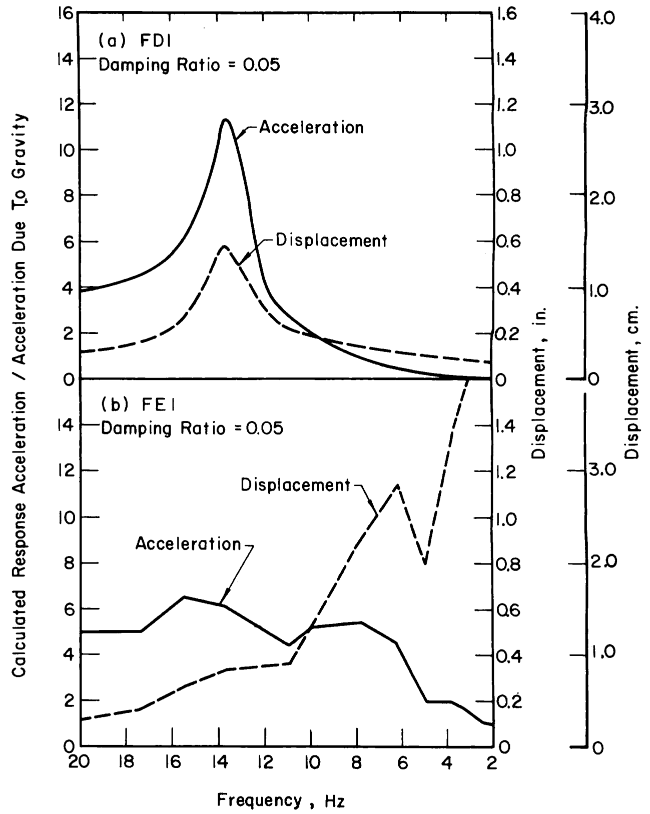


Fig. 5—Calculated ranges of acceleration and displacement response for the base motions of Frames FDI and FEI

SUBSTITUTE DAMPING

The shape of the hysteresis loop suggests that a particular loop may be approximately represented by a vibrating linear system with equivalent viscous damping.[6] This approach does contain elements of squaring the circle. As an analysis technique, it is doomed to fall short of the mark as discussed in Reference 7. However, as a vehicle to interpret the response of reinforced concrete systems with a view to design, it has considerable potential.

Quantitative values for this fictitious or substitute damping were distilled from results of dynamic tests of one-story, one-bay frames.[1] An average value of the substitute damping ratio, β_s, was obtained from each test run using Eq. (1).

$$\beta_s \left[2m\omega_o \int_o^T \dot{x}^2 \; dt \right] = -m \int_o^T \ddot{y} \; \dot{x} \; dt \tag{1}$$

Eq. (1) is based on the premise that energy input from horizontal uniaxial base motion is entirely dissipated by an imaginary viscous damper associated with horizontal velocity of mass carried by the frame. The right-hand term is energy input or product of mass, m, and the integral over the period of excitation of base acceleration, $\ddot{y}$, and mass relative velocity, $\dot{x}$. The left hand term involves two parameters. One is integral of mass velocity which represents energy dissipated by the imaginary damper multiplied by the damping coefficient. The other parameter is the critical damping coefficient for a single degree-of-freedom oscillator, $2m\omega_o$, introduced to express the substitute damping coefficient as a ratio of the critical. In reducing the data, ω_o^2 was taken as ratio of measured maximum absolute acceleration to measured maximum absolute displacement, a quantity close to the slope obtained by joining the origin to the maximum point reached on the static force displacement curve.

Fig. 6a shows values of substitute damping ratio, β_s, plotted against ratio, μ, of maximum absolute displacement to computed yield deflection of the test frames in Reference 1. Solid circles refer to Series F which included FDI and FEI. Open circles refer to Series H comprising specimens half as large as those in Series F.

