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Comparative Study of Prestressed Concrete Beams, With and Without Bond

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Tests were made of seven simple span beams of 28-ft span, and of three beams continuous over two spans of 28 ft each. The primary variables were, the presence or absence of bond; the amount of bonded unprestressed reinforcement; and the use of seven wire strand as bonded unprestressed reinforcement. The unbonded post tensioned beams with minimum recommended unprestressed bonded reinforcement had serviceability characteristics, strength, and ductility equal to, or better than, those of comparable bonded post-tensioned beams. An expression is proposed for the ultimate stress in unbonded tendons. Seven wire strand can be used effectively as unprestressed bonded reinforcement.

Keywords: beams (supports); bond (concrete to reinforcement); continuous beams; crack width and spacing; deflection; moments; post-tensioning; prestressed concrete; reinforced concrete; research; serviceability; strength; T-beams; unbonded prestressing.

■ WHEN A BONDED PRESTRESSED concrete beam is loaded, the change in strain in the tendon at the section of maximum moment is equal to the change in strain in the adjacent concrete. The increase in stress in a bonded tendon therefore depends on the local deformations at the section of maximum moment. In the case of an unbonded prestressed concrete beam, the change in strain in the tendon is equal to the average change in strain in the concrete adjacent to the tendon over the whole length of the tendon. As a result, the strain in an unbonded tendon, when the concrete develops its ultimate strain at the compression face, is less than would be the case if the tendon were bonded to the concrete. This was first recognized by Baker,[1] who proposed the use of the strain compatibility factor F in analysis and design, where:

$$F = \frac{\text{Change in tendon strain}}{\text{Change in concrete strain adjacent to tendon at failure section}}$$

The value of F is a function of several variables, but Baker[2] suggested that a safe limiting value of 0.1 be used in design. Other simple expressions, usually relating F to the neutral axis depth at ultimate, have suggested.[3-6]

The calculation of the stress in the tendon at ultimate strength f_{su}, using any of the suggested expressions for F, is usually time consuming. Simplified expressions for f_{su} have therefore been developed for design. Warwaruk et al.[6] suggested that the increase in stress in the tendon $(f_{su} - f_{se})$, is related to the parameter p/f_c'. They plotted $(f_{su} - f_{se})$ against p/f_c' and proposed the following equation for f_{su}.

$$f_{su} = f_{se} + (30{,}000 - 10^{10}\, p/f_c') \text{ psi} \qquad (1)$$

or

$$f_{su} = f_{se} + (2110 - 49.4 \text{x} 10^6\, p/f_c') \text{ kgf/cm}^2 \qquad (1A)$$

A similar plot has been made in Fig. 1, but including additional data. A line representing Eq. (1) has been drawn on Fig. 1, together with another line representing ACI 318-63,[9] Eq. (26-7).

$$f_{su} = f_{se} + 15{,}000 \text{ psi} \qquad (2)$$

or

$$f_{su} = f_{se} + 1055 \text{ kgf/cm}^2 \qquad (2A)$$

It can be seen that both Eq. (1) and (2) are very conservative, in the case of beams for which the parameter p/f_c' is small. It is appreciated that Eq. (2) was purposely made conservative, because of lack of detailed information on the behavior of unbonded prestressed beams, and because of certain undesirable behavior characteristics of unbonded prestressed beams which do not contain any bonded reinforcement.

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Based on the data available at the commencement of this study, the following equation was proposed as an alternative reasonable lower bound to the available data for simply supported, unbonded post-tensioned beams having reinforcement ratios permissible under ACI 318-63.

$$f_{su} = f_{se} + \frac{1.4 f_c'}{100p} + 10{,}000 \text{ psi} \qquad (3)$$

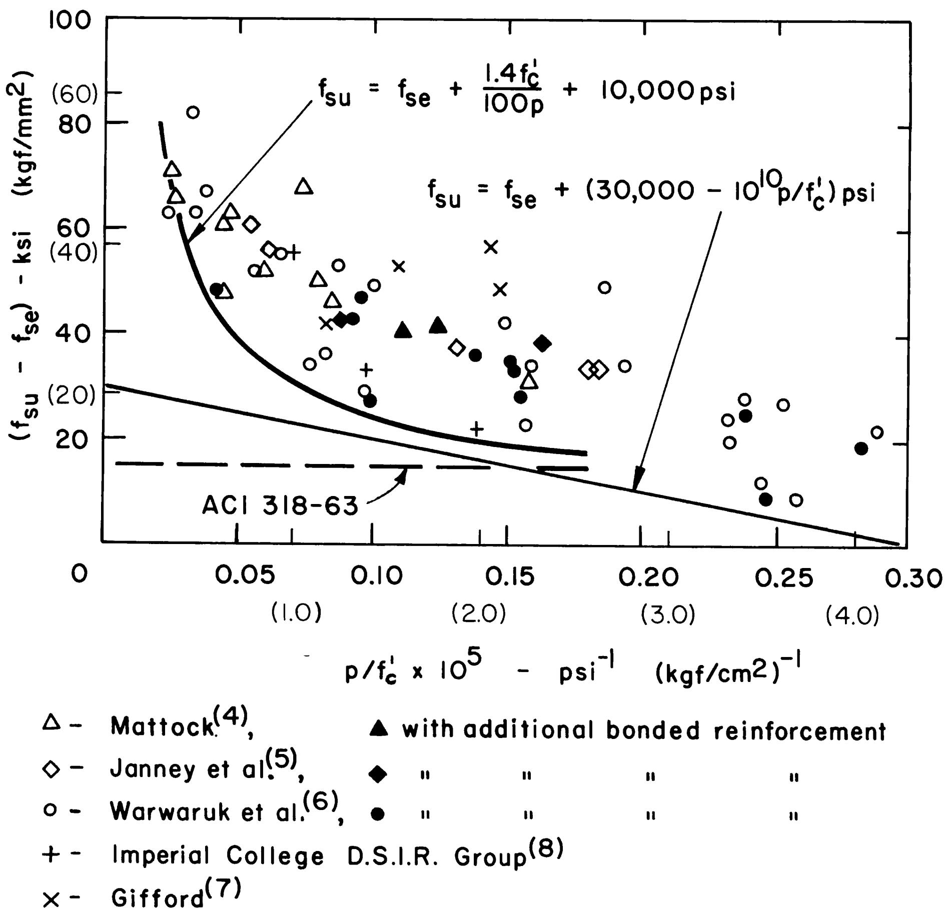

Fig. 1—Post-tensioned beams without bond, increase in tendon stress during loading to ultimate

or

$$f_{su} = f_{se} + \frac{1.4f_c'}{100p} + 700 \text{ kgf/cm}^2 \qquad (3A)$$

but, not greater than the yield strength of the tendon.

