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# Allowable Deflections

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Discusses the factors affecting the deflection of reinforced concrete members and emphasizes the importance of taking them all into consideration for an accurate estimate of deflection. Includes a table with an extensive list of situations requiring deflection limitations. These are based on $L/\Delta$ ratios and absolute values applied to the total or incremental deflections. Discusses the most significant parameters affecting the $L/D$ ratios as an indirect limit on deflections. Presents formulas and graphs for $L/D$ and gives examples for their use.

The report is divided into five chapters: Introduction, Computation of Deflections, Allowable Deflections, Allowable Span/Depth Ratios, and Correlation with Actual Building Structures.

*Keywords:* creep (materials); damping; deflection; modulus of elasticity; reinforced concrete; serviceability; shrinkage; span depth ratio; structural analysis; structural design; temperature; vibration.

## CHAPTER 1—INTRODUCTION

The purpose of this report is to: (1) discuss some practical considerations in the computation of deflections of reinforced concrete building structures, (2) suggest limits on the movements to avoid undesirable deflection or dynamic responses, and (3) suggest limits on the ratio of span to depth which will result in structural members having satisfactory stiffness in most cases.

Most of the factors affecting deflection computations and allowable limits are discussed. No attempt has been made to condense the recommendations into a form suitable for design tables or code limitations.

The committee sent questionnaires to about 300 practicing structural engineers in the United States and received over 40 replies. On the basis of these replies it appears that the design for deflection of reinforced concrete members has not kept pace with design for strength. Many engineers apparently have vague ideas on how to compute deflection and on the significance of the computations. Deflection limitations as commonly specified are rarely more sophisticated than 180 to 480 or more for span/deflection ratios and 10 to 40 for span/depth ratios. Many engineers apparently make no distinction between simple and continuous spans in setting span/depth limits.

However, there seems to be a growing awareness of the problems created by excessive deflections. The majority of the respondents indicated first hand knowledge of such problems.

It is hoped that this report will assist structural engineers to make more intelligent estimates of

**ACI Committee 435,** Deflection of Concrete Building Structures, was organized in 1957 to study research on deflection of concrete flexural members in building structures under rapid and long-term loads. Dan E. Branson was chairman from 1962 to 1968 during which time this report was prepared. This is the committee's third published report. Subcommittee 1, Allowable Deflections, consists of seven members with Russell S. Fling as chairman.

the actual deflections and to place more realistic limits on the magnitude of these deflections.

## CHAPTER 2—COMPUTATION OF DEFLECTIONS

Strength computations are usually based on the assumption that the member will never (or on a probability basis, almost never) have less strength than demanded by the superimposed loads. If the same philosophy is used in stiffness computations, structural members will result that are unduly conservative and sometimes unsatisfactory.

The engineer's natural tendency to overestimate loads and use conservative design assumptions should be resisted since an overestimate of deflections could be as troublesome as an underestimate, especially if the members are to be cambered. Stiffness computations should predict the true deflections as accurately as possible with a unit "factor of safety." The maximum expected error due to fallacious assumptions or inaccurate data on material properties or loads can be estimated and a range of deflection values established. The probable effect of deflection at the high or low end of this range can then be assessed.

Although methods for computing deflections of reinforced concrete members have been available for many years,[1,2] they have not yielded uniformly consistent results. The principal reason for this is that many factors affecting the true magnitude of the deflection have been inadvertently ignored. This has led to an unjustified lack of confidence in the available computational methods. Some of these factors are discussed in the following list.

1. For strength computations, the concrete is assumed to carry no tension; however, this assumption is unduly conservative for deflection computations. In many cases (particularly solid slabs), the concrete remains essentially uncracked and behaves as a gross transformed concrete section. Even a shallow highly stressed beam may remain uncracked over a substantial portion of its depth and length. Consequently it is somewhat stiffer than a fully cracked beam. In the fully cracked zone, the tensile strength of the concrete between cracks and near the neutral axis increases the effective stiffness.

2. Permanent and transient loads that are actually likely to be applied to the structure should be used instead of those assumed in strength computations, especially for long-term deflections. The actual live load is often substantially less than the maximum specified in building codes. Similarly, the assumed dead loads are frequently on the heavy side to be "safe" in strength computations.

3. True moments should be used rather than those computed in the design for strength, especially when empirical moment factors have been used. For example, moments based on factors given in Section 904 of ACI 318-63[3] will generally be too high (the sum of the positive and negative moments equal up to 123 percent of the static moment), while those derived from Section 2104 of ACI 318-63[3] will generally be too low (the sum of the positive and negative moments equal as little as 72 percent of the statical moment). Another source of error is the redistribution of moment due to creep. This redistribution usually decreases the moment causing high stresses and increases the moment causing low stresses. At the present state of the art, computational methods for estimating this error are not available.

4. Almost all flexural members have some end restraint even though many are designed for strength as simple spans. A small end moment will materially reduce the deflection, therefore full credit should be given the restraint afforded by masonry bearing walls, columns, and minor structural elements. When satisfactory analytical techniques are not available, logical estimates of this restraint should be made.

5. Where possible, the modulus of elasticity and modulus of rupture should be based on measured values from concrete similar to that used in the structure. However, tests to determine the modulus of elasticity must be conducted in such a way as to obtain values which are consistent with the theoretical assumptions on which the deflection computations are based. When they are computed from the cylinder compressive strength using empirical formulas, the average cylinder strength at the time load is applied should be used rather than the strength assumed in the design. The average modulus of rupture of a series of identical test specimens should be used rather than the lowest value as this will more nearly represent the true line of demarcation between a cracked and an uncracked section.

6. The conditions of loading during construction (for example, the reshoring sequence) can drastically alter the subsequent deflection. Concrete loaded at an early age will have a greater initial deflection than concrete loaded at a later age. In addition, the modulus of rupture will also be lower. Consequently, cracking is more likely with a resultant increase in deflection. Conversely, concrete that is properly cured and is protected from early stresses until high modulii of elasticity

and rupture have been gained, will ultimately prove to be much stiffer.

7. The ambient weather conditions during and after construction will affect the drying shrinkage and hence the magnitude of shrinkage warping. Concrete members cast and maintained in a damp, cool atmosphere can be expected to have less deflection than those in a hot, arid atmosphere.

