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THE SHELL VAULT OF THE EXPOSITION PALACE, PARIS

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FOREWORD

This paper was the basis for an oral presentation at the Joint ASCE-IABSE Meeting at the New York Convention, October 1958. All Joint Meeting papers published in Proceedings or in Civil Engineering will be reprinted in one volume.

SYNOPSIS

General requirements for the structure are given. The preliminary and final designs are described and the function of structural elements is explained. Principles of design, strength of materials, allowable stresses and forces affecting the roof design are presented. The investigation of local and general buckling of the shells, the influence of creep and other factors on the stability of the shell vault are discussed in detail. Research studies are described. The solution of numerous construction problems to meet design requirements are also presented.

INTRODUCTION

The Exposition Palace of the National Center of Industries and Technology (Fig. 1) is the first unit in what is to be a large urban development in the area surrounding the Place de la Defense and in which will be concentrated the professional services of the heavy industries of France.

Note.—Discussion open until June 1, 1960. Separate Discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Structural Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. ST 1, January, 1960.

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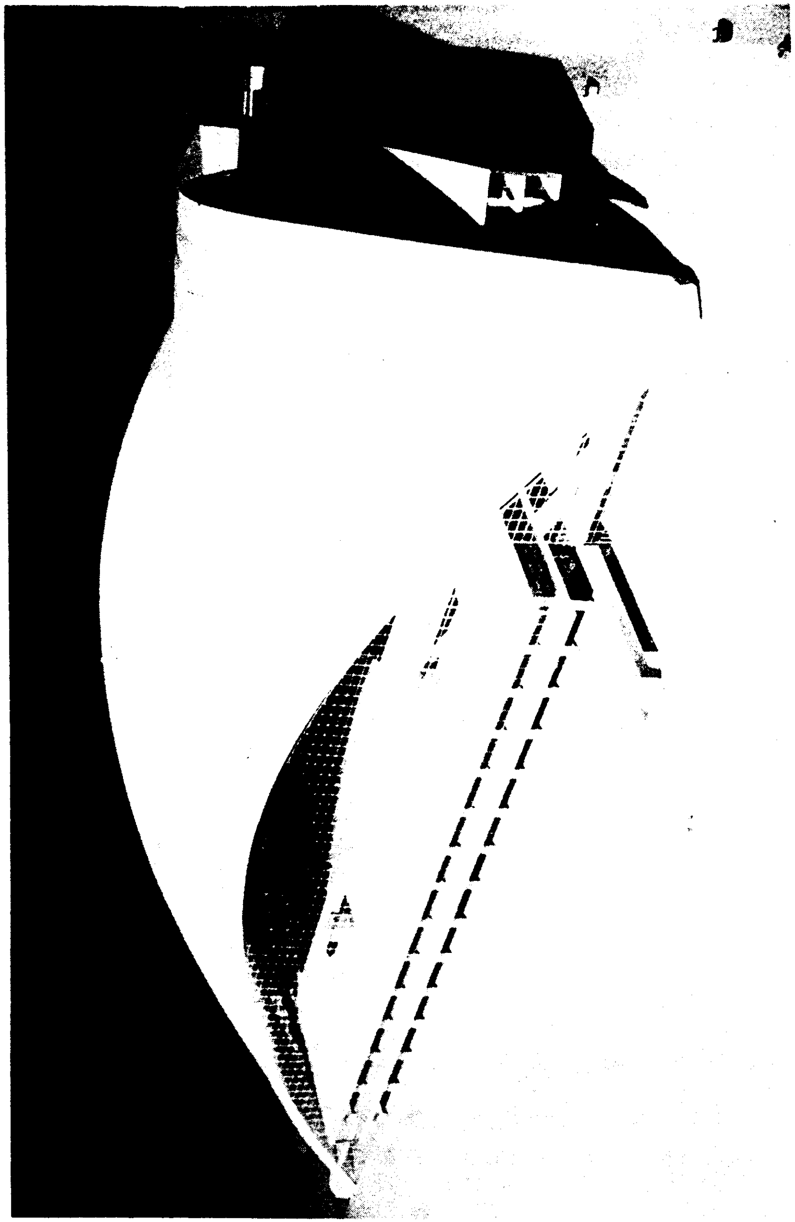

FIG. 1.—MODEL OF THE EXPOSITION PALACE OF THE NATIONAL CENTER OF INDUSTRIES AND TECHNOLOGY, PARIS, FRANCE.

From the inception of the project the originator planned to erect a monumental and striking structure as a symbol of achievement in construction and a milestone in the expansion of Paris. The scope and criteria were fixed by three architects, Messrs. Camelot, de Mailly and Zehrfuss, all Grand Prix de Rome winners, who were responsible for the triangular layout of the structure. Three civil engineering construction firms, Balency and Schuhl, Boussiron, and Coignet, collaborated in the design and construction.

The requirements set by the project director and the architects were rather exacting. They specified a 21-month construction period and set a ceiling on expenditures before completion of a preliminary design. The building was to contain 1,120,000 sq ft of floor area and a total roof surface of 290,000 sq ft (Fig. 2).

These limitations with respect to time and cost for a structure of such magnitude made it necessary for those responsible for the design to study the engineering problems and the entire project organization with unusually great care.

A number of architectural solutions of the large roof structure were studied, and the final requirements subsequently set by the architects and the project director were as follows: Build a reinforced concrete or steel roof, preferably of groined vault type, inscribed in an equilateral triangle with sides measuring 715 ft and supported at the three points of the triangle.

Conforming to the above criteria, seven solutions including three reinforced concrete designs, three structural steel designs and one composite concrete-steel design were submitted by different engineering firms.

SOLUTION OF THE GENERAL PROBLEM

Preliminary Design.—It should be emphasized that meeting the design criteria by use of a thin concrete shell called for an unprecedented span with the half-chord of each groined arch having a length of 390 ft corresponding to an arch span of 780 ft measured along the projection of two intersecting groins. The unusual plan, far more complex than a rectangular shape, raised numerous problems.

That this structure was to represent a great advance in engineering techniques and construction practice is apparent from the fact that the two thin shells of the Marignane double hangar,[2] with world record spans of 333 ft each, could be inscribed within the span of the Exposition Palace. To realize such an accomplishment, two major requirements of light weight and safety had to be considered. The difficulty consisted primarily in finding a satisfactory compromise between these contradictory requirements.

Light weight means economy of materials and, of necessity, a precise and careful design. It was decided to use a self-supporting roof without transfer of flexural or shear loads to structural members (purlins, arches, etc., Fig. 3a). Preferably the flow of the forces towards the supports should be direct and follow a minimum course (Fig. 3b). The form should be clear-cut and esthetically pleasing even to a layman.

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[2] Le hangar a deux nefs de 333 ft. de portee de l'Aeroport de Marignane - N. Esquillan - Annales de l'Institut Technique du Batiment et des Travaux Publics XX, Septembre 1952, No. 37.

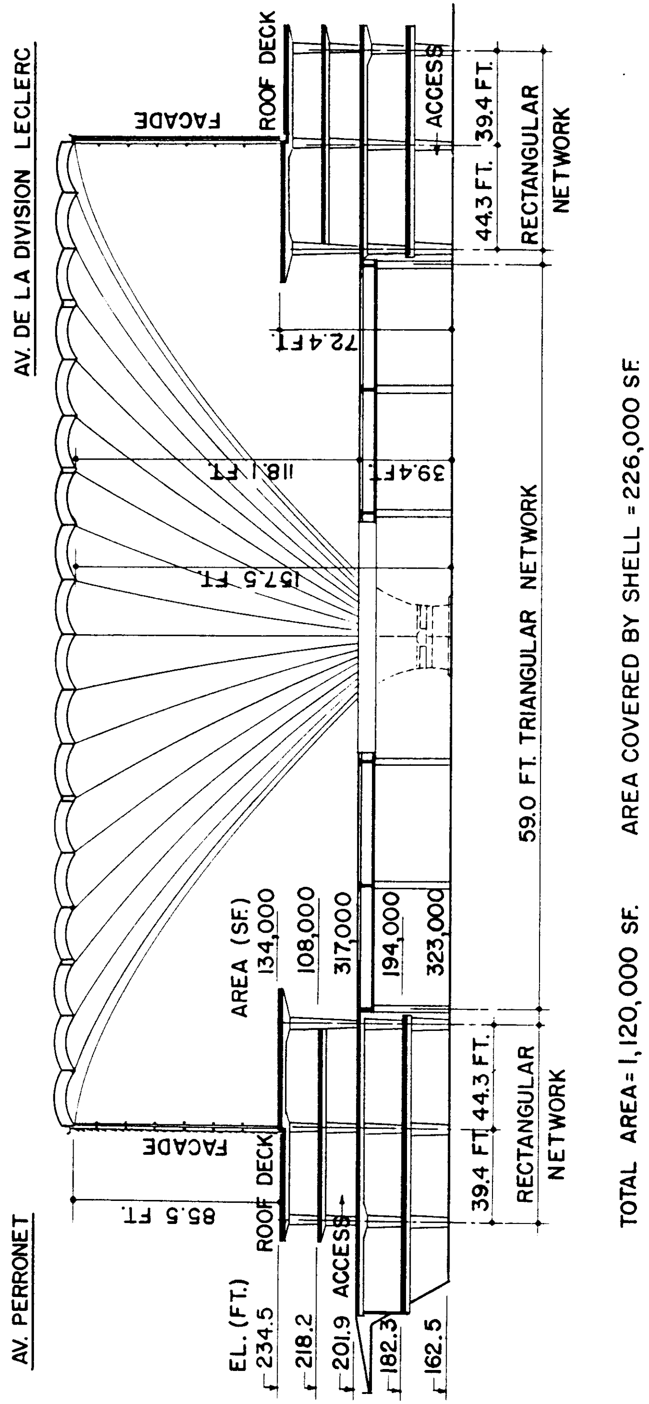

FIG. 2.—CROSS-SECTION PERPENDICULAR TO TWO ADJACENT FACADES.

Three possibilities were examined: (a) a thin single-slab shell of simple curvature; (b) a thin single-slab shell with double curvature; and (c) a vault consisting of two inter-connected thin shells of single or double curvature (Fig. 4).

The first two designs, due to the triangular shape of the roof and of their relatively low torsional resistance, particularly in the case of design (a), required heavy ribs connected to a rigid frame in the plane of the facades. In a structure of such great span, the problem of buckling is of primary importance, whether in the case of general buckling of the roof as a whole or local buckling of the thin shell. Repeated checks were made in this connection especially because the radii of curvature of the directrices of the vault would be relatively long and the stresses due to dead load would be very high owing to the unusual dimensions of the structure. These dimensions augment the effects of creep and shrinkage to such an extent that the stresses approach those allowable only for the highest quality concrete used in prestressed concrete.