It can be seen from Fig. 1, that this equation does reflect the behavior of beams for which the parameter p/f_c' is small.

Behavior of unbonded post-tensioned beams

Most tests of unbonded post-tensioned beams have been made on beams in which the unbonded tendons were the only flexural reinforcement in the beam. In most beams of this kind, if the tendons do not come into contact with the beam between the anchorages, a single wide crack will form at the section of maximum moment. After cracking, the beam behaves as a shallow tied arch, rather than as a flexural member. The crack increases rapidly in width and depth as the load increases. Such cracks usually fork at their upper ends. The deflection increases rapidly after cracking. If the tendons come into contact with the beam between the anchorages, then additional cracks will usually form. However, these cracks are normally few in number and still open much more rapidly than do cracks in bonded prestressed concrete members.

The undesirability of such behavior was recognized at an early date, and several of the experimental studies[4-6] previously referred to, included a limited number of unbonded post-tensioned beams in which was placed additional bonded unprestressed reinforcement to control cracking. It was founded that quite a moderate amount of additional bonded reinforcement resulted in crack widths and spacings similar to those found in bonded prestressed concrete beams. In addition, the stiffness of the beam after cracking, and the ultimate strength, were also increased. The increase in ultimate strength arises from the additional tensile force provided by the bonded reinforcement. The stress in the unbonded tendon at ultimate does not appear to be changed by the presence of additional bonded reinforcement, as may be seen in Fig. 1. In addition, unbonded post-tensioned beams containing additional bonded reinforcement behaved as flexural members under overloads, rather than as tied arches.

This investigation

Because of the conservatism of Eq. (2), it is in practice frequently necessary to provide a relatively large amount of additional deformed bar reinforcement in unbonded post-tensioned beams, in order to give them a theoretically adequate ultimate strength. This amount of additional unprestressed, bonded reinforcement could be reduced if the proposed Eq. (3) were used to calculate the ultimate tendon stress in design, in place of Eq. (2). However, the data on which Eq. (3) is based came from tests of beams for which the span to depth ratio was 17:1 or less. Also, most of the test beams were prestressed with straight tendons. The question was raised as to whether beams prestressed by parabolically draped tendons and having the large span to depth ratios found in practice, would behave the same way.

This experimental study was designed to answer that question. It was also planned to provide a direct comparison of the behavior of bonded and unbonded post-tensioned concrete beams, designed to have the same flexural ultimate strength according to the procedures of ACI 318-63; and to study the behavior of an unbonded post-tensioned beam in which unprestressed seven wire strand is used as the additional bonded reinforcement.

EXPERIMENTAL STUDY

In the main test program, three groups of beams were tested. The groups consisted of: three T-beams (CBI, CU1, and CU2), continuous over two spans of 28 ft (8.53 m) each; three simply supported T-beams of 28 ft (8.53 m) span (TB1, TU1, and TU2); and three simply supported rectangular section beams of 28 ft (8.53 m) span (RB1, RU1, and RU2). Each beam was prestressed by two ½ in. (12.7 mm) 270K seven wire strands, which were continuous from end to end of the beam. In the first beam of each group, the tendons were bonded to the beam by grouting after prestressing. In the other two beams of each group, the tendons were left unbonded and additional unprestressed deformed bar reinforcement was provided. All three beams in each group were designed to have the same ultimate strength. The bonded beams CB1, TB1, and RB1, and the unbonded beams CU1, TU1, and RU1 were designed according to the provisions of Chapter 26 of ACI 318-63. The unbonded beams CU2, TU2, and RU2 were designed assuming that the increase in stress in the unbonded tendons would be as given by Eq. (3), instead of 15,000 psi (1055 kgf/cm²) as stipulated in Section 2608(a)3 of ACI 318-63.

An additional unbonded post-tensioned T-beam (TU3) of 28 ft (8.53 m) span was also tested. This beam was identical to Beam TU1 and TU2, except that in place of additional deformed bar reinforcement, a single 3/8 in. (9.5 mm) diameter unprestressed seven wire strand was provided as additional bonded reinforcement.

The continuous T-beams shown in Fig. 2, represented to half scale a typical beam taken from a hypothetical slab and beam structure. The prestressing strands were draped parabolically in each span, to give an effective depth of 10.0 in. (25.4 cm) at both midspan and at the center support, and zero eccentricity at the ends of the beams. Details of the simple span beams are shown in Fig. 3. The strands were draped parabolically in all these beams, with a sag of 6.35 in. (16.1 cm) and an effective depth of 10.0 in. (25.4 cm) at midspan.

Each strand in the bonded beams was passed through a 3/4 in. (19 mm) diameter steel tube. These strands were grouted after tensioning. Each strand in the unbonded beams was supplied coated with grease and enclosed in a close fitting plastic sheath. The ultimate strength of the strands f_s' was 279.9 ksi (197.0 kgf/mm^2), and the stress at 1 percent elongation f_{sy}, was 255.1 ksi (179.4 kgf/mm^2). These stresses are based on the nominal cross-sectional area of the strand, 0.153 sq in. (0.987 cm^2). The yield strength of the #2, 3, and 4 bars were 54.7, 50.0, and 46.7 ksi, (38.5, 35.2, and 32.7 kgf/mm^2). The concrete compression strength was about 4000 psi at test.