8. Since the amount of drying shrinkage and creep is affected by the volume/surface ratio, large concrete beams will have a lower ultimate deflection than thin slabs. The difference can be as high as 40 or 50 percent.[4]

9. Nonstructural elements of the structure frequently reduce calculated deflections quite substantially. For example, partitions may span from end to end and carry almost their entire weight when the "supporting" member deflects. Deflecting members may come to rest on walls or window mullions below. For many years it has been known that high rise buildings may not deflect laterally as much as the theoretical frame computations indicate because of the stiffness contributed by walls and partitions.

10. Full allowance should be made for the flanges in tee beams even when they are on the tension side of the beam. Flanges may increase the stiffness of an uncracked section to as much as 234 percent and the stiffness of a cracked section to as much as 185 percent* of that of rectangular sections with the same over-all depths and with widths equal to the stem widths of the flanged sections. Consideration should be given to using a greater flange width than permitted by ACI 318-63[3] for strength computations. For example, the effective stiffness of a beam with pan slab joists parallel to it might include the stiffness of several adjacent joists.

11. The transformed area of reinforcing steel in uncracked sections should not always be ignored as it may increase the moment of inertia of the plain concrete as much as 30 percent.* Similarly, compression steel in cracked sections may increase the moment of inertia as much as 15 percent.

12. Concrete that is subjected to high temperatures in service [over about 100 F (37 C)] may have deflections increased significantly.[5]

13. Compression reinforcing steel will substantially reduce the long term creep deflection. In some circumstances, the long-term increase in deflection may be reduced to 20 or 30 percent of the short-term deflection.[6]

14. In continuous beams, where the flexural rigidity varies from negative moment regions to positive moment regions, the deflection must be computed by a method which takes into account the contribution of each section of the beam to the total beam deflection. For example, if the relative flexural rigidity in the positive moment region is equal to 1.0 and the relative *flexural* rigidity in the negative moment region of the same beam is equal to 2.0, the average flexural rigidity would be 1.5. However, the deflection of a beam with small or no end moments would be proportional to a flexural rigidity close to 1.0.

## CHAPTER 3—ALLOWABLE DEFLECTIONS

Before World War II, allowable bending stresses were lower than those permitted at the present time. As a result, reinforced concrete structures were often uncracked in service. The lack of cracks as well as the lower stresses resulted in small deflections. In addition, many construction materials were softer and more pliable than those used today. For example, soft brick and lime mortar versus hard brick and portland cement mortar, wood stud versus exposed concrete block partitions, wood versus steel window sash, wood versus steel doors and frames, and wood plank versus plywood paneling. Walls and partitions were more massive and helped carry structural loads, even though they were not designed to do so. Modern construction uses harder, more brittle materials, and more slender structural members. Thus, more sophisticated limits on deflection are required now instead of formerly used rules of thumb.

Excessive deflection and tendency to vibrate in structural members can be undesirable for a variety of reasons. These can be classified in four broad categories: (1) sensory acceptability, (2) serviceability of the structure, (3) effect on nonstructural elements, and (4) effect on structural elements.

Sensory acceptability tends to be a matter of personal opinion. Therefore, limits will vary considerably with the social culture and use of the structure. Deflection limits for serviceability of the structure are more easily defined since they can be related to the intended use of the structure. Movements affecting the nonstructural elements of construction, such as walls and ceilings, must be limited to prevent cracking or other damage. In some cases excessive deflection can prevent doors, folding partitions, and other movable elements from operating properly. Similarly, movements may affect the structural element itself. In this case, the movement must be limited to prevent structural behavior from being different than that assumed in the design. If this is not possible then the deflection must be considered as part of the design for strength.

Structural deflections are usually thought of as linear movements or a vertical translation of the member at midspan or in cantilever beams at the

*See Appendix.

## TABLE 3.1—DEFLECTION LIMITATIONS FOR VARIOUS EXAMPLES

| Reasons for limiting deflections | Examples | Deflection limitation[a] | Portion of total deflection on which the deflection limitation is based |
|---|---|---|---|
| 1. Sensory acceptability | | | |
| 1.1 Visual | Droopy cantilevers and sag in long span beams | By personal preference[b] | Total deflection[c] |
| 1.2 Tactile | Vibrations of floors that can be felt | $L/360$[d] | Full live load |
| | Lateral building vibrations | No recommendation[e] | Gust portion of wind |
| 1.3 Auditory | Vibrations producing audible noise | Not permitted | |
| 2. Serviceability of structure | | | |
| 2.1 Surfaces which should drain water | Roofs, outdoor decks | $L/240$[f] | Total deflection |
| 2.2 Floors which should remain plane | Gymnasia and bowling alleys | $L/360$ + camber[g,h] or<br>$L/600$[h] | Total deflection<br>Incremental deflections after floor is installed[j] |
| 2.3 Members Supporting sensitive equipment | Printing presses and certain building mechanical equipment | Manufacturer's recommendations | Incremental deflections after equipment is leveled[j] |
| 3. Effect on nonstructural elements | | | |
| 3.1 Walls | 3.1.1 masonry and plaster | $L/600$ or 0.30 in. (7.6 mm) max[k,h] or $\phi = 0.00167$ rad.[l] See Reference 7 | Incremental deflections after walls are constructed[j] |
| | 3.1.2 metal movable partitions and other temporary partitions | $L/240$ or 1 in. (25.4 mm) max[h] | Incremental deflections after walls are constructed[j] |
| | 3.1.3 lateral building movement | 0.15 in. (3.8 mm) offset[m] per story $0.002 \times$ (height) See Fig. 3.1 | Five min sustained wind load |
| | 3.1.4 vertical thermal movements | $L/300$ or 0.60 (15.2 mm) max.[n] See Reference 8 and Fig. 3.2 | Full temperature differential movement |
| 3.2 Ceilings | 3.2.1 plaster<br>3.2.2 unit ceilings such as acoustic tile | $L/360$[h,p]<br>$L/180$[h,p] | Incremental deflections after ceiling is built[j] |
| 3.3 Adjacent building elements supported by other members | Windows, walls and folding partitions on unyielding supports below the deflecting member | Absolute deflection limited by tolerances built into the element in question | Incremental deflection after building element in question is constructed[j] |