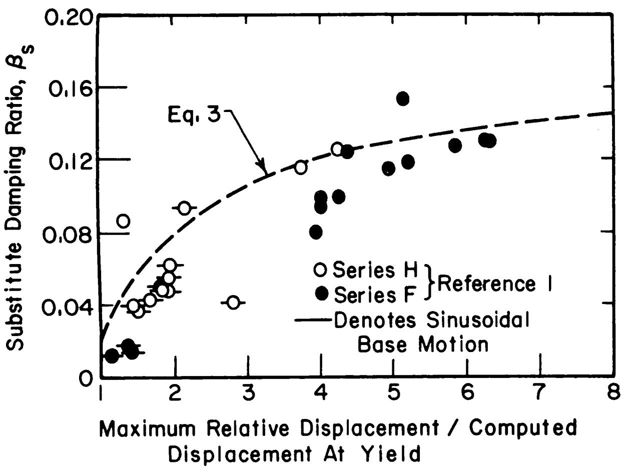


Fig. 6a—Variation of measured average substitute-damping ratio, β_s, with the ductility ratio, μ. (Reference 1)

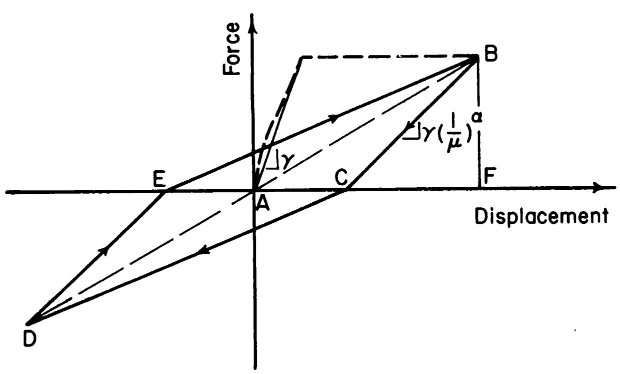


Fig. 6b—Idealized symmetrical hysteresis loop. (Reference 2)

Data from different test runs with different base motions and different specimen sizes are not strictly comparable. However, there is a discernible trend with the defined ductility ratio, a trend which is consistent with that derived from Jacobsen's approach[6] using Takeda's hysteresis.[2] Consider a symmetrical loop $EBCD$ shown in Fig. 6b. It is assumed that several cycles into the inelastic ranges have preceded it. Slope of line BC is defined as $\gamma(1/\mu)^{\alpha}$ where γ is the slope corresponding to fully cracked section for linear response, μ is the ratio of maximum to yield displacement, and α is taken as 0.5.[1] Slope AB is the effective stiffness or $m\omega_o^2$. The substitute damping ratio, β, would vary with attained ductility, μ, as ratio of area EBC to area ABF.[6] Thus,

$$\beta_s \propto (1 - 1/\sqrt{\mu}) \qquad (2)$$

If it is assumed that β_s has a threshold value of 0.02 at $\mu = 1.0$, Eq. (3) may be used to represent the data in Fig. 6a.

$$\beta_s = (1 + 10\,(1 - 1/\sqrt{\mu}))/50 \qquad (3)$$

The form of the expression above and its relationship to the data in Fig. 6a should emphasize that the value of β_s calculated for a particular μ represents a range rather than a precise quantity. Just as it would be inconsistent with the nature of reinforced concrete structures to invoke small fractions of the ductility ratio, it would be unjustifiable to obtain values of β_s having more than two decimal places, from Eq. (3).

DESIGN BASE SHEAR

Using the concept of reduced stiffness and substitute damping, a procedure based on linear response can be used to estimate effects of inelastic response for reinforced concrete structures which can be idealized as single-degree-of-freedom systems. The procedure involves the following steps.