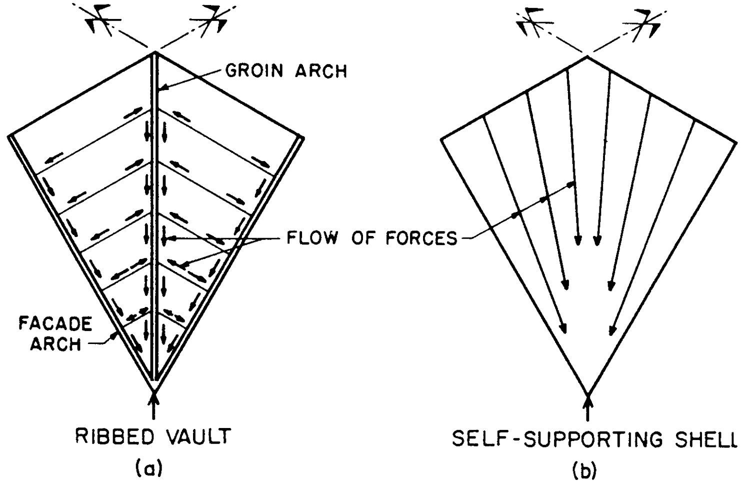

FIG. 3.—SIMPLICITY OF FLOW OF FORCES IN SELF-SUPPORTING SHELL IN CONTRAST TO A RIBBED VAULT.

Under these conditions a single shell slab is extremely sensitive to eccentricity of the centroidal axis, caused by structural defects or deformations under non-symmetrical loads. A difference of only 3/4 in. between the theoretical locations of the centroidal axis and its real location leads to a flattening of the axis over a distance of about 13 ft for a radius of curvature of 330 ft, and of 52 ft for a radius of curvature of 1300 ft. Such a difference could be caused by settling of the falsework or forms. In point of fact, the effective values of the radii of curvature of the main arch increase from 295 ft at the crown to 1,378 ft at the springing. The 1,378-ft radius is much longer than that of the Sando bridge, which had had the largest radius in the world for concrete arches of all categories, with a span of 865 ft and a radius of curvature of approximately 790 ft.

A study of critical buckling, based on tests and the theories advanced by different authorities,[3] for the three types of shell roofs under consideration,

[3] Dischinger, Flugge; Timoshenko; von Karman; Girkman; Wastlund; Chambaud.

indicated that using the same quantity of material, the safety factor for general buckling reaches a maximum of over four for a double shell vault.

With most of the material concentrated near the extreme fibers, the self-supporting double shell would have a maximum strength in flexure and torsion for a minimum amount of concrete and could safely withstand considerable eccentricity of the stress curve with respect to the centroidal axis. For this design, an error or an accidental deformation of 4 in. would cause a stress increase of only 43 psi.

For the above reasons, the initial proposals were for the use of two parallel cylindrical reinforced concrete thin shells of single curvature, connected by a system of metal beam lattices along lines radiating from the abutments and along lines perpendicular to the facades.

As the design progressed, an attempt was made to replace the beam lattice by a three-dimensional system of structural tubing placed along the ribs of tetrahedra, whose bases were alternately located in the upper and lower shell slabs. Both the lattice and the tubing appeared to be complex and costly and it seemed advisable to look for another solution.

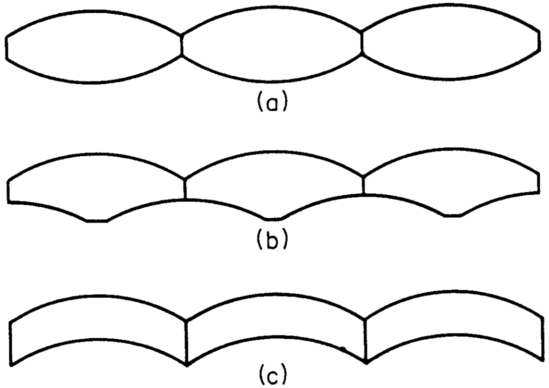

FIG. 4.—STUDY OF THREE DOUBLE SHELL DESIGN FORMS LED TO SELECTION OF CROSS-SECTION (c).

Final Design.—The structure finally adopted is entirely of reinforced concrete. Substantially simpler than the preliminary designs, it retains their best features, that is, the double shell without special supporting members (Figs. 2 and 8).

In the Marignane hangars a corrugated shell was used to take the place of ribs and diaphragms and to provide a moment of inertia which would be sufficient. In the Exposition Palace, a greater moment of inertia is provided by the double shell. Nevertheless, despite difficulties in construction, transverse corrugation of the two slabs had to be provided to prevent buckling and to avoid the use of numerous rigid structural members.

The reinforced concrete webs radiating from the abutments are more effective than the metal lattice of the preliminary composite design solutions. The vertical transverse reinforced concrete diaphragms are spaced at 29.5 ft and are perpendicular to the facades. They contribute to the rigidity of the structure and act to equalize possible differences in load between the different

sections and to distribute concentrated live loads which may be suspended from the roof.

The resulting structure has all the properties of an airplane fuselage or wing, in that it combines light weight with exceptional resistance to bending, torsion, buckling and static and dynamic stresses in all directions. The triangular shape required particularly careful attention to the application of the theories of strength of materials and the stress distribution in the vicinity of the crown.

The above general reasoning led almost automatically to the selection of the final design form (Fig. 4c), either on the basis of strength of materials, practical design considerations or construction methods which could not be overlooked.

The question of how many corrugations should be used had to be answered. This was not a free choice because (1) transverse curvature of the shell had to be sufficient to provide the required factor of safety against local buckling, (2) placing concrete without having to resort to double formwork limits the maximum slope of surface to 35°, and (3) construction by stages with movable falsework led to adoption of a number of corrugations which was a multiple of the construction stages, the number of stages itself being dictated by the time specified in the contract.

STRUCTURAL FEATURES AND THEIR FUNCTIONS

The shell roof (Figs. 2 and 5) covers an area of 226,000 sq ft and is similar to a groined vault without any ribs or structural arches (Fig. 3b). Being completely self-supporting it has a substantial moment of inertia due to its double angle. The theoretical half-spans of the six similar component sections vary from 337 ft for the sector at the facade to 390 ft for sectors at the groin. The theoretical rise is 152 ft. The height of the cross section measured along a perpendicular to the centroidal axis increases from 6.2 ft at the crown to 9.0 ft at the springing.

The abutments are massive vertical structures resting on excellent foundation soil. The project director called for maximum use of the site and located the abutments at the extreme limits of the property. Therefore, it was impossible to transmit the longitudinal force of the arch directly to the soil, because subway tunnels or underground passages for vehicular traffic would probably be constructed just beyond the abutments in the near future. Tie members between abutments in the planes of the facade were therefore deemed indispensable.

The ties had to be located at a level sufficiently low to avoid obstructing the basement entrance. This location has a considerable advantage from the standpoint of safety. Long, exposed ties would be greatly affected by high temperature in case of fire and would not fulfill the function of keeping the abutments at a constant distance under dead load. By placing them underground, they were protected from fire and other significant temperature variations.

Roof.—On either side of the central groined arch the roof consists of nine successive corrugations of which the projected width decreases from 22.5 ft at the crown to 1.85 ft at a distance of 43 ft from the springing. Similarly, the radius of curvature of the intrados of the corrugations varies from 22.5 ft at the crown to 10.8 ft at a distance of 43 ft from the springing. The decrease in width and curvature of the corrugations results in a substantially constant factor of safety against local buckling, as the stresses due to dead load practically

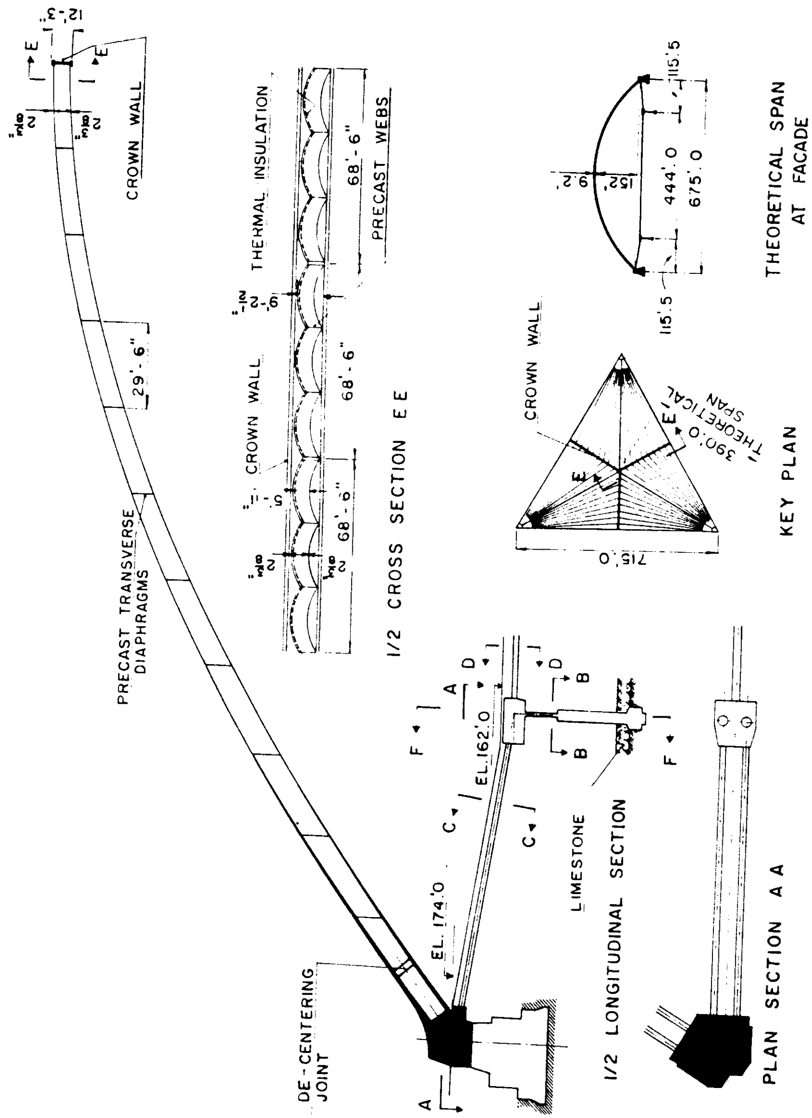

FIG. 5.—GENERAL LAYOUT OF ROOF AND ANCHORAGE FOR ABUTMENTS.

double between the crown and the springing. In the vertical plane perpendicular to the facades all the radii of curvature are identical, which permits the re-use of forms.

To economize on concrete the thickness of each shell, which is only 2-3/8 in. at the crown for the two center sections, remains at 2-1/2 in. to about 100 ft from the springing (Figs. 7 and 8). Due to reduced width, the stresses at the latter section approach the allowable limits. From this section to the abutment the thickness of the shell slabs is determined by the allowable stress and is progressively increased to 24 in. at the abutment where their width is reduced to 13 ft on either side of the groin.