Because the test beams were to half-scale, it was necessary to increase the dead weight of the test beams by 100 percent, in order that the dead weight stresses in the test beams should be the same as they would be in full scale beams. This was done by placing 8 x 8 x 25½-in. (approximately 20 x 20 x 65 cm) concrete blocks on the top flange of the T-beams at 1 ft (30.5 cm) centers, or hanging them below the rectangular beams at 2.0-ft (61 cm) centers. The blocks can be seen in Fig. 4.

Each span was subjected to four equal point loads. These were applied at points 1.5 ft (46 cm) and 5.5 ft (168 cm), each side of midspan. They were increased monotonically until failure of the test beam occurred. Measurements were made of deflection and crack width, and of strand tension in the unbonded beams.

DISCUSSION OF TESTS OF SIMPLE SPAN BEAMS

Ultimate strength and increase in tendon stress

It can be seen in Table 1 that the use of Eq. (26-4) and (26-6) of ACI 318-63 leads to a close estimate of the ultimate strength of the bonded prestressed beams. This is because Eq. (26-6) approximates quite closely[6] the variation of ten-

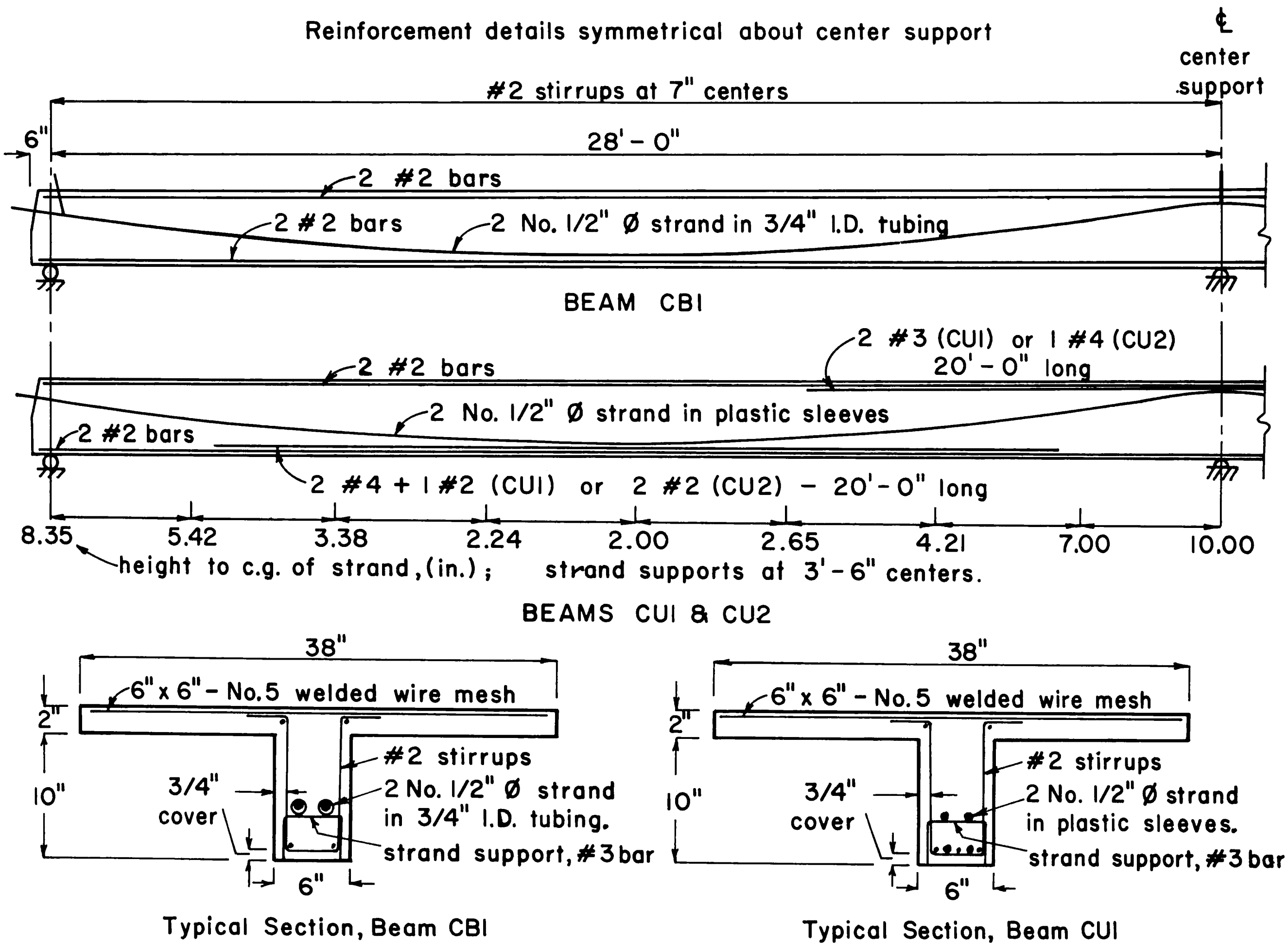

Fig. 2—Details of continuous beams

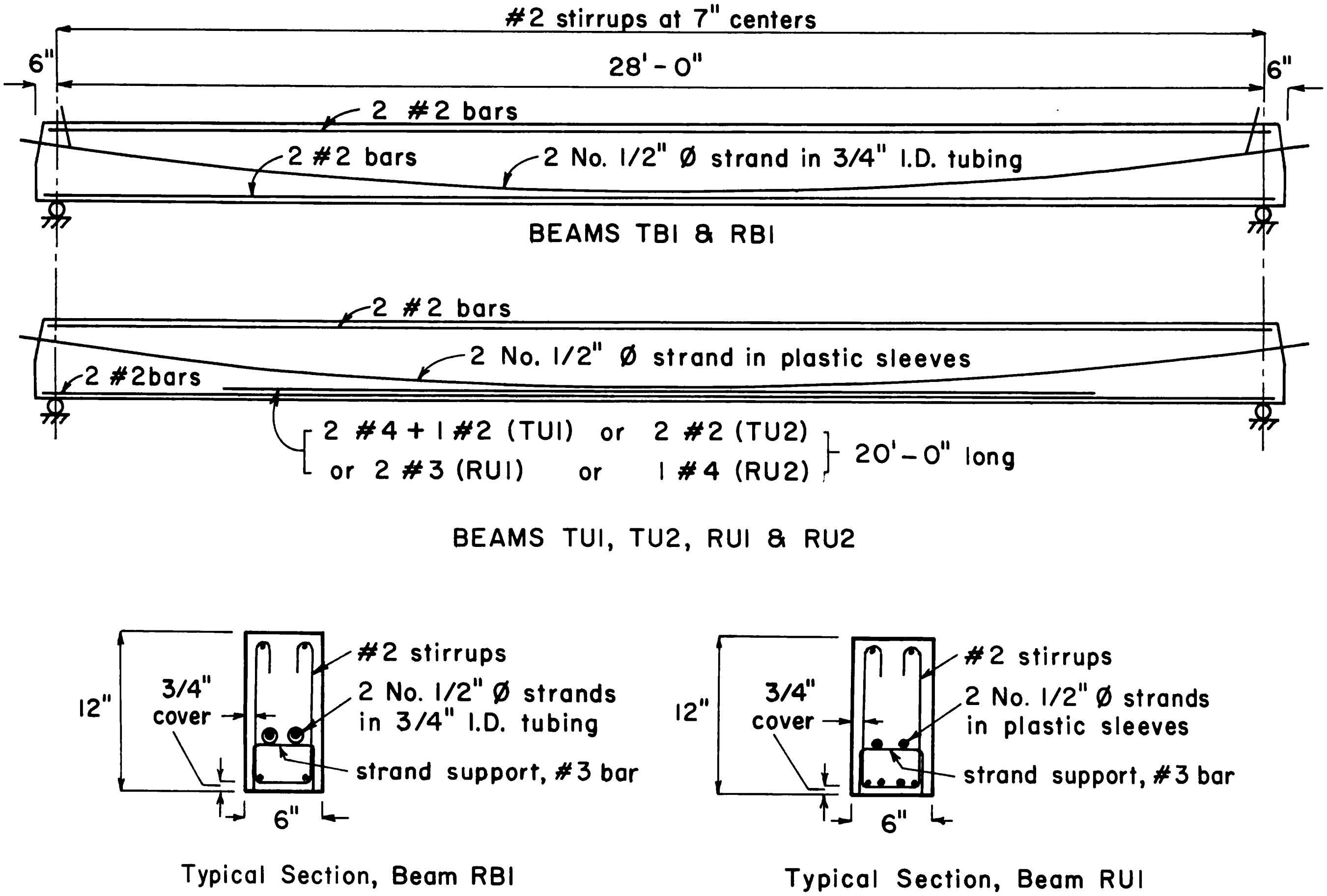

Fig. 3—Details of simple span beams

don stress at ultimate, with the ratio of prestressed reinforcement and the concrete strength. However, in the case of the unbonded beams it is found that the actual ultimate strength may be up to 30 percent greater than the ultimate strength calculated using Eq. (26-4) and (26-7) of ACI 318-63. This is because Eq. (26-7) does not reflect the variation in tendon stress at ultimate. It can be seen in Table 1 that in the case of the T-beams with low reinforcement ratios, the actual stresses in the unbonded tendons at ultimate were up to 31 percent higher than predicted by Eq. (26-7). It is clear therefore, that if two companion post-tensioned simple span beams (one bonded, one unbonded) are designed according to ACI 318-63, to have the same *design* ultimate strength, the unbonded beam may in fact have an *actual* ultimate strength up to 30 percent greater than that of the bonded beam. The margin of excess strength decreases as p/f_c' increases. It can also be seen from Table 1 that the proposed Eq. (3) satisfactorily reflects the behavior of the unbonded tendons in these simple span test beams, in which the span to effective depth ratio is 33.6.

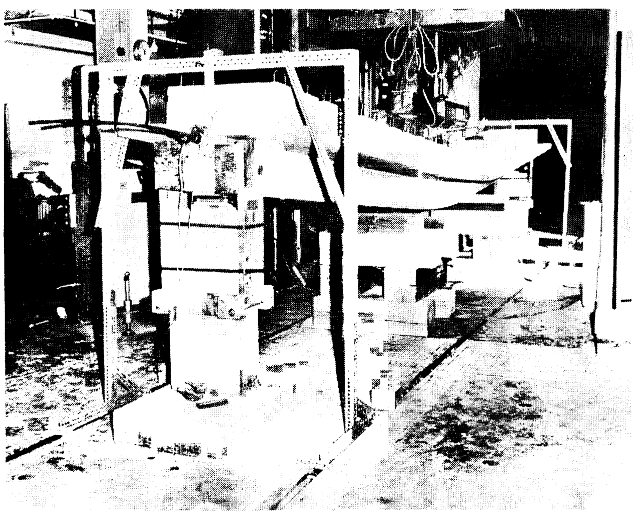

Fig. 4—Unbonded post-tensioned Beam TU1 at 90 percent of ultimate load, deflection 12 in.

In design according to ACI 318-63, it is assumed that additional bonded unprestressed reinforcement will develop its yield strength f_y. The test of Beam TU3 indicated that for beams with a low ratio of flexural reinforcement, the yield strength of bonded unprestressed seven wire strand will be attained or exceeded. If it is assumed that the maximum concrete compressive strain is 0.003, then the neutral axis factor k_u must be not more than 0.23, if the yield strain of 0.01 is to be attained at ultimate in the bonded, unprestressed strand.

