Reference 18.7

ULTIMATE STEEL STRESSES IN UNBONDED PRESTRESSED CONCRETE

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INTRODUCTION

Current design practice in the United States for unbonded post-tensioned flat plate structures is partially based on the assumption that the stress in the post-tensioned tendons at the time of flexural failure can be predicted using the equation given by the American Concrete Institute (ACI) Code (1) for stresses in beams.

However, data from recent tests of a number of thin flat plate structures and of beams having comparable span-depth ratios show that this is generally not valid if the span-thickness ratio is about 45, or more. In many instances the increase in steel stress, Δf_s, beyond the prestress level, f_{se}, was much smaller than the value predicted using the ACI Code. In no case did the stress in multipanel flat plate specimens reach the expected values. In some of the beam specimens the expected values were reached or exceeded, while in others the increases were less than half that expected.

The purpose of this paper is to present the test data and to demonstrate that an additional factor, the span-depth ratio, l/d, needs to be introduced into the prediction equations for ultimate steel stress in unbonded post-tensioned members. Such an expression is not developed in this paper, but recommendations for a series of tests to develop the needed test data are presented.

CURRENT DESIGN VALUES

When an unbonded prestressed concrete beam or flat plate is loaded, compatibility of strain between concrete and steel cannot be assumed. Slipping occurs between steel and concrete because of the lack of bond. It is difficult to calculate the stress in the tendon, f_{ps}, at failure of the beam. Simplified expressions for f_{ps} have therefore been developed for design. The equation used in the 1971 ACI Code (1) is:

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$$f_{ps} = f_{se} + 10{,}000 + \frac{f'_c}{100\rho_p}\ \text{psi} \quad (1)$$

but not more than f_{py} or $(f_{se} + 60{,}000)$ psi. Or, in SI units, $f_{ps} = f_{se} + 68.9 + [f'_c/(100\rho_p)]$ N/mm², but not more than f_{py} or $(f_{se} + 414)$ N/mm². This

TABLE 1.—Basic Geometry and Information on Loadings

Slab (1)	Reference (2)	Number of spans (3)	Thickness, in inches (4)	Length of each span, in inches (5)	Width of each span, in inches (6)	Loading (7)
Slab A	16	3	2.94	120	55	Four load points
Slab B	6	3	2.75	120	55	for each span
		Panels				
Flat Plate I	2	9	2.90	120	120	16 load points for
Flat Plate II	10	9	3.00	120	120	panel, assumed
Flat Plate III	3	4	2.75	120	120	equal to uniform load
S-1	9	2	4	180	40	Single load
S-2	9	2	4	180	40	Single load
S-3	9	2	4	180	40	Single load
C-I	7	2	6	180	24	Single load
C-II	7	2	6	180	24	Single load
Z-I	8	2	4	180	40	Single load
Z-II	8	2	4	180	40	Single load
		Panels				
Scordelis	13	4	3	84	84	Uniform loading
RU_1	11	1	12	336	6	Four equal point
RU_2	11	1	12	336	6	loads
		Panels				
Mark 4	5	8	3	144	108	Uniformly distributed load

Note: 1 in. = 25.4 mm.

expression is based on Mattock's tests (11) of beams.

Warwaruk, Sozen, and Siess (17) proposed the following equation for f_{ps}:

$$f_{ps} = f_{se} + 30{,}000 - \frac{\rho_p}{f'_c}10^{10}\ \text{psi} \geq f_{se} \quad (2)$$

In SI units: $f_{ps} = f_{se} + 210 - (4.76\ \rho_p/f'_c)\ 10^5$ N/mm² $\geq f_{se}$. The SI expression is that used for design in the 1974 Australian Code (12), and is not an exact conversion.

In the case of beams for which the parameter l/d is small, the tendon stress values predicted by Eqs. 1 and 2 are generally less than the test f_{ps} values. When the parameter l/d has a large value, e.g., l/d = 40 or more, the tendon

stress values predicted by Eqs. 1 and 2 are generally larger than the test f_{ps} values.

Data from tests of a number of thin specimens are summarized in Tables 1 and 2. Table 1 gives basic geometry and information on the loadings, while Table 2 summarizes the results of tests and various calculated values. Five of the tests were on multipanel flat plate structures. The other 11 were on beams of various depths and widths, and several of these were planned to be essentially companions to some of the flat plates.

The flat plates all eventually failed in punching shear at a column, but the full flexural capacity was reached, judging from load-deflection curves, in all but Flat Plate II and the shear failures were secondary.