TABLE 3.1 (Cont.)—DEFLECTION LIMITATIONS FOR VARIOUS EXAMPLES

| Reasons for limiting deflections | Examples | Deflection limitation[a] | Portion of total deflection on which the deflection limitation is based |
|---|---|---|---|
| 4. Effect on structural elements | | | |
| 4.1 Deflections causing instability of primary structure | Arches and shells Long columns | Effect of deflections on the stresses and stability of the structure should be taken into account in the structural design of the element | |
| 4.2 Deflections causing different force system or change in stresses in some other element | Beam bearing rotation on masonry wall | Effect of deflections on the stresses and stability of the structure should be taken into account in the structural design of the element | |
| 4.3 Deflections causing dynamic effects | Resonant vibrations which increase static deflections and stresses such as those produced by wind, dancing, moving loads and machinery | Dynamic deflections should be added to static deflections and the total should be less than the limitations imposed for other reasons | |

[a] Deflection limitations are given for members supported at both ends and for cantilevers, except as noted. It is assumed that the supports do not move.

[b] For excessive deflections, correct total deflection by camber. Overcorrection by camber is desirable.

[c] Total deflection is the sum of all individual computed deflections due to all loadings plus those due to time dependent effects.

[d] Noticeable floor vibrations depend upon the frequency as well as the amplitude. In addition, the damping efficiency is importance since vibrations which die out within a few cycles will be felt only as a single movement, if at all.[9]

[e] Limitations may be based on the lateral acceleration (or deceleration) of the building. Secondary vibrations caused by the wind sway might also be a problem and require limitation in some manner.[10] More research in this area is necessary.

[f] Surfaces should be sloped or total deflections corrected by camber to prevent ponding of water.

[g] The deflection of floors which should remain plane may be partially compensated by camber so that any possible incremental deflection as well as the camber itself does not produce a floor which deviates from plane by more than $L/360$ either upward or downward.

[h] The span $L$ should be considered either parallel to or perpendicular to the direction of stress, whichever is shorter. (see Fig. 3.3).

[j] Incremental deflection is the change in elastic deflection over a period of time produced by the addition of new loads or subtraction of existing loads during that time, plus the change in deflection produced by creep and shrinkage during that time.

[k] This limitation assumes that the deflection computation does not give credit to composite action between the member and the wall, or what is commonly called "arching action." If composite action is computed,[11] the deflection limitation should be considerably less than that tabulated. The tabulated limits might still allow some visible damage to weak fragile walls and walls which cannot act compositely with the structural member (e.g., walls with large openings in the span.)

[l] Rotation of any member supporting a wall, at the point of support of the wall. This is equivalent to a limitation of 1/600th of the height of the wall.

[m] This limitation applies to the lateral deflection between adjacent floors caused by the story wind shear but does not include the deflection caused by axial lengthening and shortening of the columns. It also applies to the vertical offset deflection between two shear walls in the same plane (see Fig. 3.1).

[n] $L$ is the distance between the exterior column and the first interior column (see Fig. 3.2 and Reference 12).

[p] Deflection of cantilevers may be twice as high as the tabulated ratios.

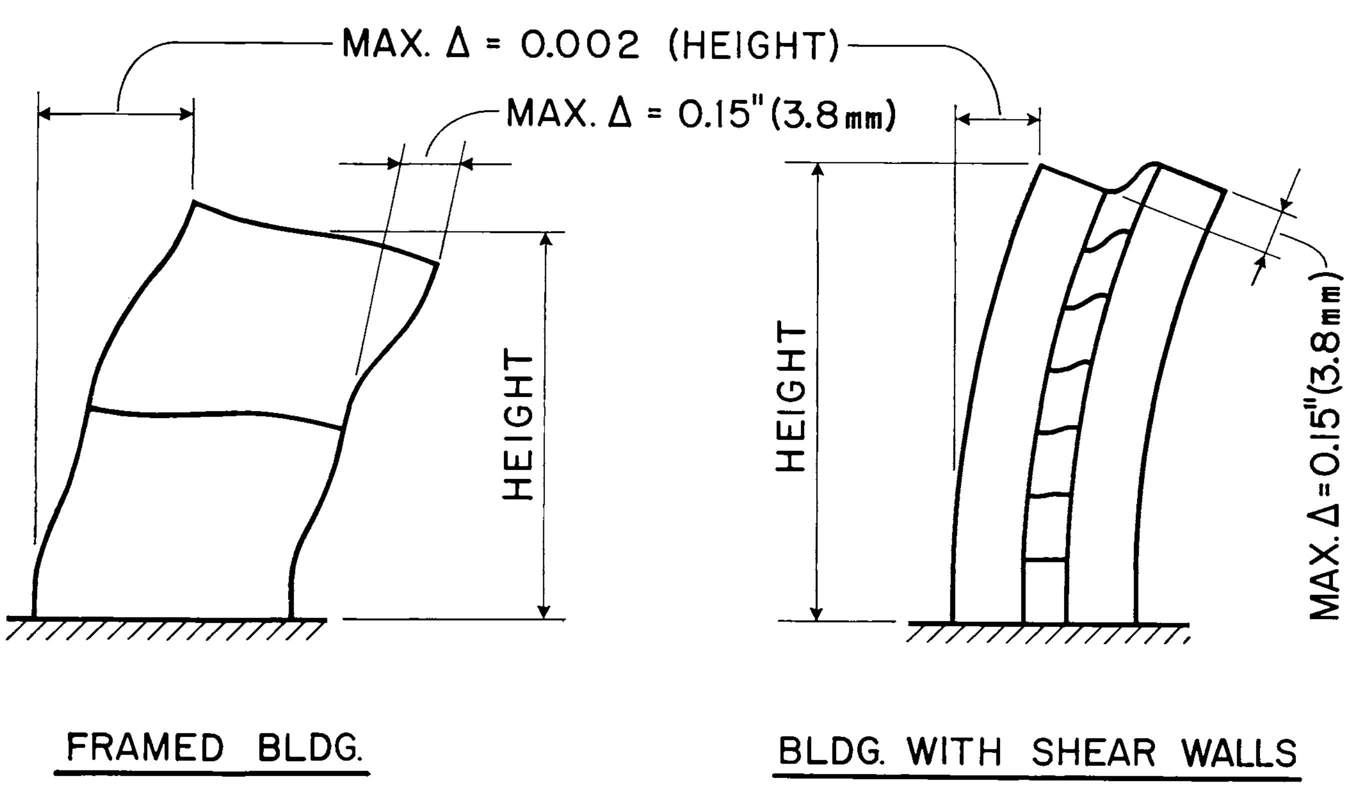


Fig. 3.1—Lateral deflection limitations for tall buildings (see Table 3.1, Section 3.1.3)