TABLE 1—TEST RESULTS (SIMPLE SPAN BEAMS)

Beam	TB1	TU1	TU2	TU3‡	RB1	RU1	RU2
M_u (test), in.-kips	973	1105	905	1031	827	707	689
M_u (ACI),* in.-kips	950§	861	698	862	810§	678	654
M_u (calc.),† in.-kips	—	1009	853	1011	—	696	670
$\frac{M_u \text{ (test)}}{M_u \text{ (ACI)}}$	0.99	1.28	1.30	1.20	1.02	1.04	1.05
$\frac{M_u \text{ (test)}}{M_u \text{ (calc.)}}$	—	1.09	1.06	1.02	—	1.02	1.03
f_{se}, ksi	182.6	182.9	181.6	188.2	188.2	183.1	186.6
f_{su} (test), ksi	—	260.3	253.5	259.9	—	208.1	205.2
f_{su} (ACI),* ksi	273.4	197.9	196.6	203.2	238.0	198.1	201.6
f_{su} (calc.),† ksi	—	251.3	250.8	255.1	—	206.7	208.3
$\frac{f_{su} \text{ (test)}}{f_{su} \text{ (ACI)}}$	—	1.31	1.29	1.28	—	1.05	1.02
$\frac{f_{su} \text{ (test)}}{f_{su} \text{ (calc.)}}$	—	1.04	1.01	1.02	—	1.01	0.98
Average crack spacing, in.	5.50	4.25	5.35	5.21	5.50	4.24	4.58

* Calculated using f_{su} (ACI) $= f_{se} + 15000$ psi.

† Calculated using:

f_{su} (calc.) $= f_{se} + \frac{1.4f_c'}{100p} + 10{,}000$ psi, but not greater than f_{sy}.

‡ The bonded unprestressed strand assumed to develop its yield stress of 242 ksi.

§ Includes contribution of duct tubing acting as additional bonded reinforcement. This contribution was not included in the strength calculation when these beams were designed and is the reason for the high values of M_u in the bonded beams.

Note: in. × 2.54 = cm; in.-kips × 11.52 = kg-m; ksi × 0.703 = kgf/mm².

Equating the total tension in the prestressed and unprestressed reinforcement to the compression in the concrete for the limiting case; it follows that, for the unprestressed strand to reach its yield strength, we have the condition:

$$\frac{pf_{su}}{f_c'} + \frac{p_u f_y}{f_c'} \text{ must be} \leqq 0.195k_1$$

where p and p_u are the ratios of prestressed and additional unprestressed reinforcement respectively, and k_1 is the rectangular stress block factor defined in Section 1503(g) of ACI 318-63.

In the case of Beam TU3; $0.195k_1 = 0.166$, and the actual value of:

$$\frac{pf_{su}}{f_c'} + \frac{p_u f_y}{f_c'} = 0.065$$

If the amount of reinforcement is such that the above requirement is not satisfied, then the stress at ultimate in the bonded, unprestressed strand must be calculated, taking into account compatibility of strains in the beam and the stress-strain curve of the strand.

Ductility and serviceability

It can be seen in Fig. 5 that all four T-beams behaved in a very ductile manner, the maximum midspan deflections exceeding 15 in. in all cases. Beam TU1 exhibited the greatest ductility, attaining a maximum midspan deflection of 23 in. just before failure. The less ductile behavior of the rectangular beams was to be expected, since in these beams:

$$\frac{pf_{su}}{f_c'} + \frac{p_u f_y}{f_c'} \approx 0.26$$

The widths of the cracks in Beams TU1 and TU2, and RU1 and RU2, were less than or about equal to those which occurred in Bonded Beams TB1 and RB1, respectively, for loads up to about twice a hypothetical service live load based on the load factors and ultimate strength equations of ACI 318-63. The cracks in Beam TU3 and the fact that fewer bonded reinforcing elements were used in TU3 increased in width more rapidly than was the case with the other T-beams. This was probably due to a combination of the higher stress which would exist in the bonded strand reinforcement of Beam TU3 than in the other beams. However, the maximum crack width in Beam TU3 did not exceed 0.01 in., until the applied load reached about twice the hypothetical service load (see Fig. 6).

The distribution of the cracks in the unbonded beams was as good as, or better than, the distribution of the cracks in the bonded beams. This may be seen in the crack patterns for the beams shown in Part 2; and in Table 1 where the average crack spacings at the end of each test are recorded.

The behavior of the beams in this study indicates that if even the minimum amount[10] of additional bonded reinforcement is provided, then an unbonded post-tensioned member will behave as a flexural member subject to a combination of transverse and axial forces, and not as a tied arch.

DISCUSSION OF TESTS OF CONTINUOUS BEAMS

Moment distribution and ultimate strength

It can be seen in Fig. 7 that Beam CB1 essentially behaved elastically, showing moment redistribution due to inelastic behavior only in the last load increment. Inelastic behavior is evident in Beams CU1 and CU2 at loads above about 23 kips, being most pronounced in Beam CU1. The support