The two shells (Figs. 5 and 8) are connected by means of precast reinforced concrete webs 2-3/8 in. thick, which enable the system to function as assumed in the design calculations and provide sufficient resistance to shear. The last section of the groin web at the crown is cast-in-place and is 3-1/8 in. thick. Due to the construction procedure by separate sections, webs were provided on either side of each section in order to better resist torsion, flexure and shear.

The cast-in-place web at the facade is 4-3/4 in. thick and includes a projecting concrete gutter which channels rain water above the plate glass facade. The connections between the steel mullions of the facade and the vault transmit wind loads in the direction perpendicular to the facade, but permit sufficient vertical and horizontal displacement of the vault so that the two systems are fully independent. The wind load is transferred directly to the ends of the 2-3/4 in. thick transverse vertical reinforced concrete diaphragms which are perpendicular to the facade and spaced at 29.5-ft intervals. These diaphragms distribute the concentrated non-symmetrical forces to the various shell sections and also stiffen the shells which act as thin membranes between the four thin walls (webs and transverse diaphragms) for the transmission of local forces.

The junction of the precast webs and diaphragms was detailed very carefully to eliminate all possible danger of rupture due to shear. Lapping of reinforcement was given special attention and the panels were assembled and made rigid by cast-in-place joints. All panels are provided with openings for ventilation, for equalization of temperature between the two shells and for access from cell to cell. These openings also permitted the transfer of shoring from one section to another during construction.

Watertightness of the roof is obtained solely from the density of the concrete which is suitably vibrated and the surface is enriched with cement and troweled. Since the shell is in compression throughout and is double curved, cracks are not likely to occur. The inner shell doubles the assurance of no leakage.

A light color polyester-base paint was applied to the exterior of the upper shell. This adds to the watertightness, makes the outer surface smoother, seals all pores and achieves uniform appearance of concrete of different ages, prevents damage due to flue gases from nearby industrial plants and reduces heating of the concrete surface.

All rain water drains towards the abutments. From the enormous roof area of 75,000 sq ft tributary to each abutment, a veritable torrent of water occurs at the base, as much as 5,600 gpm during storms. It was necessary to provide a sort of ski-jump discharge similar to those of overflow spillways on dams, and large reinforced concrete basins at the ends of the abutments. A 32-in. diameter pipe connects the bottom of each of these receptacles to the sewer system.

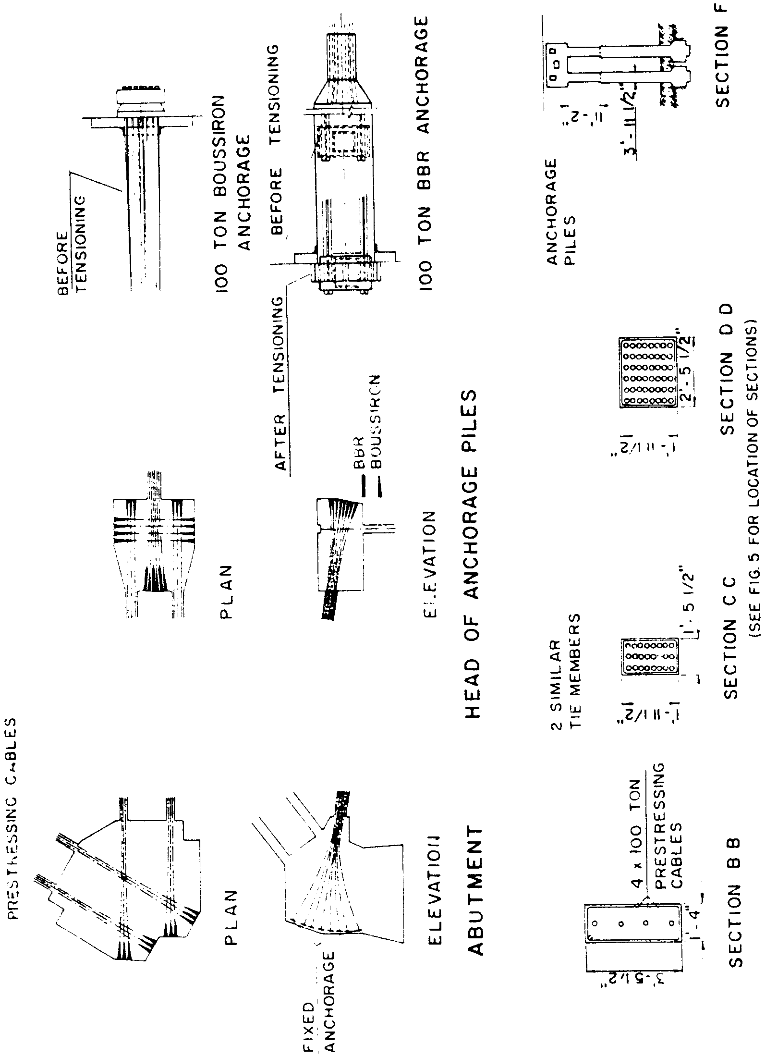

FIG. 6.—SECTIONS AND DETAILS OF ANCHORAGE FOR ABUTMENTS.

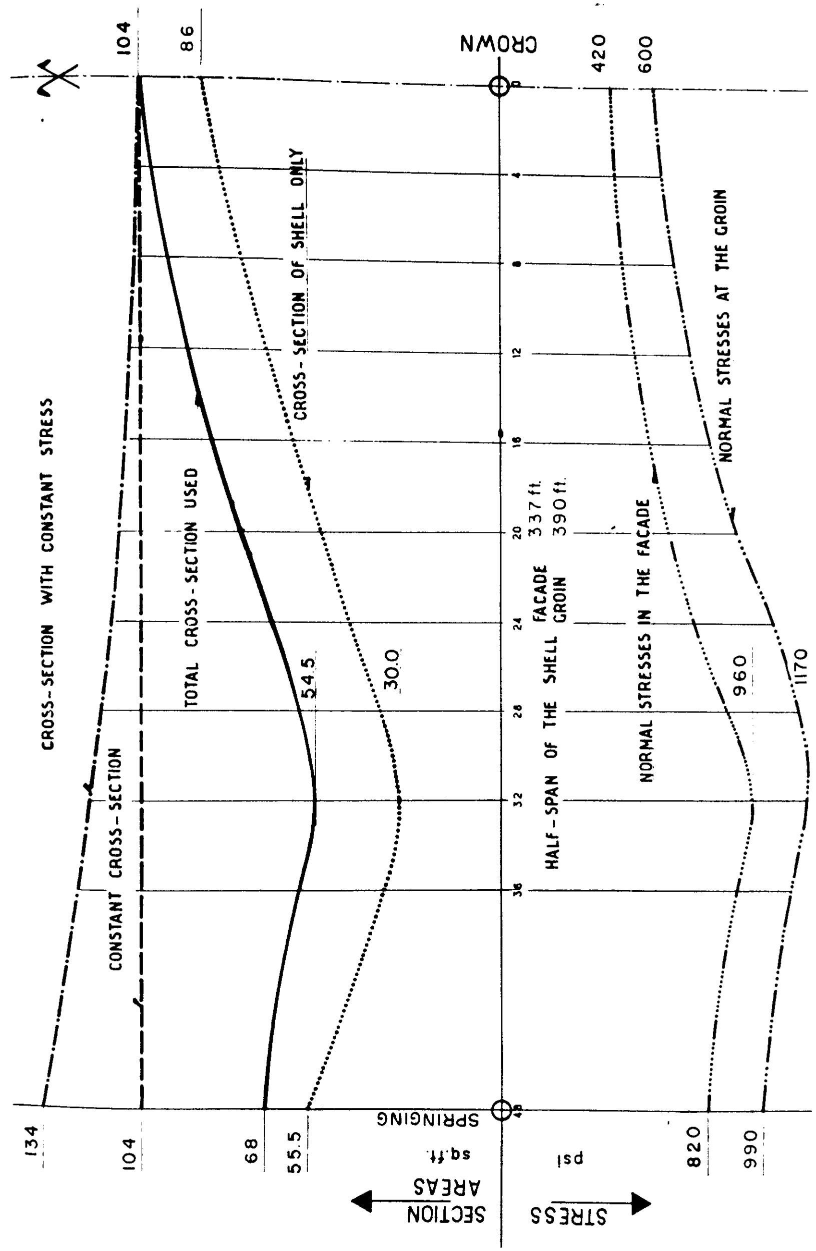

FIG. 7.—COMPARISON OF THE QUANTITY OF CONCRETE REQUIRED ON THE BASIS OF THE HYPOTHETICAL DIMENSIONS AND STRESSES. UPPER CURVES SHOW NORMAL SECTIONS PERPENDICULAR TO THE PLANE OF THE FACADE; LOWER CURVES SHOW STRESSES UNDER NORMAL DEAD LOAD (THRUST UNDER DEAD LOAD ONLY, ASSUMING THAT EACH SEGMENT HAS A CATENARY LOAD CURVE).

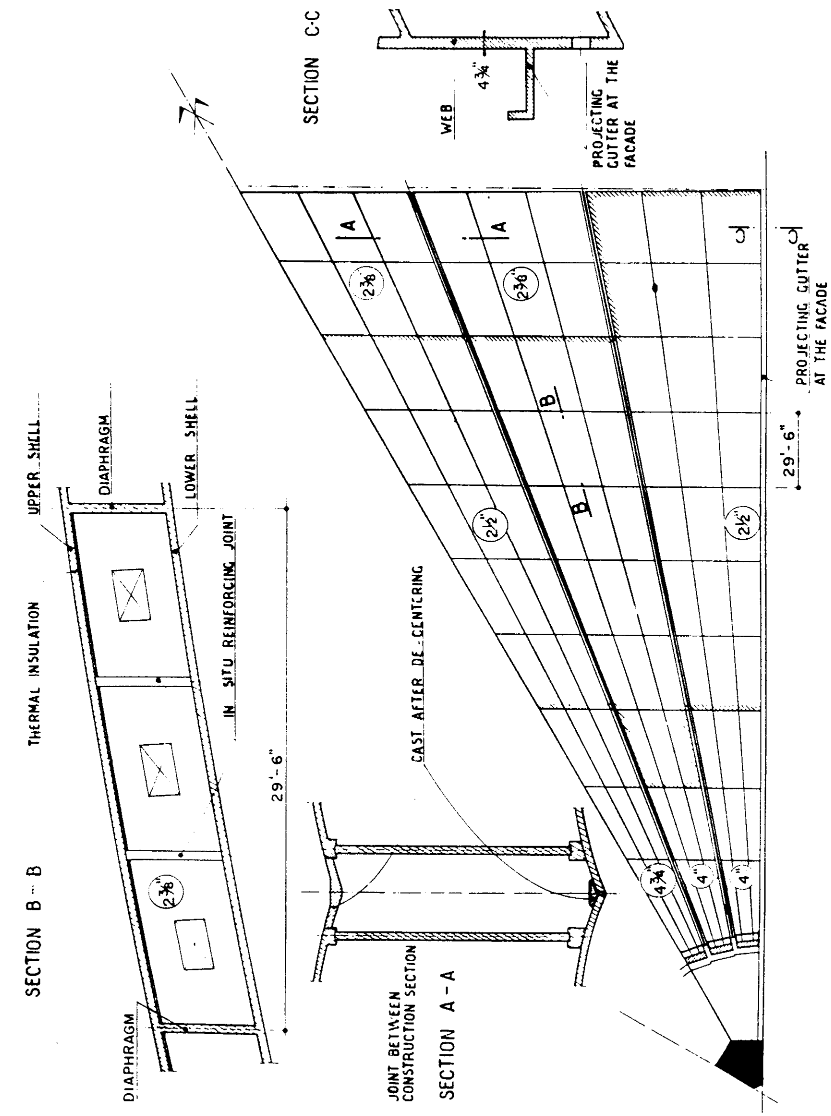

FIG. 8.—PLAN AND SECTION OF A ONE-SIXTH SECTOR OF ROOF. THICKNESS OF TOP AND BOTTOM SHELLS SHOWN IN CIRCLES.