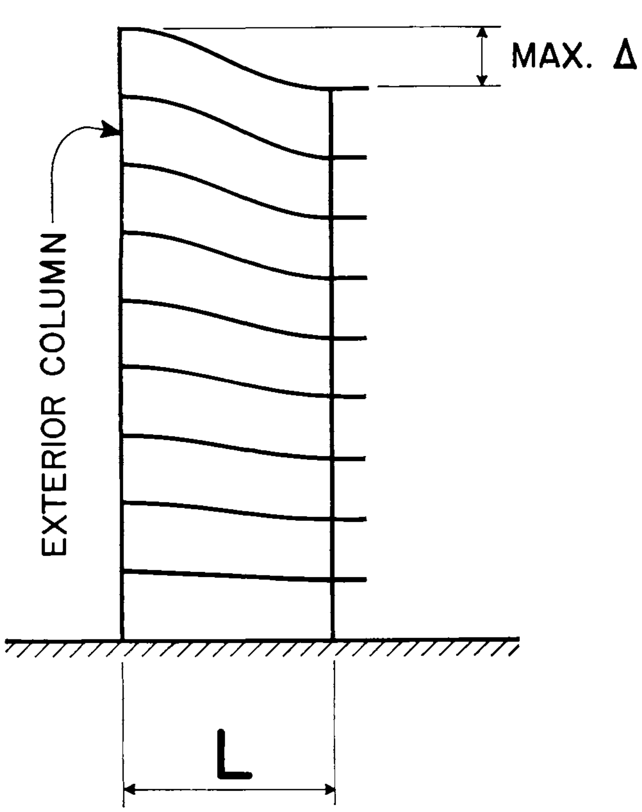


Fig. 3.2—Vertical thermal movement limitations for tall buildings with exposed columns (see Table 3.1, Section 3.1.4)

unsupported end. Generally, these movements do cause the most serious problems. However, angular distortion or rotations without linear movements can also cause similar problems. An example of such a rotational problem is a wall crossing the end of a beam subject to large end rotations, where wall and beam are at right angles. In this case, rotation could crack the wall. In some cases, such as plaster ceilings, the linear movement, per se, does not cause damage. Here beam curvature is the proper measure of potential damage. Linear movement, rather than beam curvature, is usually the proper index of damage to walls and partitions because they usually cannot follow the beam curvature due to their excessive rigidity.

Linear movements may be limited to an absolute value or to a fraction of the span depending upon the reasons for the limitation. In other cases, the permissible movement may be a function of the frequency of vibration or to the rate of damping.

In Table 3.1 numerical limits on deflection are suggested for a variety of situations in accordance with the preceding discussion. In some cases, specific limits cannot be given. Such instances are indicated.

The deflection limitations in Table 3.1 do not apply to movements caused by earthquakes since most building codes permit some damage and unpleasantness when severe earthquakes occur.

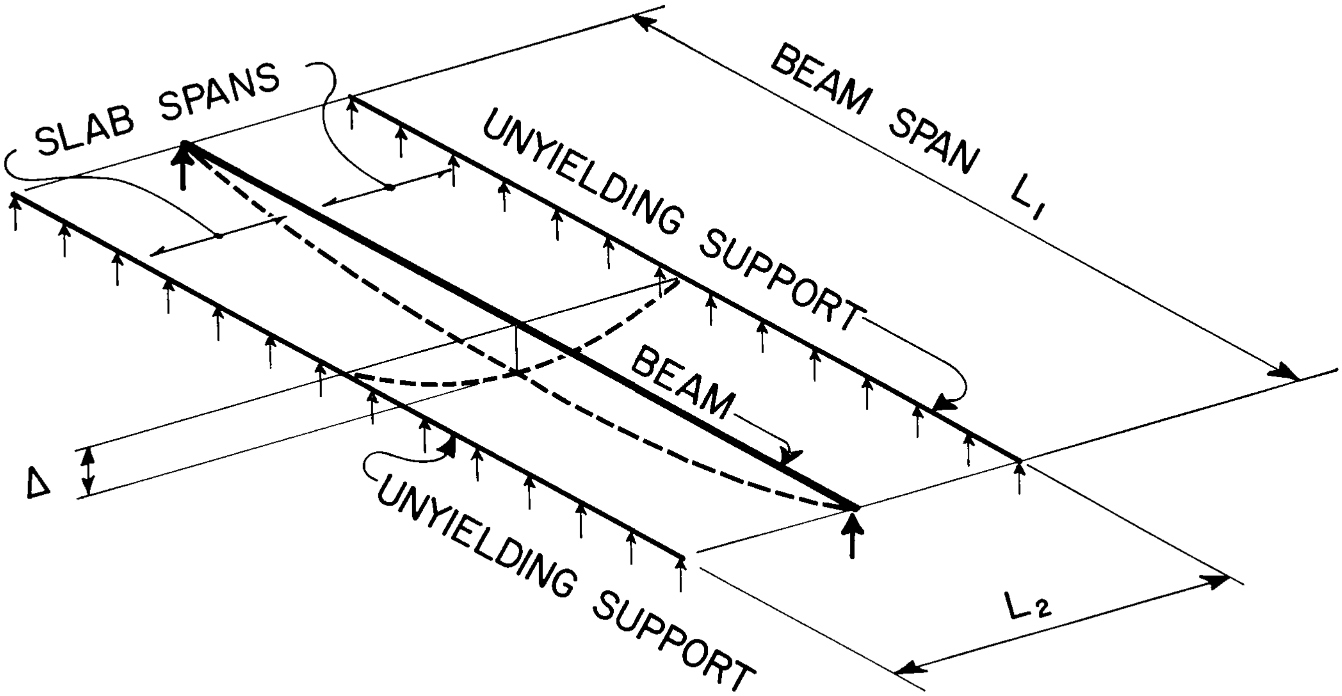


Fig. 3.3—Beam deflection limitation. The deflection/span limitation for the beam is based on dimension $L_2$ when the bowling alley, wall, ceiling or other nonstructural element is oriented in the direction of the slab span. (see Table 3.1, Sections 2.2, 3.1.1, 3.1.2, 3.2, and Footnote h.)