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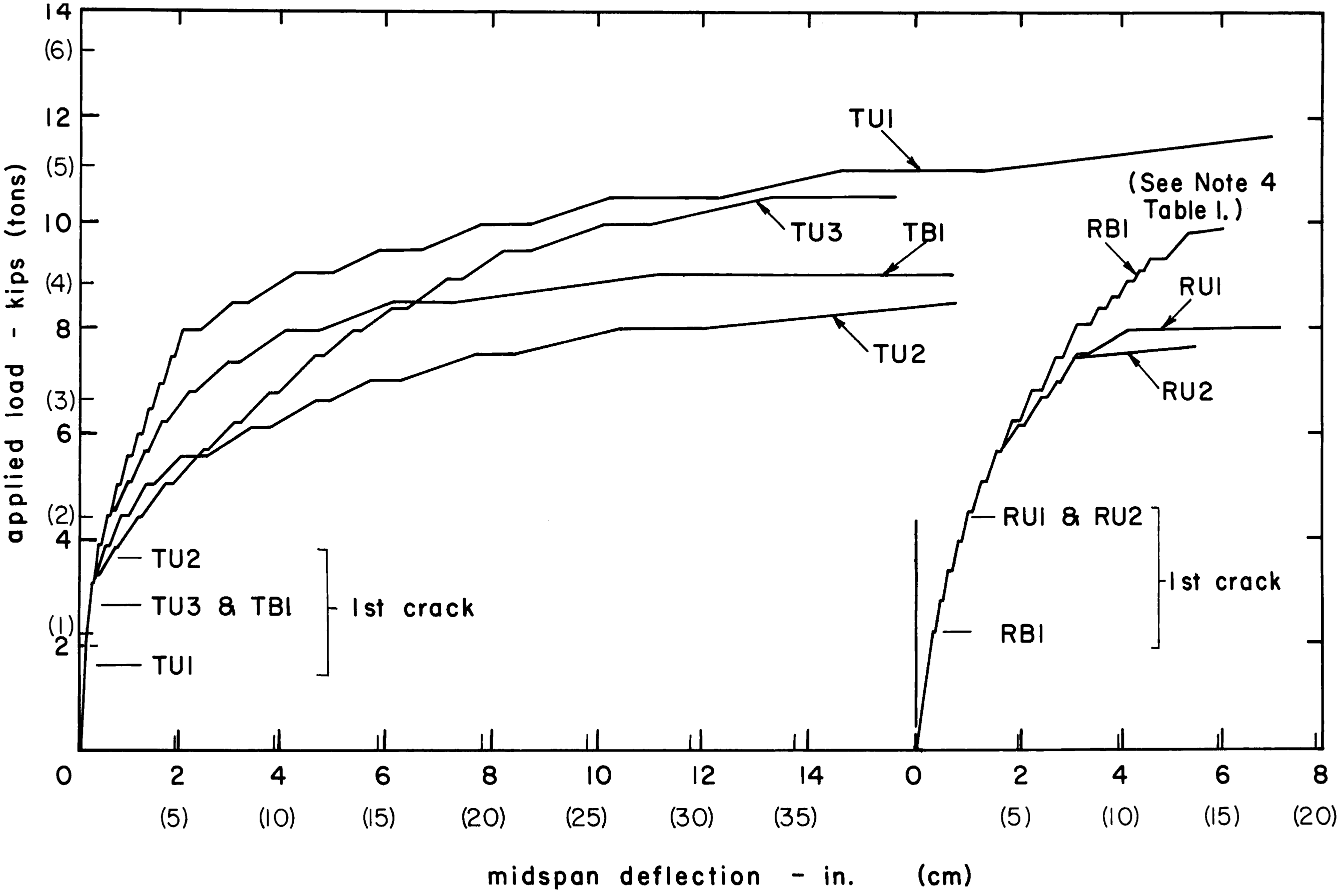