The upper shell concrete was placed directly against organically inert panel forms 1-7/8 in. thick composed of wood fibers in a cement binder, the roughness of which insures good bond to concrete. The resulting thermal insulation is substantially better than that required by the design specifications. The double shell construction and the insulating panels combine to give an overall heat transfer coefficient, K, of approximately 0.17 BTU per hr per degree Fahrenheit, a level often not attained in residential buildings. Thus the Palace will not be uncomfortable in the summer even at midday.

The double shell and insulating panels also provide good sound insulation, for instance, against the noise of a hail storm, and prevent condensation on the lower shell intrados.

Crown Wall.—At the crown it was necessary to provide a special transverse diaphragm of great strength. The primary role of this element, which is essential to the stability of the roof structure, is to receive all the forces produced by the shells and webs which abut against it at angles becoming more acute as the center of the vault is approached. Each of the six 9-sector sections attains the maximum width of 205 ft at the crown and the monolithic action of each section must be assured in this zone where the deflections would be maximum. The crown wall may be considered as a horizontal extension of the groin web. The moments produced in these groin webs acting as half arches must be transmitted and distributed by torsion to the nine opposite radiating sectors. The crown wall is designed for this purpose. Compression in the wall varies from zero at the facades to a maximum of 4440 kips at the center of the vault. Its height is 12 ft 3 in. It consists of an 11-in. web and two flanges with a constant thickness of 15 in. and width varying from 23-1/2 in. to 49 in. (Fig. 15).

Design of the crown wall presented numerous problems, such as:

(1) Its position at the section of maximum thrust necessitated allowable stresses of 1,900 psi in order to reduce the concrete cross section as much as possible. Even with this relatively high allowable stress the maximum compression was such that a flange width much greater than 49 in. would have been required. In order to keep this dimension it was necessary to use an exceptionally high percentage of hard grade deformed steel in the flanges; approximately 4%, or more than 18 lb per cu ft of concrete.

(2) In the course of the different stages of construction, special arrangements had to be made to insure transmission of compressive stresses while permitting complete freedom of vertical deformation.

(3) For various reasons (increased weight due to the doubling of the wall at the crown, local thickening of the shell slabs to transmit jack pressure, etc.) it was impossible to employ usual de-centering methods by means of horizontal jacks located at the crown.

Decentering Walls.—In order to decenter each completed section of the roof, gaps were left at the springings of the arches approximately 36 ft from the center of the abutments in which jacks were provided (Figs. 5 and 8). On either side of this 3-ft wide gap, the diaphragms are capable of distributing the concentrated loads that would be produced during decentering operations.

Location of the gaps in this unusual position made it possible to take advantage of the relatively large shell thickness of 8 in., thus avoiding the need of reinforcement to resist the thrust of the jacks acting along the centroidal axis. Grouping the jacks also produces easier and quicker distribution of forces in the shells.

Abutments and Foundations.—The abutments are massive, prismatic concrete supports 20 ft 8 in. high, for which the dimensions and shape were designed to insure proper anchorage of the tie members and rigid fixing of the vault. The step-shaped foundations extend 20 ft 4 in. below the base of the abutment. The bottom 8-ft high step of the foundation was cast directly in the excavation insuring better bond with the excellent foundation material of marl and Lutetian limestone.

Tie Members and Cable Anchorages.—The tie members (Figs. 5 and 6) are located in the vertical plane of the facades. The central sections 440 ft long are of concrete, 24 in. by 30 in. in cross section. Each tie consists of forty-four cables and each cable has a working strength of 185 kips. Beyond the central sections the ties are separated into two inclined members 24 in. by 18 in. in cross sections, with twenty-two cables in each member. Each tie member works at 8,160 kips to resist the permanent load thrust of the dome. Four additional void ducts are left for possible future installation of cables.

At the point where the ties are separated, underground concrete pads, prestressed in three directions, are placed upon two prestressed cable anchorage shafts designed to withstand a vertical force of 1,764 kips. These shafts are continued downward through a layer of limestone and flared beneath it. To permit a slight horizontal movement during successive prestressing of the cables without producing excessive stresses in the anchorages, elastic sections of prestressed concrete, 16 in. by 41 in. by 11.5 ft, are positioned between the bottom of the anchorage block and the top of the circular shafts.

The location and intersection of the cables in the abutments and the steel reinforcement were studied extensively to arrive at a suitable distribution of the considerable forces acting in the anchorage.

All the cables are of a type used in the Boussiron-BBR prestressed concrete system, consisting of approximately 1/4-in. diameter wires fixed at the ends by steel button heads (Fig. 6).

Facades.—The large facades consist entirely of panels of tempered plate glass. The panels are supported by a metal frame whose lightness contrasts with the imposing mass of the vault and of the concrete floors. The main vertical members are H-shaped. The maintenance walkways contribute to horizontal rigidity; and the muntins for the glass panels, at about 7 ft 6 in. on center, are of thin hollow stainless steel.

GENERAL PRINCIPLES OF DESIGN

External Forces and Internal Reactions.—The principal forces affecting the roof are those usually considered in the design of roof structures: dead load, live load (service load), wind and snow loads, volume changes due to temperature, shrinkage and creep.

The dimensions of the structure and the great risks involved were such that the following effects were also considered: method of construction; errors in shell thickness; variation in concrete density; deviations between theoretical and actual points of application of forces due to possible construction errors affecting the center line of the vault, or the axes of its various component parts; temperature differences and differences in shrinkage and elastic modulus between the two shells; second degree deformations; and compensating errors during decentering of the arch and operation of the jacks.

Dead Load.—As already mentioned, the vault contains no ribs or facade arches. It consists of a double self-supporting shell with two basic characteristics: (1) the average thickness of the concrete is constant along each horizontal generatrix in the planes perpendicular to the facades; and (2) the centroidal axis selected for a facade, for instance BC, is a catenary curve of the dead load of triangle BCH (Fig. 9a).

From these two basic characteristics, it follows that any sector, such as the elementary triangle BFG cut out of the shell and projected onto a vertical plane along its length, is a catenary curve of the dead load. If sector BDE is taken along the facade with DE = FG = dy, the projection of the elementary sector BDE is a catenary curve of its dead load, having as its centroidal axis that of the facade, and the loads are proportional to those of sector BDE.

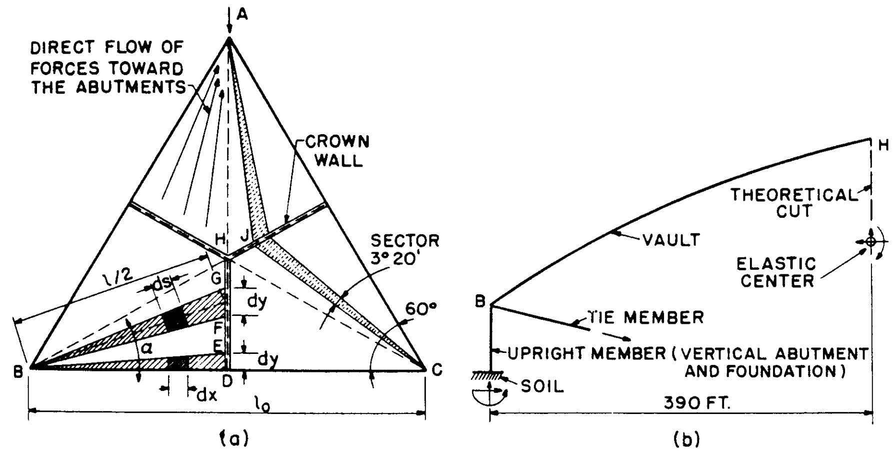

FIG. 9.—(a) ANY SECTOR OF THE ROOF IS A CATENARY CURVE OF THE DEAD LOAD. (b) DEFORMATION OF THE ROOF STRUCTURAL SYSTEM, CONSISTING OF THREE INDETERMINATE HALF-ARCHES, IS RESTRAINED BY TIES BETWEEN MASSIVE ABUTMENTS.

Sectors BDE and BFG have the same rise. Their spans are respectively l_o and $= \dfrac{l_o}{\cos\alpha}$. The total dead loads are equal and can be resolved into separate equal loads applied along the length of the lines dx and $ds = \dfrac{dx}{\cos\alpha}$. The thrusts are thus Q_o and $Q = \dfrac{Q_o}{\cos\alpha}$.

The different sections BFG, each representing a catenary curve of its load, the sum of which represents the triangle BDH (one-sixth of the vault), are in independent equilibrium. The equilibrium of all these sectors requires a transverse reaction at the crown (parallel to the generatrices and directed along DH) which represents the transverse component of the thrust. The transverse components of each of the triangles BCH, ABH and ACH transmitted at point H to the crown wall are in equilibrium at that point.

In practice, the presence of the crown wall, variations in the height of the webs and other factors make it impossible to plot each sector in accordance with this strictly defined catenary curve. It was, therefore, necessary to find a compromise solution which took into account all the various factors. The centroidal axis of an isolated sector forming an angle of 3°20' approaches the ideal curve. The centroidal axis finally adopted is a curve of the fourth order valid for all the radiating sectors.