## CHAPTER 4—ALLOWABLE SPAN/DEPTH RATIOS

Placing limits on the span/depth ratio of flexural members is an indirect method of limiting their deflection. Realistic $L/D$ limits for reinforced concrete members are far more complicated than for simple span, prismatic beams of homogeneous, isotropic materials. This chapter discusses the determination of span/depth limits which will result in satisfactory stiffness in most cases.

The allowable span/depth ratio for a concrete member is determined primarily by its flexural rigidity $EI$, the permissible deflection, and the conditions of end restraint. However, this statement oversimplifies the problem as there are many ramifications to these three factors.

The general formula* for the span/depth ratio is:

$$\frac{L}{D} = \frac{C_m}{C_\Delta} \cdot \frac{\Delta}{L} \cdot \frac{E_c}{F} \cdot K_1 \tag{1}$$

For uncracked sections:

$$K_1 = \frac{C_y}{f} \tag{2}$$

For fully cracked sections (closely spaced cracks extend to the neutral axis for full length of the beam):

$$K_1 = \frac{n\,(1-k)}{f_s} \cdot \frac{d}{D} \tag{3}$$

in which

$C_m$ = moment coefficient
$C_\Delta$ = deflection coefficient
$F$ = ratio between long-time deflection and short-time deflection
$C_y$ = $y_t/D$
$f$ = bending tensile stress in an uncracked concrete flexural member

The other symbols have their usual meaning.

To interpolate between fully cracked and uncracked sections:

$$\frac{L}{D} = \left(\frac{L}{D}\right)_{cr} + \left(\frac{M_{cr}}{M_{max}}\right)^3\left[\left(\frac{L}{D}\right)_{g} - \left(\frac{L}{D}\right)_{cr}\right] \tag{4}$$

Eq. (4) is an adaptation of Eq. (26) from the report on Deflection of Reinforced Concrete Flexural Members.[1]

Fig. 4.1 is based on Eq. (1) using typical values for $E_c$, $F$, and $d/D$. For other values of $F$ and $d/D$, the span/depth ratio can be approximated by simple ratio. However, a new graph must be prepared for each value of $E_c$ because the flexural rigidity is a varying function of the modulus of elasticity.

The following is a discussion of the more significant parameters affecting the span/depth ratio:

Concrete flexural stresses under working loads have a profound influence on the allowable span/depth ratio since they are inversely proportional to the ratio and because they establish the effec-

*See Appendix for derivations.

tive flexural rigidity. Fig. 4.1 illustrates that $L/D$ may vary by a factor of 10 depending on the magnitude of the concrete flexural stresses as measured by $M/bd^2$. At low stresses the member remains relatively uncracked with a high flexural rigidity. At high stresses, the member becomes more nearly fully cracked with a consequent reduction in flexural rigidity. The ratio of uncracked to fully cracked flexural rigidities varies from about 1.0 to 4.5 for most slabs and rectangular beams, from about 1.0 to 2.5 for the positive moment sections of T-beams and from about 3 to 10 for the negative moment sections of T-beams.

Due to their light loading and their configuration, solid slabs are most likely to remain uncracked and hence are usually the stiffest structural members for a given depth. On the other hand, T-beams designed to use the concrete at maximum stress by ultimate strength design will be almost fully cracked hence have a relatively low flexural rigidity.

For equivalent sizes, prestressed concrete members tend to be stiffer than reinforced concrete members because the prestressing usually prevents or reduces cracking. In addition, prestressing reduces the creep deflection because moments produced by sustained external loading are counteracted by the prestressing moment.

The allowable span/depth ratio is inversely proportional to the permissible span/deflection ratio. For a more complete discussion of deflection limitations, see Chapter 3.

The support conditions and end movements have an important influence on the allowable slenderness ratio although not as large an influence as might be assumed from a cursory examination of coefficients in deflection formulae. This is because continuous beams are usually more heavily loaded than simple beams of equivalent size to take advantage of the greater moment capacity. Also, continuous beams may have spans unequally or alternately loaded resulting in deflections which are larger than deflections of similar beams subjected to continuous uniform load.

The length of span also affects the allowable span/depth ratio because the longer spans require higher flexural stresses which tend to reduce the flexural rigidity due to increased cracking. Also, in many cases, the limiting deflection is an absolute value rather than a fraction of the span. This makes the span/depth ratio inversely proportional to the span [see Eq. (1)] rather than independent of the span.

The allowable span/depth ratio is inversely proportional to long-term creep and shrinkage deflection as measured by the parameter $F$. Therefore, anything that tends to reduce the incremental long-term deflection tends to increase the $L/D$ ratio, as for example, the addition of compressive reinforcement or a delay in loading until a portion of the creep and shrinkage deflection has taken place.

The span/depth ratio is directly proportional to the modulus of elasticity only for uncracked sections, whereas the flexural rigidity of cracked sections is only slightly affected by variations in $E_c$. Since many shallow beams are almost fully cracked, variations in $E_c$ are usually not significant for them. However, many slabs remain relatively uncracked, hence a lower $E_c$ would require a proportionately lower $L/D$ ratio.

For everyday design office use, the general formula for the span/depth ratio may be arranged as follows and graphs may be prepared using the product of $C_w$ and $L$ as a parameter.

$$\frac{L}{D} = \sqrt{\frac{\Delta}{L} \cdot \frac{E_c}{F} \cdot \frac{12\,K_2}{C_\Delta W_c} \cdot \frac{1}{C_w L}} \qquad (5)^*$$

For uncracked sections:

$$K_2 = C_1 \qquad (6)$$

*See Appendix for derivation.