Fig. 5—Deflections of simple span beams

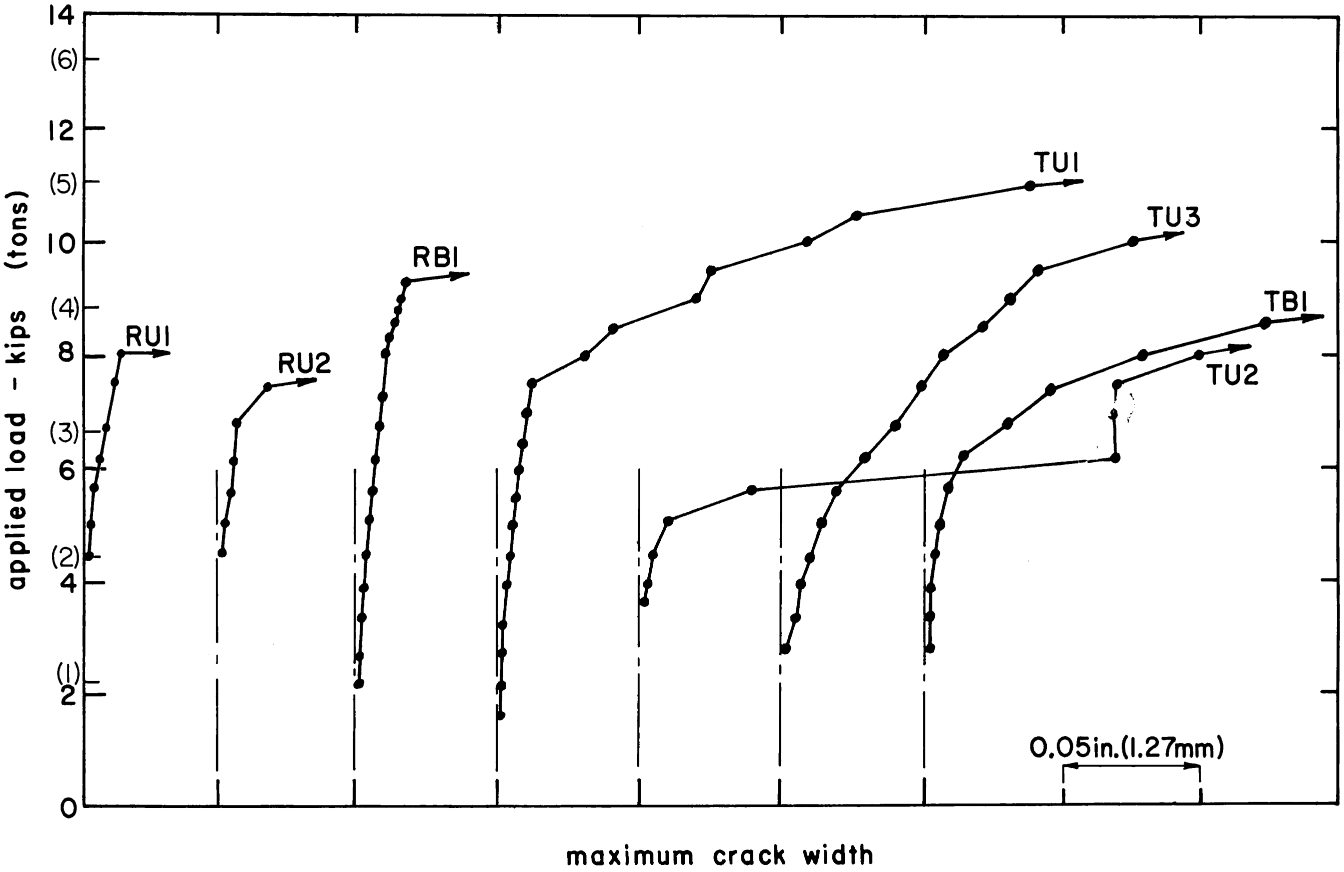

Fig. 6—Maximum crack widths in simple span beams

moments at ultimate load in Beams CB1, CU1, and CU2 are respectively 70.6, 56.2 and 69.1 percent of the elastic theory support moments due to dead load and ultimate applied load. A major part of the support moment redistribution of 29.4, 43.8, and 30.9 percent occurring in the three beams was due to the action of the nonconcordant tendons.

When a hinging region develops in a two span continuous beams, it becomes statically determinate. Under these conditions, secondary moments due to prestress in a nonconcordant tendon must disappear—since all tendon profiles are concordant in a statically determinate structure. However, the nonconcordant tendon will produce rotation in the hinging region due to elastic deformations along the length of the beam. (It is the suppression of these rotations in an elastic continuous beam that causes the secondary moments to occur.) In the case of a downwards transformed tendon, the rotation in the hinging region will be of opposite sign to that which must occur to allow redistribution of support moments due to dead and live load. The net amount of inelastic rotation necessary at a hinging region in order that a given amount of moment redistribution can occur is therefore reduced. Alternatively, if a given amount of inelastic rotational capacity is available in a support hinging region, then the amount of moment redistribution possible is increased. The amount of the increase is equal to the positive secondary moment caused in the elastic continuous beam. In the limiting case of no net inelastic rotation occurring, the beam behaves elastically up to failure and the support moment at failure would be the elastic theory support moment due to dead

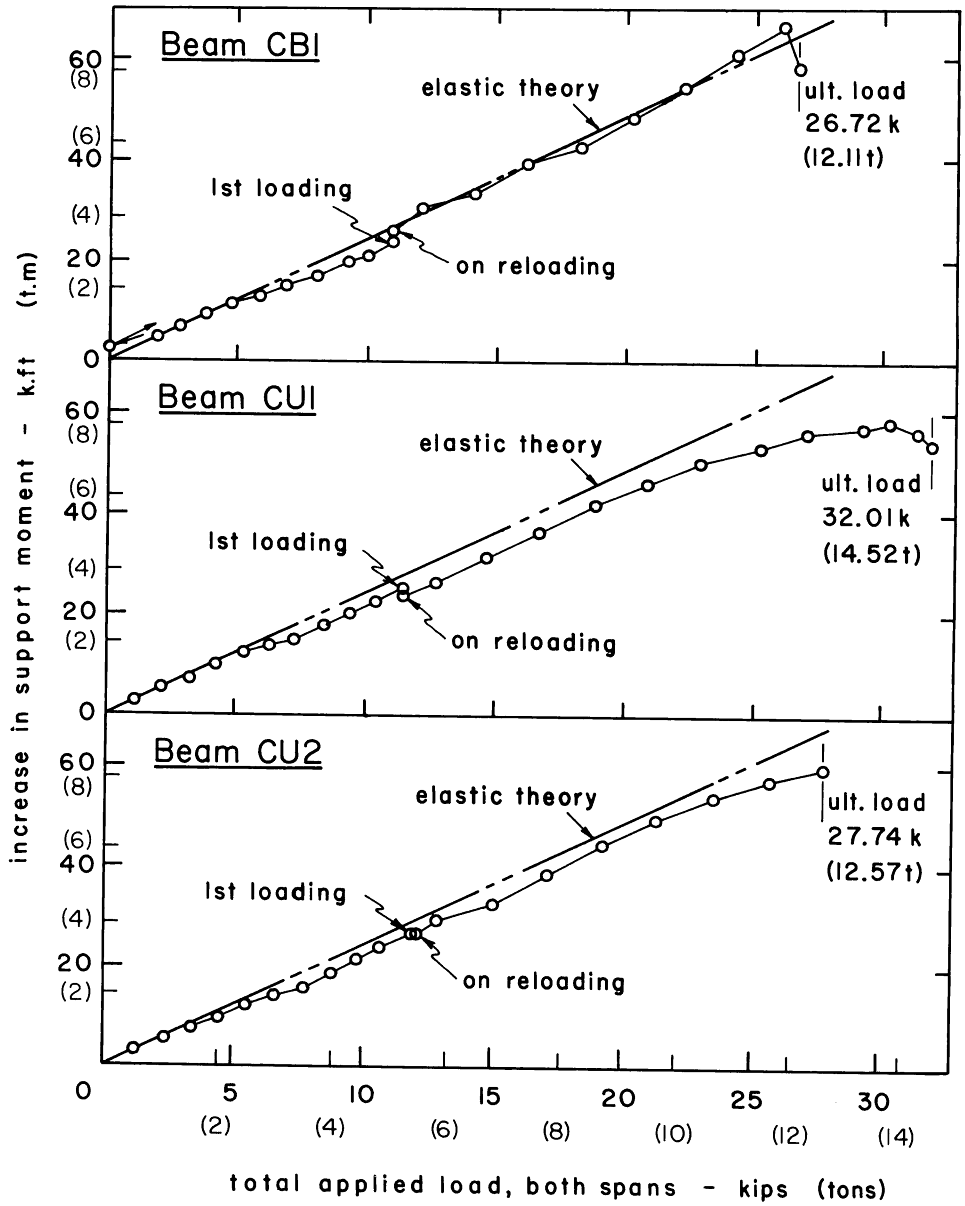

Fig. 7—Variation of center support moment with applied load

and live loads, less the positive secondary moment due to prestressing. This ties in with the alternate concept of such a beam remaining elastic until the instant before failure of the support section, in which case it would be statically indeterminate and the secondary prestress moment would continue to act up to failure. In the general case, the support moment at failure would be the elastic theory support moment due to dead and live loads, less the positive secondary moment due to prestressing, less additional redistribution of moment possible as a result of inelastic behavior of the beam adjacent to the support.