TABLE 2.—Changes in Stress in Unbonded Post-Tensioned Strands in Several Slab and Shallow Beam Tests

Slab (1)	Reference (2)	l/h (3)	$l/$eff d (4)	Δf_s, in kips per square inch: Observed (5)	ACI 71 (6)	Warwaruk's equation (7)
Slab A	16	44	52	19.5	39.9	26.7
Slab B	6	44	53	21.0	53.4	27.7
Flat Plate I	2	44	51	6–25	29.0	24.7
Flat Plate II	10	44	51	9–15	38.5	28.1
Flat Plate III	3	44	51	5–23	28.3	24.5
S-1	9	45	60	24.5–32.3	26.05	23.8
S-2	9	45	60	34.4–35.0	26.6	24.0
S-3	9	45	60	17.9–22.8	20.3	20.3
C-I	7	30	36	20.4–27.0	20.5	20.5
C-II	7	30	36	24.0–55.8	27.8	22.9
Z-I	8	45	60	13.2–27.9	20.8	20.7
Z-II	8	45	60	19.7–33.2	20.3	20.3
Scordelis	13	28	34	22.0	51.6	27.6
RU1	11	28	33.6	25	19.7	19.7
RU2	11	28	33.6	18.6	18.3	18.0
Mark 4	5	48	57.6	25.8	32.2	25.4

Note: 1 ksi = 6.9 N/mm².

The beam and wide slab specimens all failed in flexural modes. In many of the tests, a range of Δf_s values is listed. Strand forces were measured by means of load cells at the ends of at least some strands, and these are the lower values. Strand forces were also determined in several specimens using strain gages on the strands, and in others friction losses between the critical sections and the load cells were estimated. These forces are the higher values in the tables.

The observed values of Δf_s shown in Table 2 are in most cases lower than either the Eq. 1 or 2 values. There are exceptions, but these are very erratic. For example, slabs C-I and C-II had about the same span-depth ratios as used in beams, and while one developed Δf_s about 6 ksi (41 N/mm²) higher than the ACI Code value, the other developed 28 ksi (190 N/mm^2) more than the Code prediction. The two beam specimens, RU1 and RU2, both developed Δf_s values slightly higher than computed using the ACI Code provisions, but this is to be expected since these beams were in the body of data considered when the Code expression was developed.

In other cases, and especially Slab B, Flat Plates I and III, and the slab tested by Scordelis, the measured Δf_s values are markedly lower than the ACI Code expectations. The Δf_s values given by Eq. 2 are often considerably lower than the ACI values, but even these values are too large for several of these slabs. The low Δf_s value from the slab tested by Scordelis is particularly disturbing, since it was a relatively thick flat plate by current standards, with a span-depth ratio comparable to that found in many beams.

It must be recognized that a deficiency in Δf_s does not lead to a proportionate deficiency in moment capacity since the moment is related to the total stress, $f_{ps} = f_{se} + \Delta f_s$. The initial stress, f_{se}, is generally very high relative to Δf_s, but if Δf_s is lower than expected, so is f_{ps}.

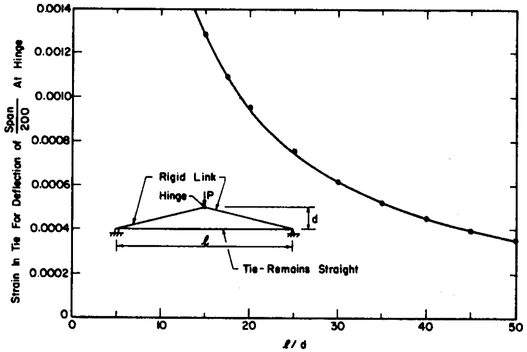

FIG. 1.—Steel Strain Versus Span/Depth Ratio for Deflection of Span/200

Several of the structures tested reached their required ultimate loads before failure, despite the fact that Δf_s was somewhat lower than expected. This occurred partially because they had reinforcement areas in excess of that required by the nominal strength calculations, because the design had been governed by allowable stress considerations at service load. If the strength requirements had governed the design, and they apparently seldom will in flat plates designed to current specifications and practices in the United States, the failure loads would not have been as large as the required ultimate loads.

A simple truss model can be used to represent a post-tensioned structure, as shown in Fig. 1. It was assumed that the tie remained straight and the strain on the tie was found, for a hinge deflection = δ. Fig. 1 shows the strain in the tie versus l/d for a deflection of $l/200$. If it is assumed that the center of the tie also has the same deflection, δ, the strain in the tie will be slightly larger.

The truss model is a conceptual representation of a cracked unbonded post-tensioned beam, and the numerical values of strain in the tie should not

be assumed to represent any real case. Other more sophisticated models are also possible, but it is believed that similar trends will be obtained. However, the trend of decreasing strain with increasing values of l/d should be a realistic representation of the increase in steel strain beyond the prestress value, f_{se}, in real cases, and it can be seen that the difference between $l/d = 25$ and $l/d = 50$ is quite important. This l/d effect has been ignored in American code development.