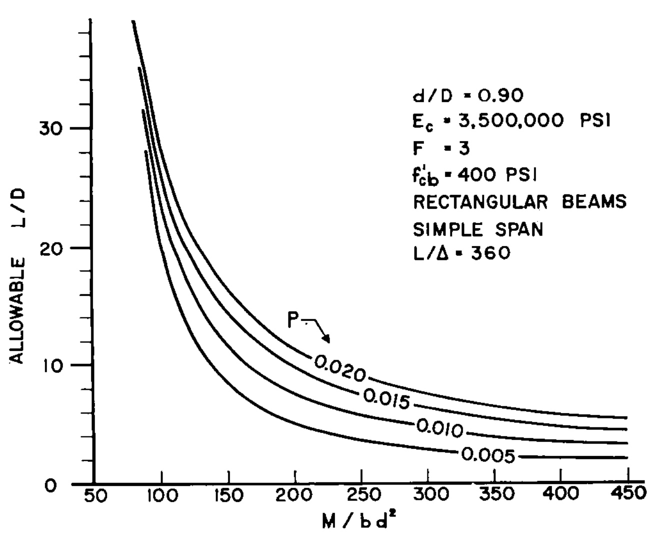


Fig. 4.1—Allowable span-depth curves

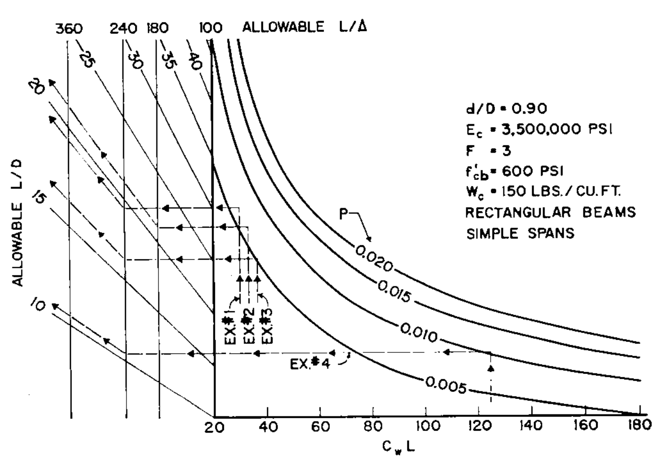


Fig. 4.2—Allowable span-depth curves

For cracked sections:

$$K_2 = 12\, n\, (1 - k)\, pj \left(\frac{d}{D}\right)^3 \qquad (7)$$

in which

$W_c$ = weight of concrete, lb per cu ft
$C_w$ = ratio of the total load to weight of concrete beam (stem) alone
$C_I$ = ratio of $I$ for T-beam to $I$ for stem alone

The other symbols have their usual meaning.

Although Eq. (5) appears to place different emphasis on the parameters affecting the span/depth ratio than does Eq. (1), it should be noted that the number of parameters in Eq. (5) is greater. The additional parameters are interrelated in such a way as to make it possible to reduce Eq. (5) to Eq. (1). For example, $\Delta$ is proportional to $C_w$; hence, these two parameters tend to cancel each other.

Fig. 4.2 is an example of a graph prepared from Eq. (5). The following are some examples for its use.

### EXAMPLE 1

On a 15 ft (4.57 m) simple span, what thickness slab is required to support a superimposed load of 125 psf (610 kg/m$^2$) with a maximum deflection = $L/240$?

Try 8 in. (20.3 cm) slab, $p$ = 0.75 percent, dead load = 100 psf (488 kg/m$^2$).

$$C_w = \frac{100 + 125}{100} = 2.25, \quad C_wL = 33.75$$

From Fig. 4.2, the required $L/D$ = 18.5.

Then the required $D \approx 15 \times 12/18.5 = 9.75$ in. (28.4 cm).

Since this is more than the assumed depth of slab try a 9½ in (24.2 cm) thick slab.

$$C_w = \frac{119 + 125}{119} = 2.05, \quad C_wL = 30.5$$

From Fig. 4.2, the required $L/D$ = 19.5. Then the required $D \approx 15 \times 12/19.5 = 9.25$ in. (23.5 cm)

### EXAMPLE 2

Same conditions as Example 1, except that the allowable deflection = $L/180$. Try $C_wL$ = 33.75. From Fig. 4.2, $L/D$ = 21. Then the required $D \approx 15 \times 12/21$ = 8.5 in. (21.7 cm).

### EXAMPLE 3

Same conditions as Example 1, except that the slab is continuous on both ends. Assume $p \approx 0.75 \times 2/3$ = 0.5 percent. Try 8 in. (20.3 cm).

$$C_w = 2.25, \quad C_wL = 33.75$$

From Fig. 4.2, the required $L/D$ = 17.25 for a simple span. Referring to Eq. (5), the required:

$$L/D \approx 17.25 \sqrt{\frac{5}{384} \times \frac{384}{1.5}} = 31.5$$

for a continuous span.

Then the required $D \approx 15 \times 12/31.5 = 5.7$ in. (14.5 cm). Since this is less than the assumed depth of slab, try a 6.5 in. (16.5 cm) slab.

$$p \approx 0.60 \text{ percent}, \quad C_w = 2.54, \quad C_wL = 38$$

From Fig. 4.2, $L/D$ = 16.25 for a simple span and $L/D = 16.25\sqrt{5/1.5} = 29.6$ for a continuous span. The required $D \approx 15 \times 12/29.6 = 6.1$ in. (15.5 cm)

### EXAMPLE 4

On a 25 ft (7.61 m) span with one end continuous, what depth of isolated rectangular beam is required to carry a wall weighing 1000 lb per ft (14.9 kg/cm) with a maximum deflection = $L/240$?

Try a 12 x 20 in. (30.5 x 50.8 cm) beam, $p$ = 1.2 percent, weight = 250 lb per ft (3.73 kg/cm)

$$C_w = \frac{1250}{250} = 5.0, \quad C_wL = 125.0$$

From Fig. 4.2, the required $L/D \approx 10.5$ for a simple span. Referring to Eq. (5), the required:

$$L/D \approx 10.5 \sqrt{\frac{5}{384} \times \frac{384}{2.1}} = 16.0$$

for a continuous span.

Then the required $D \approx 25 \times 12/16 = 18.75$ (47.7 cm)

Alternately, if the distance between inflection points = $0.75L$, $C_w(0.75L)$ = 94. From Fig. 4.2, the required $L/D$ = 12.