1. Assume an admissible value of μ.
2. Calculate stiffness based on cracked section.
3. Determine natural period T.
4. Calculate β_s corresponding to assumed value of μ Eq. (3).
5. Obtain base shear (response acceleration x mass) and maximum displacement by entering spectral response diagram with an increased natural period of $T\sqrt{\mu}$ (or a reduced natural frequency) and a damping ratio equal to β_s determined in Step 4.

The above procedure is explicitly a design method and not an iterative analysis technique. The object is to provide sufficient strength so that an assumed displacement limit is not exceeded. It is suitable for direct use with spectral response diagrams having the characteristics shown in Fig. 5b which are representative of a large class of earthquake ground motions.[8] The direct method would not be suitable for the type of response shown in Fig. 5a, although the procedure could be used by "scanning" a series of frequencies or iteratively.

COMPARISON WITH "EXACT" ANALYSIS

A series of response-history analyses were made for simple oscillators with hysteretic response as defined in Reference 1, ignoring the tensile strength of concrete and assuming the slope of the force displacement curve after yielding to be equal to five percent of that before yielding.[9] Natural periods (based on initial slope) of the oscillators are listed in Table 1. For each period, calculations were made for three different levels of yield force. Each oscillator was "subjected" to two base motions: NS component of El Centro 1940 and EW component of Managua 1973 record. Maximum attained values of μ are compared in Table 1 with those required by the approximate method described in the preceding section. The approximate substitute damping method was used iteratively to obtain the required values of μ.

The favorable comparison between the "exact" and approximate values of μ suggests that the

TABLE 1—COMPARISON OF CALCULATED DUCTILITY RATIOS

Initial period sec.	Base shear coefficient*	Values of ductility ratio, μ			
		El Centro 1940, N		Managua 1972, E	
		"Exact"†	Approx.‡	"Exact"†	Approx.‡
0.15	0.16	28.5	26	28.8	26
	0.32	4.1	10	13.0	10
	0.48	3.3	5	5.2	6
0.50	0.16	8.0	6	5.6	6
	0.32	2.9	3	2.8	3
	0.48	2.0	2	2.0	2
1.0	0.08	4.9	6	5.6	6
	0.16	2.5	2	3.0	2
	0.24	1.5	1.5	1.3	1.5
2.0	0.04	4.6	5	4.5	4
	0.08	2.0	3	1.9	2
	0.12	1.5	2	1.4	2

*Lateral yield force divided by weight of structure.
†Attained in response-history analysis.
‡Required by iterations using the substitute-damping method.