Under dead load the total vertical reaction at each support is 10,100 kips and the force in one tie member is 7,800 kips. Axial compressive stresses due to dead load are relatively moderate. For the groined arch, for example, these stresses increase from 575 psi at the crown to a maximum of 1,120 psi at three-tenths of the half-span from the springing and decrease down to 925 psi at the springing. (See Table 1 for allowable stresses.) These stresses are substantially modified by the appreciable moments due to dead load. These

TABLE 1.—PROPERTIES OF MATERIALS

(a) Concrete

Location	Cement content lb per cu. ft.	Ultimate strength, f'_c, psi[a]	Allowable Stress, psi[b]		
			Compression	Tension: Prestressed	Tension: Cast-in-place
Abutments	22	5,600	1,560	130	100
Shells	25	6,100	1,700	140	115
Crown Walls	25	6,800	1,900	160	130

(b) Reinforcing and Prestressing Steel

Type or Purpose	Elastic limit, f_{el}, psi	Ultimate strength, f_{ult}, psi	Elongation at rupture, ϵ %	Allowable stress, f_s, psi[b]
Tor, Caron (std. European deformed bars) or equal	57,000	85,000	12	30,000
Mild	35,000	60,000	16	20,000
Semi-hard for welded mesh	77,000	92,000	—	34,000
Stress relieved, self-unrolling, for prestressing cables	178,000	206,000	6	164,000
For tie cables	164,000	206,000	—	128,000

[a] 90-day strength of 8-in. cubes. [b] The above allowable stresses may be increased 8% for a simultaneous accumulation of live load stresses and stresses due to possible construction errors, shrinkage, creep, etc.

moments are produced by: (a) the deviation between the adopted fourth degree curve and the actual catenary curves of the different radiating sectors; (b) the weight of shoring for the upper shell which is removed after decentering; (c) the weight of joints between the construction stages, these joints being cast several months after decentering; and (d) allowance for possible construction errors in shell thickness (± 0.1 in. for each shell).

All the above factors bring the fiber stresses at the extrados of the groined arch to 435 psi at the crown and 1,150 psi at the springing. The total shear at the abutment is 575 kips.

Live Loads.—The self-supporting shell has a high moment of inertia. The structural system may be described as three indeterminate half-arches fixed

on the vertical abutments which in turn are supported in the soil acting as an elastic foundation. The tie members offer an elastic restraint to the deformations of the system and the degree of restraint is determined by the stress in the cables and in the concrete which surrounds them (Fig. 9b).

The deformation coefficients of the soil, determined previously by tests, permit calculation of the theoretical height to which the vertical member of the abutment should be extended to produce the same displacement in the system as would occur by soil deformations. As a precaution, two possible extreme values of the soil deformation coefficient were considered in design.

Experience with the Marignane hangars[2] and other structures showed that shell structures of this type function like arches with respect to principal stresses. This hypothesis is valid provided the distribution and transmission of stresses in different parts of the structure are thoroughly investigated in the second phase of design calculations. The behavior of the different construction sections in the course of decentering and during a non-symmetrical load test demonstrated the validity of this analogy. The principle of resolution of a statically indeterminate system is classical: a determinate system is obtained by taking a section along the generatrices at the crown and at the connections of the tie members to the abutments. Thus three similar sections will be obtained. Forces are applied to every section so that the sum of displacements caused by horizontal and vertical flexure and torsion is zero. The number of statically indeterminate variables in the entire system is eight. To facilitate solution, cases of symmetric loads related to axes of symmetry of the roof were considered. Influence lines are traced for each sector.

Anticipated live loads.—(1) Snow—10 psf uniformly distributed without drifting, or 5 psf in least favorable locations for each section with drifting.

(2) Service load—3 psf distributed in a least favorable manner, for each section.

(3) Wind—basic dynamic pressure corresponding to a wind velocity of 75 mph. Design calculations were based on the two least favorable cases determined by wind tunnel tests on a reduced-scale model: (a) with wind blowing perpendicular to a facade and three facades in place; and (b) with wind blowing perpendicular to a facade, this facade being destroyed and the other two still standing.

(4) Possible construction errors in thicknesses corresponding to 2.5 psf, assumed in least favorable locations.

(5) Variations of temperature of the whole structure of ± 36° F. with a modulus of elasticity $E = 4.25 \times 10^6$ psi.

Taking into account all live loads, the compressive stresses at the intrados of the groined arch may reach 2,060 psi at the springings, 1,660 psi at two-tenths of the half-span and 1,000 psi at the crown (extrados 1,190 psi). Tensile stresses of 90 psi at the crown can result for the hypothetical case of wind blowing onto a destroyed facade.

A verification carried out by doubling all live loads demonstrated the high degree of safety on the roof structure since even under such extreme conditions the series of stresses at the sections mentioned above would not exceed the following:

Compression: 2650 psi, 2390 psi, 1590 psi $<$ 6100 psi*
Tension: 560 psi $<$ 640 psi

* See Table 1 - Ultimate compressive strength of concrete in shells.

The stresses are conservative considering the quality of concrete and the available reinforcement to resist tension.

Volume Changes.—The volume changes in the concrete result from elastic deformations and creep under dead load, and from shrinkage and temperature changes. In order to minimize, if not eliminate, the forces which result from such volume changes, the arch was decentered by jacks located in cuts near the springings.

Elastic deformation under dead load was automatically cancelled at the moment of decentering. Creep, however, causes the crown to deflect downward with time following a relationship determined previously by numerous tests of long duration on prisms of the same concrete and subjected to the identical stresses that occur in the shells. The major part of creep and shrinkage is attained after four to five months. During this period the successively constructed sections remain supported by jacks. The total creep and shrinkage, part of which will occur after concreting the joints, can be easily estimated from the deflection of the crown so it is possible to compensate for all the volume change when the joints are filled. The shell will then be in a most favorable condition from the standpoint of strength.

It is not possible to compensate for certain effects, however, and they must be considered in design calculations. The two shells are concreted at different times and therefore the modulus of elasticity cannot be identical. Also, the two shells may be exposed to different thermal and hygrometric ambient conditions. With respect to shrinkage, for instance, a relatively long rainy period can delay contraction of the upper shell and can even produce expansion of the concrete; on the other hand, the inner shell would undergo accelerated shrinkage if the structure were to be heated during the same period. Similarly, solar radiation may produce appreciable temperature differences in the summer between the extrados of the upper shell and the intrados of the lower shell. When the outer shell is subjected to low temperature and the interior is heated, the opposite effect is produced.

For this reason the roof design was checked assuming a differential temperature of $\pm 18^{\circ}$ F. between the two shells, and differential residual shrinkage of 0.5×10^{-4} in. per in. and finally a difference in the moduli of elasticity of 10%.

Buckling and Creep.—It is the writer's opinion that shear and buckling should be most carefully investigated in thin shell structures of large curvature. Failures from such causes are particularly dangerous because no outward indications precede their occurrence; failure is almost instantaneous with the appearance of shear cracks or of a first light fold due to buckling. A crucial problem for shells of this type is the relation of creep to buckling.

General buckling of the roof as well as of the three individual sections was investigated by determining the critical load by Euler's formula for buckling:

$$P_{cr} = \frac{n \pi^2 E I}{L^2}$$

in which n was taken as 6 considering the rise-span ratio of the arch and the not perfectly fixed end conditions. $E = 1.42 \times 10^6$ psi was used for dead loads due to creep so as to obtain a factor of safety greater than 4. Other checks were made since it was reasonable to expect that an arch will not fail abruptly due to Euler's buckling under the effect of a practically constant thrust. In the particular case of the Palace roof, it was deemed prudent to anticipate an increase in the actual deformations over those initially computed because of the

maximum total deviation of the neutral axis under the effects of possible structural errors or of eccentricity of forces caused by non-symmetrical live loads and corresponding deformations of the second order. All these calculations were carried out to determine whether the deformed roof was stable and still would provide a safety factor of 2 with respect to rupture.

Local buckling due to blister or wave deformation was the subject of extensive research and tests on cylindrical straight shells of constant cross-section, subjected to axial compression. All authors appear to agree that the general formula for determining critical stress is as follows:

$$n_{crit} = k\ E\ \frac{e}{R}$$

where E is the modulus of elasticity of the material used, e the thickness of the shell, R its radius of curvature and k an empirical coefficient. Timoshenko and Girkmann give the value of $k = 0.6$, but it appears that they studied only buckling by wavy folding. The GALCIT (Guggenheim Aeronautical Laboratory, California Institute of Technology) Report No. 130, 1939, after tests made on a sheet metal cylinder, studied the effect of variation of ratios $\frac{R}{e}$ and $\frac{L}{R}$, where L is the distance between transverse diaphragms. For $1.2 < \frac{L}{R} < 3$, which is the condition in the Palace roof,

$$n_{crit} = 0.3 \times 0.6\ E\ \frac{e}{R} = 0.18\ E\ \frac{e}{R}$$

Theodore von Karman[4] and Michidsen and Kempner[5] give values between 0.182 and 0.195 for k. Their studies also concern diamond-pattern buckling, such as may occur in the thin shells being discussed in this paper.

Tests were also made on cylinders made of bristol board and of steel, and on cylindrical shells of steel and reinforced concrete with stiffened edges. The ratios e/R varied between $\frac{1}{100}$ and $\frac{1}{500}$. The mean probable value determined experimentally for k is approximately 0.18.

Another important factor which must be considered in a study of buckling in a reinforced concrete shell structure is the value of E under sustained load and higher than usual stresses. Tests were conducted for a period of more than two years but cannot be fully discussed here due to space limitations. They demonstrated the importance of creep in buckling, particularly at early ages (Fig. 10). The principal observations[6] applying to the quality of concrete used in the shell are as follows:

(1) Shrinkage increases rapidly during the first two months. It becomes stabilized more rapidly than creep and appears to attain an equilibrium value after two years. It becomes more rapid as the transverse dimensions of the elements decrease in size (case of the thin shell).

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4 The Buckling of Thin Cylindrical Shells Under Axial Compression, Journal of the Aeronautical Sciences, June 1941, No. 8.

5 Journal of the Aeronautical Sciences, December 1948.

6 These observations were made by Delarue of the Casablanca Laboratories on a beam of the prestressed bridge in Rabat and on prisms of the same concrete, which were made in the laboratory. His observations agree with the writer's evidence.

(2) Up to approximately three months, contraction due to creep appears to be proportional to the square root of time.

(3) For periods greater than three months, the variation appears to adjust itself in accordance with a law of proportionality of the type $\log \frac{t}{t_o}$ where t_o is the age at which the load is applied.

(4) With initial elastic moduli higher than 6.4×10^6 psi, the apparent moduli of elasticity as a function of duration of applied load tend towards rather lower values, of the order of 1.4×10^6 psi, while the instantaneous modulus of elasticity increases towards 7.1×10^6 psi.