Then, the required $D \approx 25 \times 12 \times 0.75/12 = 18.75$ in. (47.7 cm).

## CHAPTER 5—CORRELATION WITH ACTUAL BUILDING STRUCTURES

Almost no reports on measurements of long-term deflection of actual building structures in North America have come to the attention of the committee. Apparently, some long-term deflection measurement and analysis have been performed in Germany and reported in Reference 13. However, a reasonable amount of peripheral information is available such as short-term deflection measurements of test loaded structures, observations of structures with excessive deflections, and partial information on attempts at long-term deflection measurements.

Preparation of this report, especially Chapter 4, depended heavily on such observations. The following points should be emphasized.

1. Loading the structure at too early an age may cause deflections as high as two or three times as great as might otherwise be the case. This is partly because the concrete has a lower modulus of elasticity and a higher rate of creep at an early age. More importantly, however, the structure may become fully cracked due to a low modulus of rupture where otherwise it might never become more than partially cracked. The flexural rigidity of a fully cracked section is usually 30 to 50 percent of flexural rigidity of an uncracked section.

2. Many concrete sections, especially slabs and massive concrete beams, hardly ever reach a fully

cracked condition. The influence of this lack of cracking on the ultimate deflection of the member can hardly be overemphasized.

3. To obtain consistently reliable estimates of the probable ultimate deflection of concrete members it is necessary to include the effect of all factors even though some of them may influence the final deflection to a lesser degree than the final margin of error.

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# APPENDIX

## NOTATION

$A_s$ = area of tensile steel
$A_s'$ = area of compressive steel
$b$ = width of flange in T-sections and width of rectangular sections
$b'$ = width of web in T-sections
$C_I$ = ratio of $I$ for T-beam to $I$ for stem alone
$C_m$ = moment coefficient
$cr$ = subscript denoting a cracked section
$C_w$ = ratio of the total load to weight of concrete beam (stem) alone
$C_y$ = $y_t/D$
$C_\Delta$ = deflection coefficient
$D$ = total depth of beam
$d$ = effective depth of reinforced concrete beam
$d''$ = distance from the tension face of the beam to the centroid of the tension reinforcing steel
$E_c$ = modulus of elasticity of concrete, short duration of loading
$E_s$ = modulus of elasticity of steel
$F$ = ratio between long-time deflection and short-time deflection
$f$ = bending tensile stress in an uncracked concrete flexural member
$f'_{cb}$ = modulus of rupture of concrete
$f_s$ = stress in steel reinforcement
$g$ = subscript denoting an uncracked section
$I$ = moment of inertia = $bD^3/12$ for rectangular sections and $b'D^3/12$ for T-sections
$I_c$ = moment of inertia of gross plain concrete sections
$I_{cs}$ = moment of inertia of gross transformed reinforced concrete section
$jd$ = internal couple lever arm in elastic theory
$K$ = $M/bd^2$ or $M/b'd^2$
$k$ = height of compressive stress block in elastic theory
$L$ = span length
$M$ = bending moment
$M_{cr}$ = bending moment at nominal first crack
$n$ = modular ratio defined as $E_s/E_c$
$p$ = $A_s/b'd$ for T-sections or $A_s/bd$ for rectangular sections
$w$ = uniformly distributed load
$W_c$ = weight of concrete, lb per cu ft
$y$ = distance from extreme fiber to neutral axis
$y_t$ = distance from neutral axis to the extreme fiber in tension
$\Delta$ = maximum deflection

## CONTRIBUTION OF TRANSFORMED AREA OF REINFORCING STEEL TO STIFFNESS OF REINFORCED CONCRETE BEAM (see Fig. A1)

Assume $A_s = A_s'$. $0 < d''/D < 0.5$
For plain concrete

$$I_c = bD^3/12$$

For transformed steel area,

$$I_{cs} = 2(n-1)A_s\left(\frac{D}{2} - d''\right)^2$$

$$= 2(n-1)pbD\left(\frac{D}{2} - d''\right)^2$$

$$\frac{I_{cs}}{I_c} = 24(n-1)p\left(1 - \frac{d''}{D}\right)\left(0.5 - \frac{d''}{D}\right)^2$$

For $n = 10$, $p = 1$ percent, and $d''/D = 0.1$

$$\frac{I_{cs}}{I_c} = 0.31104$$

## CONTRIBUTION OF FLANGE TO STIFFNESS OF REINFORCED CONCRETE BEAM

### Uncracked section stiffness

From Fig. A2 $b/b' = 9$.

$$8 \times 10 = 80 \text{ sq in.} \times 5 = 400$$
$$4 \times 64 = 256 \text{ sq in.} \times 2 = 512$$
$$336 \text{ sq in.} \qquad 912$$
$$\bar{y} = 2.7 \text{ in.}$$

Moment of inertia:

$$80 \times (10)^2/12 = 667$$
$$80 \times (2.3)^2 = 423$$
$$256 \times 4^2/12 = 341$$
$$256 \times (0.7)^2 = 125$$
$$I = 1556$$

Increase in moment of inertia due to flange = 1556/667 = 2.34.

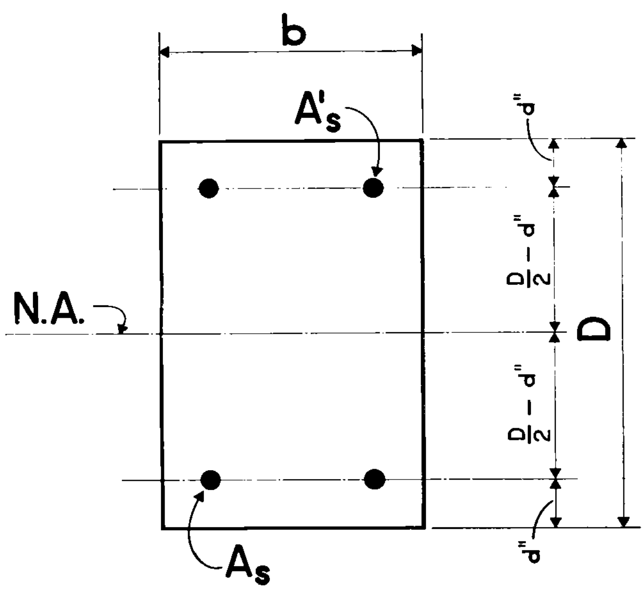


Fig. A1—Rectangular section

### Cracked section stiffness

Section as in Fig. A2, $A_s = 2.00$ sq in., $d = 8$ in., $E_c = 3 \times 10^6$, and $n = 10$.