The British Code of Practice CP110 (4) relates Δf_s to l/d in addition to a $\rho_p f_{se}/f'_c$ term. The tables in the code extend only to $l/d = 30$, and thus

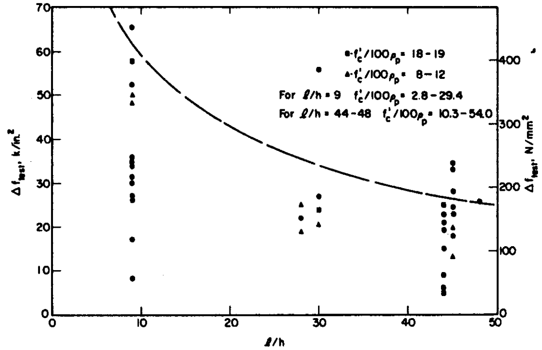

FIG. 2.—Measured Increase in Steel Stress Versus Span/Thickness Ratio

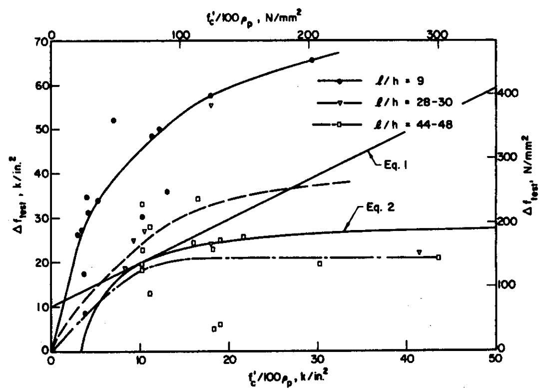

FIG. 3.—Increase in Steel Stress Versus Reinforcement Index from Beam Tests

are not applicable to most slabs. However, the trend is that the Δf_s value decreases sharply as l/d increases. Linear extrapolation of the values given cannot be justified, but if done leads to $\Delta f_s = 0$ when $l/d \simeq 50$, for all values of ρ_p.

Recent tests of unbonded beams by Tam and Pannell (14) show the influence of the l/d ratio, but all beams were heavily reinforced and thus the results may not apply directly to lightly reinforced slab structures.

Fig. 2 shows the measured steel stress increase versus l/h, with the data being predominately in three groups of l/h values, at 9, about 30, and about 45. In general, the higher f'_c/ρ_p, the higher Δf_{test}, but the purpose of the figure is to illustrate a general dependence of Δf_{test} on l/h. It can be seen that a reasonable upper bound curve to the test data has the same general shape as the curve obtained for the model, as shown in Fig. 1. This is also true for the particular case of $f'_c/(100\ \rho_p) = 8$ to 12, plotted as triangles in Fig. 2. The data presented include those from the references listed in Table 1 plus data from Ref. 17 for the beams with $l/h = 9$.

The same Δf_{test} data are plotted in Fig. 3 versus $f'_c/(100\ \rho_p)$. There is considerable scatter in the test data from the three groups of specimens with different l/h values, but it is clear that the members with $l/h = 9$ behaved quite differently from the members with $l/h \simeq 45$. The curves for the three groups of data are simple visual fits.

The values of Δf_s predicted by Eqs. 1 and 2 are also plotted in Fig. 3. Both equations overestimate the Δf_s values for lightly reinforced members having l/h ratios of 45, and both give values smaller than most of the test values for thicker members.

The data presented in Figs. 2 and 3 should make it clear that the span-depth ratio, l/d, should be included in some manner as a parameter when determining the steel stress at failure of an unbonded post-tensioned concrete member.

Conclusions and Recommendations

The data presented in this paper demonstrate conclusively that the ultimate steel stress in thin unbonded post-tensioned concrete members cannot consistently be safely predicted by the ACI Code expression. This is true for both beams and multipanel flat plate structures, as can be seen from the data in Tables 1 and 2. Consequently, considerable caution should be exercised by designers in the application of the 1971 ACI Code expression for f_{ps} to thin unbonded post-tensioned members. This is especially true in those cases where the strength requirements govern the steel area.

The increase in steel stress, Δf_s, above the post-tensioning stress, f_{se}, can be shown to be influenced by the span-depth ratio, l/d, in addition to the steel ratio and material strength factors now included in the code expression. A simple model is presented to demonstrate the sensitivity of Δf_s to the span-depth ratio, and there are strong similarities between trends from the model and from test results.

In order to provide a better understanding of the questions existing in this problem, a series of tests in which the major variable is the span-depth ratio should be carried out. The range of l/d should extend from about 20 to 50 or more, and the specimens should be as simple as possible so that all extraneous sources of uncertainty will be removed or minimized. For example, the test specimens should be statically determinate so that there can be no question about the applied moment at any section. The initial tests should not include bonded auxiliary steel, so that another source of uncertainty about the sources of resistance is eliminated. Only after a better understanding of the simplest structures has been reached should more complex and realistic specimens be tested.

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