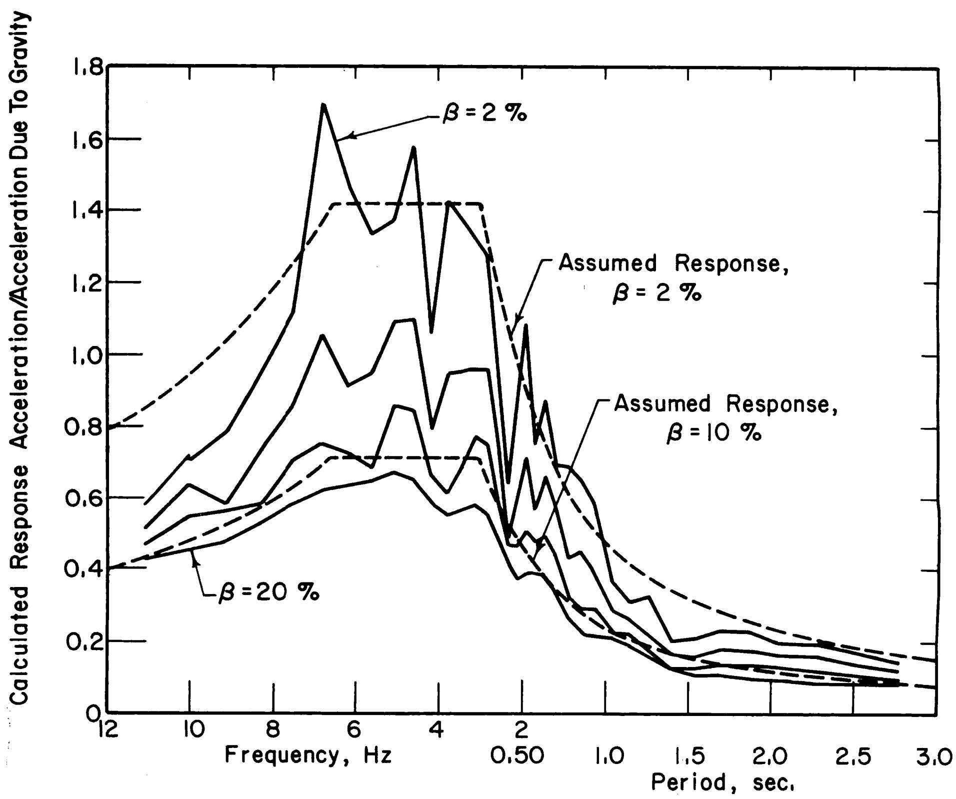


Fig. 7—Spectral acceleration response, Managua 1972, E-W. (Reference 9)

substitute-damping method may be used successfully in the domain covered by the exact analysis. In design applications, the spectral response curves should be smoothed.[8,10] A safety factor may be introduced in selecting the smoothed curves, by modifying β_s Eq. (3), or explicitly at the end of the design process.

EXAMPLE

Consider a single degree-of-freedom structure having a period of 1.0 sec based on cracked section stiffness. Admit $\mu = 3$ which modifies the period to 1.7 sec. For $\mu = 3$, Eq. (3) gives a substitute-damping ratio of approximately 10 percent. For a particular ground motion, obtain design base shear by entering Fig. 7 with $T_m = 1.7$ and $\beta_s = 0.10$. The answer is 0.14 g or 14 percent of the weight of the system. Note that the structure must be proportioned to be stable and free of adjoining structures at a deflection corresponding to the application of a lateral load equal to the base shear with the stiffness reduced by a factor of $\mu = 3$. Furthermore, the system must be detailed to permit a series of oscillations to the maximum calculated deflection without critical decay in strength.

SUMMARY

The response of reinforced concrete structures to strong earthquake motions is influenced by two basic phenomena: reduction in stiffness and increase in energy-dissipation capacity. As earthquake motion excites the structure to larger displacements, its stiffness decreases and its capacity to dissipate energy increases. Both effects can be related to the displacement attained (interpreted as strain, curvature, rotation, or deflection) or, nondimensionally, to ductility defined as the ratio of maximum to yield displacement.

Experiments with one-story, one-bay frames confirm that the maximum dynamic response of reinforced concrete structures, which can be represented by single degree-of-freedom systems, can be approximated by linear response analysis using a reduced *stiffness* and a "substitute damping" related to hysteretic properties of reinforced concrete. On the basis of this observation, a procedure is presented to determine the design base shear for an assumed ductility and a given ground motion or "design spectrum."

The main purpose of the proposed "substitute damping method" is to provide a simple vehicle for understanding the overall effects of inelastic response (increase in displacement and possible reduction in force) in reinforced concrete. It also emphasizes that "ductility" by itself is not sufficient to interpret the behavior of reinforced concrete structures. Two systems having the same ductility, defined in reference to a load deformation curve obtained under load monotonically increased to failure, may not have the same response to strong ground motion if the hysteretic properties of the two systems differ.

ACKNOWLEDGMENT

This paper was based on research supported at the University of Illinois Civil Engineering Department by Grant G130760X of the Research Applied to National Needs program of the National Science Foundation. Acknowledgment is due Professor A. Shibata of Tohoku University, Sendai, and Professor S. Otani of the University of Illinois, Urbana, whose critical discussions have influenced this paper.

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This paper was received by the Institute Jan. 28, 1974.