(5) After three months of load application producing an average stress of 1,000 psi, the apparent modulus of elasticity (shrinkage subtracted) drops to

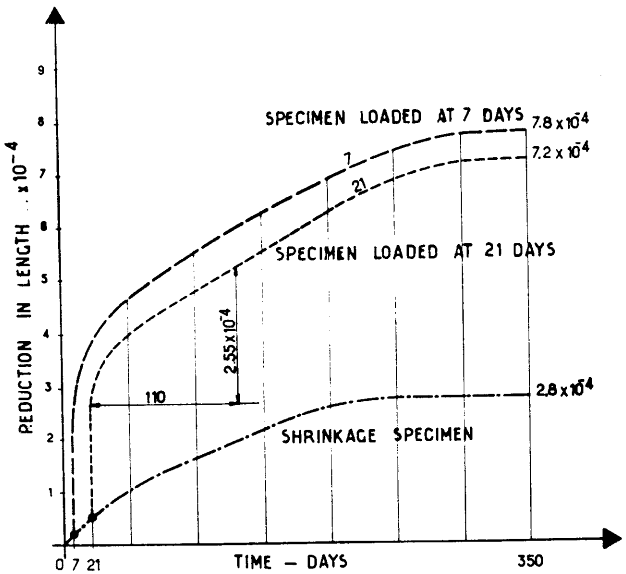

FIG. 10.—CREEP AND SHRINKAGE CURVES OF THE CONCRETE IN THE SHELLS. THE CREEP COEFFICIENT OF 2.6×10^{-4} CETERMINED BY MEASURING THE DEFLECTION OF THE CROWN AT THE END OF 110 DAYS UNDER LOAD AGREED REMARKABLY WELL WITH THE VALUE OF 2.55×10^{-4} OBTAINED FROM THE TEST SPECIMENS AT THE SAME AGE.

52% of its initial value for a load application at 7 days, and to 64% of its initial value for a load application at 21 days. From the curves it appears that the minimum apparent modulus after several decades will be between 23% and 27% of the initial value of E.

Experiments on reinforced concrete shells ($\frac{e}{R} = \frac{1}{100}$ which is approximately the average value for the Palace roof and concrete of comparable quality to that used) prove that buckling will not occur when the stress in the shell multiplied by $\frac{R}{e}$ gives $k\,E \leq 255{,}000$ psi. Thus reinforced concrete shells 1/2 in. thick (reduction to one-fifth of actual shells) subjected to 3,840 psi more than two show no adverse effects.

Many problems remain to be solved in this area such as buckling under eccentric compression, load application between 1 and 7 days and effect of secondary curvature in the shells. It is most fortunate to have the precedent of the Marignane hangars built in a relatively dry climate and subject to considerable contraction. Since 1951 their behavior has been satisfactory; the limits provided for slow deformations have not been exceeded. A scale model test conducted in 1949 to study the corrugated shell of this hangar showed that the 0.47-in. thick test shell resisted without apparent fatigue 2.25 times the maximum stresses of the real hangar roof (dead load being doubled and snow load quadrupled). It was possible to derive a probable

$$k\,E \geq 2.25\,n \times \frac{R}{e} = 2{,}100 \text{ psi} \times \frac{58.0 \text{ in.}}{0.47 \text{ in.}}$$

$$k\,E \geq 259{,}000 \text{ psi}$$

The close agreement of this value of L E and that given previously should be noted.

The first section of the roof of the Exposition Palace, which was decentered March 7, 1958, demonstrates the accuracy of design provisions concerning creep and shrinkage (Fig. 10). On July 27, 1958, the actual deflection at the crown was 2.9 in., which corresponds to a modulus of elasticity of 1.8×10^6 psi.

Other Tests.—In addition to the tests on buckling of shells and creep of the concrete, others were made including:

(a) Tests to determine differential shrinkage between cubes of same age located outdoors in normal atmospheric conditions and indoors under normal residential conditions (heating in winter, ventilation in summer);

(b) Tests to determine variation of the coefficient of deformation of soils as a function of applied unit pressure;

(c) Aerodynamic studies in wind tunnels on a model of 1/200 scale for the different stages of construction (one shell with or without floor and with one, two or three facades);

(d) Photo-elastic studies of Araldite models at 1/200 scale of a thin shell composed of three cylindrical sections tested in tension to eliminate buckling and to determine the pressure line and the effect of massive ribs along the generatrices of the crown;

(e) Bending tests of glass for the panels of the facades;

(f) Static tests of prestressing steel and prestressed concrete.

The tests of prestressing steel and concrete were made in order to apply theories of probability to evaluation of the safety of the shell and of the ties. Particularly in the case of the ties an attempt was made to determine most suitable steel stresses. Robert Levi[7] has concluded, using methods as described in numerous reports and publications, that for a probability of tolerable failure of 10^{-5} a stress of 140,000 psi in the steel was acceptable. A. Pacz,[8] following Torroja's procedure, which includes some of the financial aspects of the problem and the risks to visitors and exhibited property, arrived at a stress of 132,000 psi. Finally, a limit of 128,000 psi was adopted. The reduced stress was used to provide an additional margin of safety against possible corrosion.

[7] La Sécurité dans les Constructions, par R. Levi, Extrait de Travaux, August, 1956.

[8] La Determinacion del Coeficiente de Seguridad en las Distintas Obras, par Alfredo Paez Balaca, Instituto Technico de la Construccion y del Cemento, Madrid.

This was considered prudent because grouting the cables in the ducts, after application of stress, had to be delayed until the three-stage construction of the roof was completed. As a further precuation to avoid corrosion a soluble oil was sprayed into the cable ducts for temporary protection. Before final grouting, the oil was washed out with water.

CONSTRUCTION METHODS

The methods of construction and the sequence of operations of necessity were closely related to the design of the structure. It was not possible to recruit the 800 skilled workers required to complete the work by usual methods in the 21 months allowed from the date the site was available. Moreover, the space available for casting all elements on the site was totally inadequate. It was decided to do the precasting of large elements weighing 2.2 to 6.6 tons off site and deliver the elements to the site with well organized transport facilities. The labor force thus did not exceed 300 workers on the site with another 50 men at the precasting plant during half of the construction period.

The central batching plant was part of a precasting plant located on the river Seine three miles from the job. All precast floor elements were fabricated in this plant which was provided with fully automatic mixing equipment (Coignet process) and steam heating to accelerate hardening of the concrete. The total weight of transported precast elements, reinforcement and concrete in truck-mixers totaled 83,000 tons.

Construction Stages and Methods.—The floors were completed before the roof structure so as to reduce the height of centering needed for the vault, to provide a firm support and to minimize settling of the scaffolding (Fig. 11).

The central triangular network floor, with an area of 125,000 sq ft without an expansion joint, was built first. This floor consists of a precast double slab, prestressed at two levels in three directions. It is supported by columns at the apexes of equilateral triangles, the sides of which measure 60 ft. It so happened that the 200-psf live load specified for the floor corresponded exactly to the weight of the two shells, forms and scaffolding.

The sequence of construction of the vault was planned to maintain equilibrium of forces and deformations in the shell by careful consideration of the assumptions made in design and also to complete the structure in the specified time.

Tie Members.—Construction of the underground tie members with their anchorages and the abutments was begun along with placing of the floor. The special high-strength steel tie members were temporarily left without prestress. The cables were later tensioned in groups to resist the thrust transmitted to the abutments as each roof section was decentered. Because of the 187-kip force in each cable and the division of the tie into two members at the ends of the horizontal central section, the operation was rather a delicate one. It was necessary to induce the stress in each tie member and then grout in the decentering joints to maintain the abutment position.

Stage 1 - Springings of the Vault.—The floor was completed near the abutments first in order to permit construction of the shell from the springings to the decentering joints. Because of the concentration of forces in these areas as the width of the vault narrows toward the abutments, the shells had to be very thick. The great weight of the concrete and the proximity of the fixed abutments made it preferable in these areas to use forms supported by wood

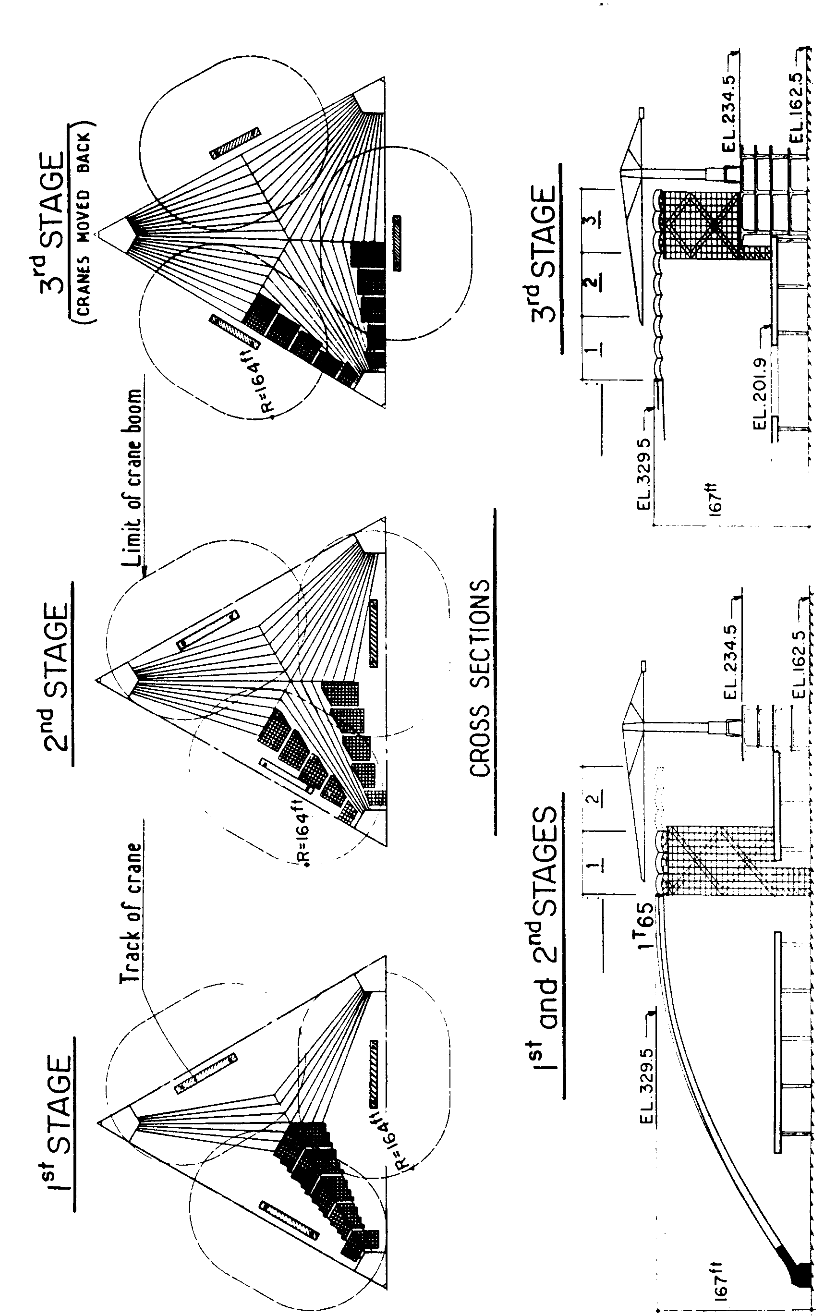

FIG. 11.—ARRANGEMENT OF TUBULAR SCAFFOLDING AND CRANES FOR THREE CONSTRUCTION STAGES.

framed scaffolding with cement mortar joints not subject to settling and having a low deformation factor, in order to minimize the danger of cracking the concrete.