The redistribution of support moment at ultimate made possible by the action of the nonconcordant tendon represented 85, 54, and 85 percent respectively of the total redistribution of moments which occurred in Beams CB1, CU1, and CU2. This amount of redistribution occurs without inelastic action and therefore no special reinforcement limitations are necessary if this type of redistribution is recognized in design.

The behavior of Beam CU1 indicates that it is also possible to achieve considerable redistribution of moments by inelastic action in continuous beams prestressed with unbonded tendons.

It can be seen in Table 2 that the maximum moment at the face of the center support exceeded the calculated moment capacity in all cases. This is probably due to a combination of effects, such as strain hardening of the unprestressed reinforcement and the development of a higher than calculated tendon force. It can be seen in Table 2 that the moment capacity of the midspan cross sections was closely approached in all three beams, and most closely approached in the unbonded beams. The measured tendon stresses at ultimate in Beams CU1 and CU2 were 239 ksi (168 kgf/mm^2) and 243 ksi (171 kgf/mm^2) respectively, as compared to the calculated value of 255 ksi (179 kgf/mm^2), obtained in both cases, using Eq. (3). It is reasonable to believe that if a modest amount of compression reinforcement had been provided near the center support, then the full ultimate moment capacities of the midspan cross sections would have been developed.

The load carrying capacity of Unbonded Beam CU1 was 20 percent greater than that of Bonded Beam CB1, although both beams were designed according to ACI 318-63 to have the same ultimate strength at all critical sections. Unbonded Beam CU2 had an ultimate load carrying capacity 4 percent greater than that of Bonded Beam CB1. The critical sections of Beam CU2 were designed assuming the tendon stress at ultimate would be as given by Eq. (3). In actual fact, the unbonded tendon stress at the center support section cannot be very different from that occurring in the midspan regions. In Beams CU1 and CU2 the actual stress was intermediate between the two calculated values, approaching the calculated stress for the midspan sections.

As a result of the redistribution of moments which occurred, and the high moment capacities

TABLE 2—BEHAVIOR OF CONTINUOUS BEAMS

	Beam		
	CB1	CU1	CU2
Center support moment† due to dead load* and prestress, in.-kips	−107	− 75	− 91
Midspan moment‡ due to dead load* and prestress, in.-kips	282	296	289
Maximum moment at center support† due to all loads, in-kips	−902	−774	−808
Maximum moment at face of center support, in.-kips	−861	−731	−767
Moment at center support† at ultimate load, in.-kips	−804	−725	−808
Moment at face of center support at ultimate load, in.-kips	−764	−682	−767
Midspan moment‡ at ultimate load, in.-kips	812	1014	843
Calculated ultimate moment of resistance at center support, in.-kips			
(1) ACI 318-63	655	677	647
(2) Using Eq. (3)	—	691	657
Calculated ultimate moment of resistance at midspan, in.-kips			
(1) ACI 318-63	950	890	730
(2) Using Eq. (3)	—	1037	869
Total load applied by rams at ultimate (both spans), kips	26.72	32.01	27.74
Average crack spacing in.	9.26	6.67	9.08

* Including weight of loading equipment, 0.443 kips per span.
† Calculated assuming a knife edge center support.
‡ Moment under the load points 1.5 ft from center of spans, measured toward the ends of the beam.
Note: in. × 2.54 = cm; in-kips × 11.52 = kg-m

developed at the center support, all three beams had ultimate carrying capacities considerably in excess of those which would be recognized by ACI 318-63.

Ductility and serviceability

The ductility of these beams was intermediate between that of the simply supported T-beams and that of the simply supported rectangular beams. The ductility was curtailed by the failure of the center support section in all three beams. It is probable that if a modest amount of compression reinforcement had been provided near the supports, than the ductility of these beams would have been of the same order as that of the simple span T-beams.

It can be seen from the crack spacings given in Table 2 and other cracking data given in Part 2, that the cracking behavior of Unbonded Beams CU1 and CU2 was as good as or better than that of the Bonded Beam CB1.

The behavior of these beams again demonstrated that the provision of the minimum required amount[10] of bonded unprestressed reinforcement, will ensure that an unbonded post-tensioned beam will behave as a flexural member, and not as one or more shallow tied arches.

CONCLUSIONS

1. Simple span and fully loaded continuous, unbonded, post-tensioned beams containing additional unprestressed bonded reinforcement and designed according to the provisions of ACI 318-63, will have serviceability characteristics, ductility and strength, equal to or better than those of comparable bonded post-tensioned beams.

2. Eq. (3) yields a close estimate of the ultimate stress in unbonded tendons, in simple span beams having span to depth ratios found in practice, with straight or parabolically draped tendons, and with at least a minimum amount of additional bonded upestressed reinforcement.

3. Eq. (3) could be used in the design of continuous beams if the tendon stress were taken as the average of the values calculated at all critical sections.

4. Seven wire strand can be used effectively as additional unprestressed bonded reinforcement in unbonded post-tensioned beams.

5. Redistribution of design support ultimate moments by an amount equal to the positive secondary prestress moment should be allowed in design, without a special limitation on amount of reinforcement.

6. The minimum amount of bonded unprestressed reinforcement that should be provided in an unbonded post-tensioned beam is 0.4 percent of the area of that part of the beam section between the flexural tension face and the neutral axis of the gross section.

7. Provision of the minimum amount of bonded unprestressed reinforcement, specified in conclusion 6 above, will ensure satisfactory serviceability characteristics in beams with unbonded tendons; it will also ensure that the beam will behave as a flexural member after cracking and not as a shallow tied arch.

ACKNOWLEDGMENTS

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