There was an advantage in completing this part of the structure as soon as possible for the entire arc of 60°. This portion sustains the highest stresses, and the longer curing period enabled the concrete to attain maximum strength at the time stresses were applied by the decentering jacks. Concrete was placed progressively from the opening for the jacks toward the crown.

Stages 2, 3 and 4 - Double Units of the Three Radiating Sectors.—The various radiating sectors were concreted on forms supported by tubular scaffolding rolling on the main floor in a direction perpendicular to the facades (Fig. 12). Construction by successive stages resulted in considerable economies in forms and scaffolding. Nevertheless, time requirements limited the operation to three construction stages for the vault instead of the four which were originally planned. Each stage consisted of three radiating sectors, each constituting one-sixth of the roof.

With a careful study of scaffolding needs and the use of new 1-1/2-in. tubes in perfectly straight sections, spacing of as much as 7 ft 4 in. between vertical tubular supports was possible. Despite preliminary construction of the main floor which resulted in a 39-ft reduction in height of scaffolds, and despite construction by stages, more than 185 miles of tubular scaffolding were needed.

At the top, the tubing formed a fork regulated by screws. These forks received the lower chords of the wood trusses, the upper members of which followed the curvature of a corrugation in a plane perpendicular to the facade. This curvature, as already indicated, remains identical for nine successive corrugations. Rafters placed on the trusses supported the plywood forms.

Each time the scaffolding was moved it was necessary to dismantle and adjust these forms, as the angle between the rafters and the trusses varied in plan approaching a right angle. The trusses could slide in their respective grooves (forks) to the desired position. This was possible, however, only by having a strong lower scaffold which could support a concentrated load at any point.

The intrados of each shell was cast in form-sections 29-1/2 ft in length, leaving a space for each web and diaphragm. To keep the vibrated concrete in place on a maximum slope of 35° a very dry mix was used. The quantity of mixing water was reduced to a minimum with the aid of a plasticizer. To prevent crazing, finishing of the upper surface was followed immediately by application of a curing compound.

The 2-3/8-in. thick webs and diaphragms with maximum height of 9 ft 2 in. would have been difficult to cast in place between vertical forms. These thin plates were therefore precast in a sheltered area in the finished basement of the Palace. Transportation was thus reduced to a minimum and cracks due to trucking and repeated handling were eliminated.

The plates were divided into elements of a length such that their weight did not exceed 1.65 tons, so that they could be handled by the large 165-ft boom-cranes. The panels were joined by cast-in-place concrete. Similar construction joints were provided between the panels and the lower shell.

A simple wood shoring was then placed on the lower shell to support the fiberboard panels which served as the bottom form for the upper shell. Reinforcement was then placed and the upper shell was cast from the springing towards the crown, proceeding by sections 59 ft long. The extrados was vibrated, floated and enriched with cement to insure watertightness.

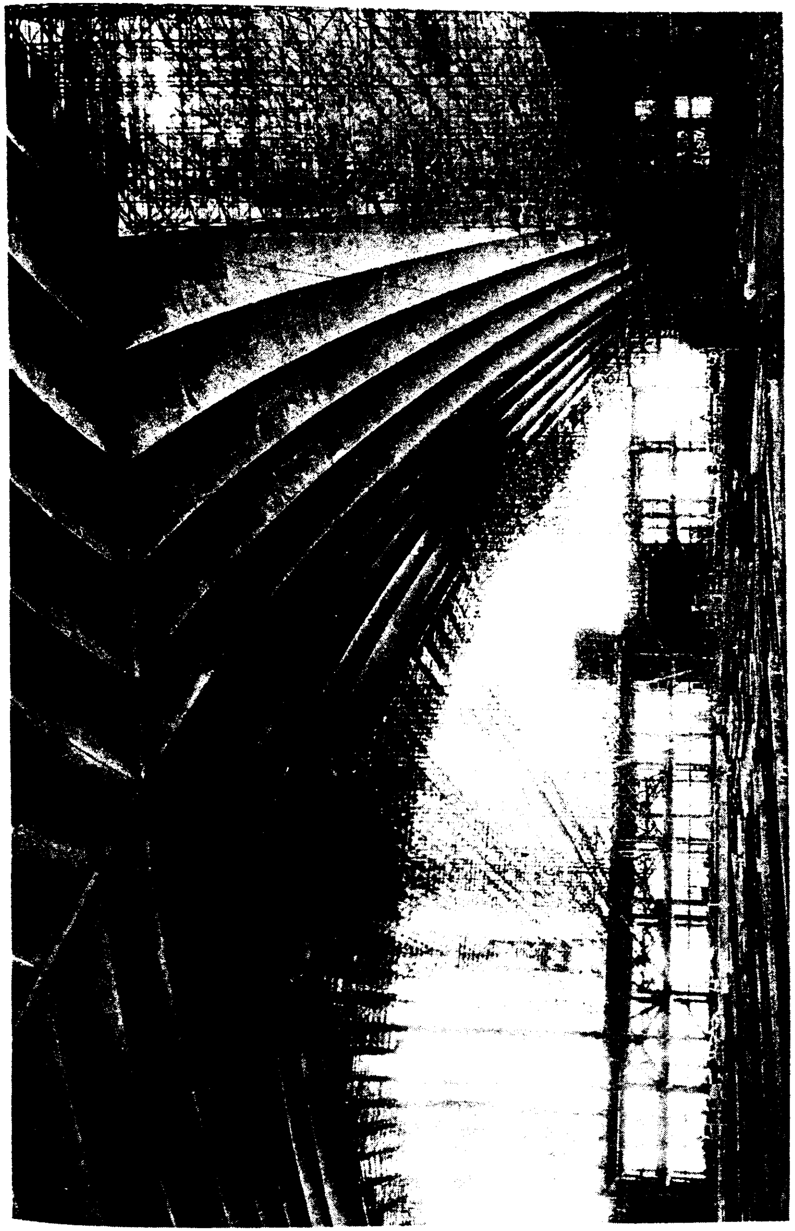

FIG. 12.—FORMS FOR EACH CONSTRUCTION STAGE, CONSTITUTING ONE-SIXTH OF THE ROOF, WERE SUPPORTED ON TUBULAR SCAFFOLDING.

It should be noted that the vertical tubing of the falsework which was supported on the main floor was subject to elastic contractions only. The load due to scaffolding, forms and concrete was the same as the live loads assumed in design of the floor, which was thus tested automatically at no additional cost.

Decentering.—The different stages were constructed with a complete separation between them following the line of the directrix (Figs. 13 and 14). This procedure offered three advantages:

(1) Narrow groups of six or three radiating sectors came close to the basic hypothesis of the theory of strength of materials. The effect of the roof as a whole, after the sections were joined, would come into consideration only for the live loads which would be relatively small in comparison to the dead load.

(2) Flow of forces follows closely the directions assumed in the design calculations.

(3) Complete freedom of deformation of one section with respect to the next during decentering and during creep and shrinkage action in the months to follow.

This independence of sections raised a problem at the crown wall where compression had to be transmitted to the center of the crown in spite of the separation between construction sections. To transmit the compressive force while retaining freedom of vertical displacement between adjacent construction sections, an arrangement of steel plates and rollers was placed in the joint. Behind each plate a vertical prestressing cable prevented failure of the crown wall under the effect of force distribution (Fig. 15).

Although the concrete strength would have permitted decentering as early as 7 days after placing, decentering by jacks took place at 21 days for each construction section so as to reduce creep effects and simultaneously increase safety against buckling.

In order to be sure of the fixity of the abutments during decentering, equilibrium was constantly maintained between the forces induced by the screw jacks and the tension in the ties. The decentering procedure was conducted in successive stages consisting of the following operations:

(1) The jacks in each of the three decentering joints were extended to pick up the vertical load tributary to each abutment in increments such that the difference between the theoretical thrust and the force exerted by the tensioned cables would never exceed 176 kips, which could easily be transmitted through the abutment to the foundation soil.

(2) Four inclined cables, two at each end of the ties in each facade (see Fig. 5), were tensioned sufficiently to resist the thrust due to each incremented load as the jacks were extended.

(3) Two horizontal cables extending between the tie anchorages in each facade were then tensioned to 185 kips as the final operation of each stage.

These decentering operations were repeated until the entire permanent load of each construction stage was transferred from the scaffolding to the abutments.

Due to the successive tensioning of the inclined and horizontal cables, the anchorage blocks were subjected to a force of 370 kips in alternate directions. The corresponding displacement of the anchorage blocks was possible without damage because of the flexibility of the relatively thin prestressed concrete member at the top of the anchorage structures (see Figs. 5 and 6).

In order to transmit the weight of the upper shell to the lower shell without causing abnormal deflections which could damage the lower shell, the shoring

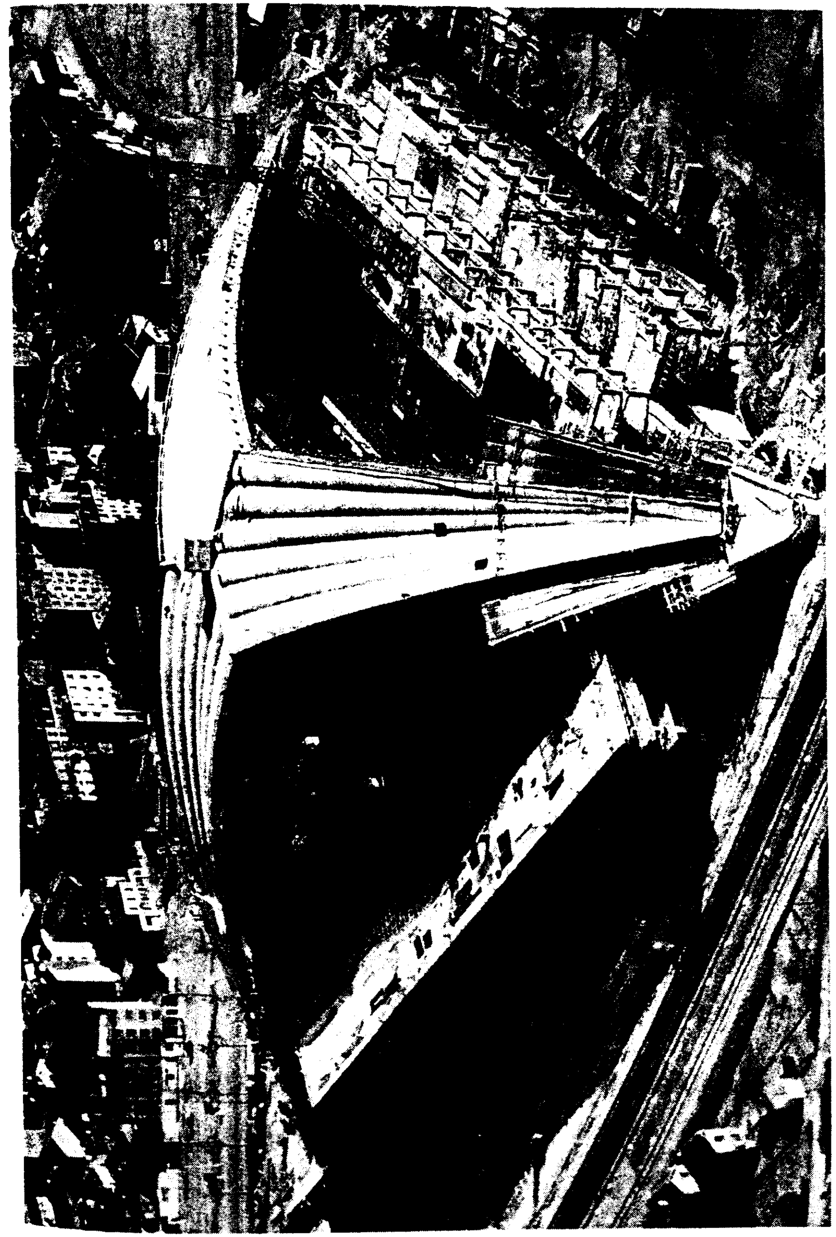

FIG. 13.—THE FIRST CONSTRUCTION SECTION OF THE ROOF WAS COMPLETED IN MARCH 1958.

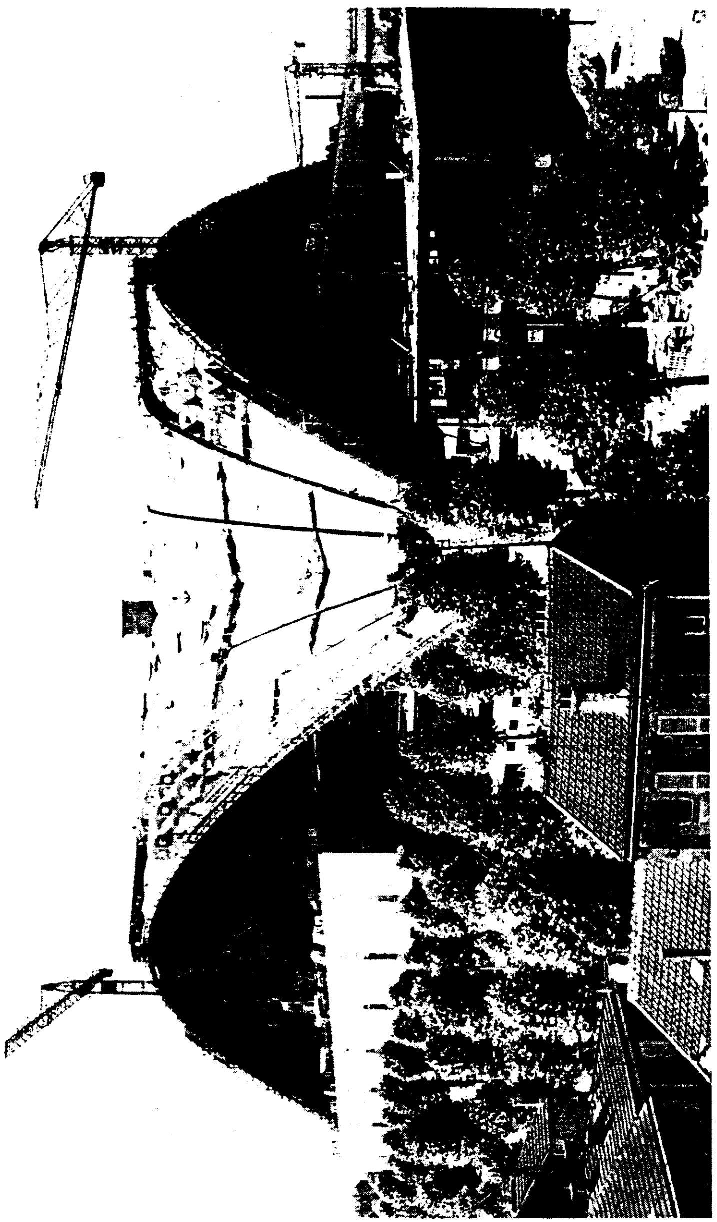

FIG. 14.—THE SPACES BETWEEN CONSTRUCTION SECTIONS WERE FILLED WITH CAST-IN-PLACE CONCRETE SEVERAL MONTHS AFTER DECENTERING TO PERMIT MOST OF THE CREEP AND SHRINKAGE TO TAKE PLACE. FINALLY THE STRUCTURE IS COMPLETELY MONOLITHIC.

between the shells was removed when between one-half and two-thirds of the compression was taken up by the jacks.

A neutral condition, that is, a condition corresponding to the elastic contraction of the concrete under working load, was plainly revealed by the changing course of deformations (Fig. 16), particularly at the crown, as well as by jack pressure which remained substantially constant. Starting from the moment when the neutral condition was established, which marked the beginning of the second decentering stage, that of compensation for shrinkage and creep, the roof was easily freed from the scaffolding without the need of special manipulation of the forks supporting the forms or of the wedges at the foot of the scaffolds. In this respect, decentering of the arch proceeding from the springing by spreading of the decentering joint is much more efficient than by having

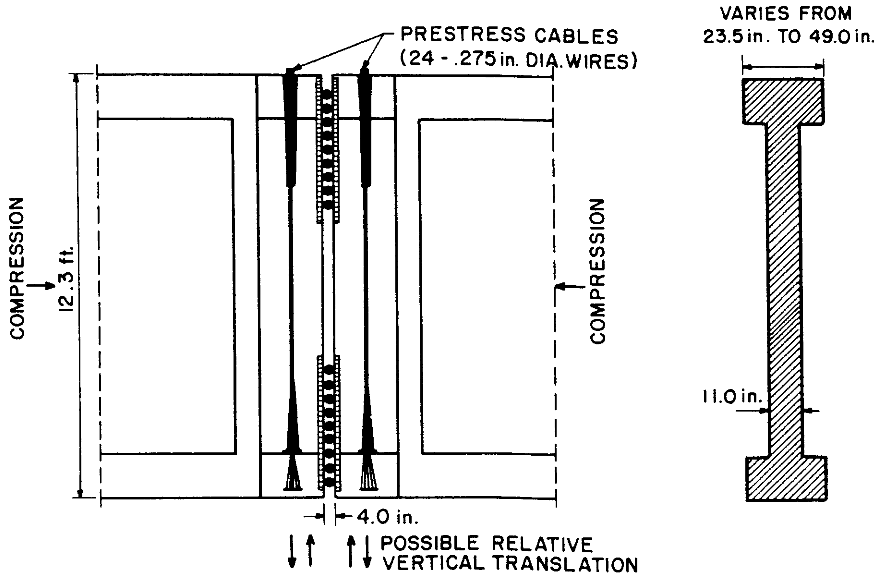

FIG. 15.—ARRANGEMENT OF ROLLERS AT JOINTS BETWEEN SECTIONS OF THE CROWN WALL.

the jacks located in openings at the crown. By the latter procedure the springings of the arch, which are fixed, never become disengaged due to elasticity of the centering.

There remained the removal of vertical scaffolding perpendicular to the facades by using rollers and winches. This was done without lowering the centering after having previously placed the forms to clear the lowest point of the corrugations. These operations were carried out rather rapidly.

Before filling the decentering joints with concrete placed between the jacks, the position of the vault section was carefully checked so that it could be corrected if necessary.

The total pressure exerted by the hydraulic jacks in the three joints at the springings amounted to 19,200 tons for the last two sections.

A period of several months was allowed for the differences in creep deformation of two adjacent construction sections to become sufficiently small to permit placing concrete in longitudinal joints and thereby producing a completely monolithic structure.

The first construction section (Fig. 13) was tested under a non-symmetrical live load by applying 88 tons to one of the legs. The deformations corresponded to those assumed in design. Stress development was followed closely from the start of construction and during all successive stages by means of 85 stable strain indicators cast in the concrete of the abutments, shells and crown wall.

On September 2, 1958, the centering was removed from the third construction section, and the joints between the three sections were cast a few months later, thus completing construction of the vault.

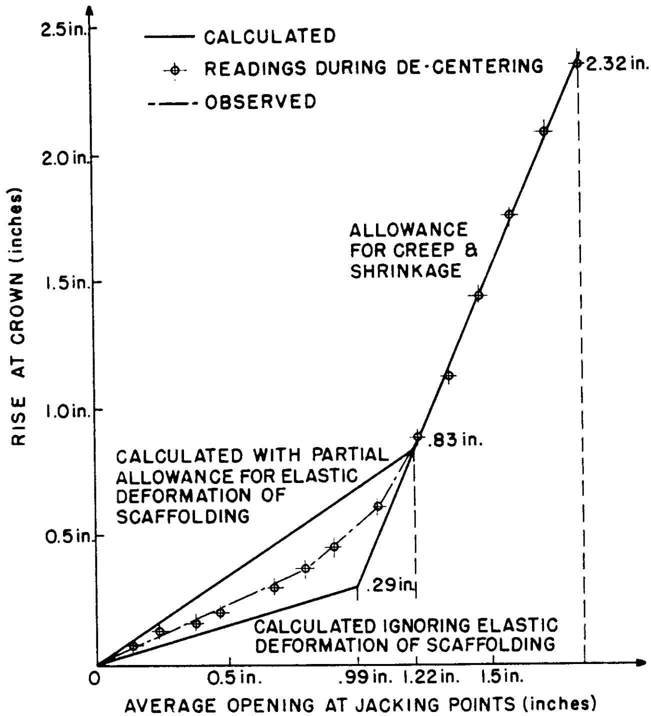

FIG. 16.—DEFORMATIONS DURING REMOVAL OF CENTERING OF THE FIRST CONSTRUCTION STAGE.

This roof structure of exceptional shape and design holds two world records at the time of completion:

- the largest surface supported at a single point—75,350 sq ft
- the longest spans for a thin shell vaulted structure—675 ft at the facades and 781 ft at the groin.