For stem only:

$$p = \frac{2.00}{8 \times 8} = 0.0312$$

$(1 - k)j = 0.380$ (from table)

For stem and flange:

$$p = \frac{2.00}{8 \times 72}\ 0.00348$$

$(1 - k)j = 0.709$ (from table)

Increase in moment of inertia due to flange:

$$\frac{E_sA_s[(1 - k)j]_{flge}d^2}{E_sA_s[(1 - k)j]_{stem}d^2} = \frac{0.709}{0.380} = 1.86$$

The above is Eq. (14) from Reference 1.

## DERIVATION OF EQ. (1), (2), AND (3)

### Uncracked sections

$$M = C_m wL^2 = \frac{fC_I I}{C_y D}$$

and

$$\Delta = \frac{C_\Delta FwL^4}{E_cC_II} = \frac{C_\Delta}{C_m} \cdot \frac{FML^2}{E_cC_II} = \frac{C_\Delta}{C_m} \cdot \frac{FfL^2}{E_cC_yD}$$

or, rearranging:

$$\frac{L}{D} = \frac{C_m}{C_\Delta} \cdot \frac{\Delta}{L} \cdot \frac{E_c}{F} \cdot K_1 \qquad (1)$$

where

$$K_1 = \frac{C_y}{f} \qquad (2)$$

### Cracked sections

$$M = f_sA_sjd = \frac{f_s\,[E_sA_s(1 - k)jd^2]}{E_s(1 - k)d} = \frac{f_s(EI)_{cr}}{E_s(1 - k)d}$$

[see Eq. (14) from Reference 1] and:

$$\Delta = \frac{C_\Delta FwL^4}{(EI)_{cr}} = \frac{C_\Delta}{C_m} \cdot \frac{FML^2}{(EI)_{cr}} = \frac{C_\Delta}{C_m} \cdot \frac{Ff_sL^2}{E_s(1 - k)d}$$

Rearranging and introducing $d = D(d/D)$ and $E_s = nE_c$:

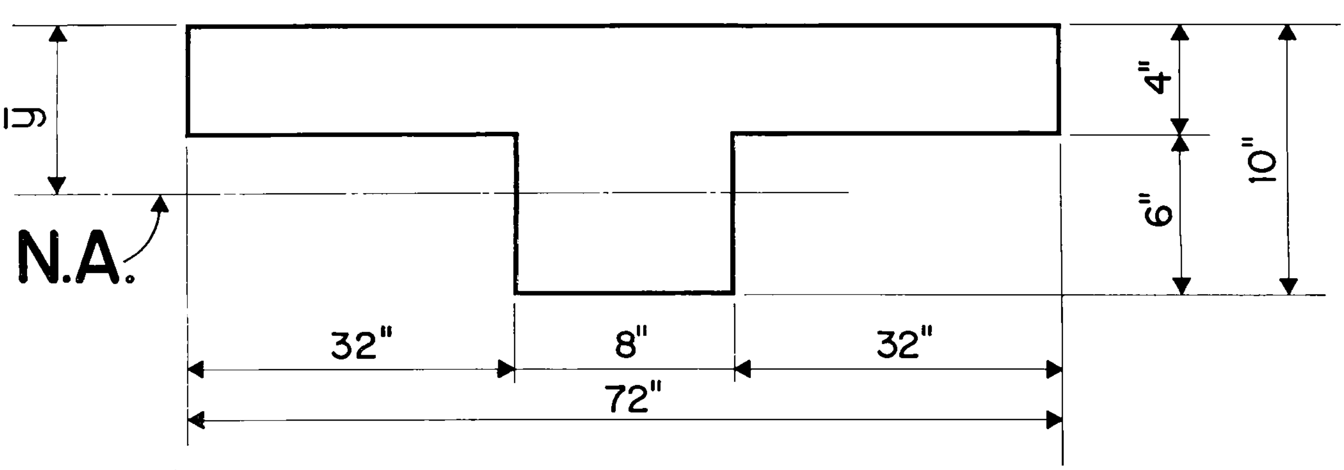


Fig. A2—T-section

$$\frac{L}{D} = \frac{C_m}{C_\Delta} \cdot \frac{\Delta}{L} \cdot \frac{E_c}{F} \cdot K_1 \qquad (1)$$

where

$$K_1 = \frac{n(1-k)}{f_s} \cdot \frac{d}{D} \qquad (3)$$

## DERIVATION OF EQ. (5), (6), AND (7)

### Uncracked sections

$$M = C_m w L^2 = C_m (C_w b D W_c) L^2$$

where

$$w = C_w b D W_c$$

$$f = \frac{C_y D M}{I} = \frac{C_y D M 12}{C_I b D^3} = \frac{12 C_y M}{C_I b D^2}$$

Substituting for $M$:

$$f = \frac{12 C_y C_m C_w b D W_c L^2}{C_I b D^2} = \frac{12 C_y C_m C_w W_c L^2}{C_I D}$$

From Eq. (1) and (2):

$$\frac{L}{D} = \frac{C_m}{C_\Delta} \cdot \frac{\Delta}{L} \cdot \frac{E_c}{F} \cdot \frac{C_y C_I D}{12 C_y C_m C_w W_c L^2}$$

$$= \frac{\Delta}{L} \cdot \frac{E_c C_I}{12 F C_\Delta C_w W_c L} \cdot \frac{D}{L}$$

$$\frac{L}{D} = \sqrt{\frac{\Delta}{L} \cdot \frac{E_c C_I}{12 F C_\Delta C_w W_c L}}$$

$$= \sqrt{\frac{\Delta}{L} \cdot \frac{E_c}{F} \cdot \frac{12 K_2}{C_\Delta W_c} \cdot \frac{1}{C_w L}} \qquad (5)$$

where

$$K_2 = C_I \qquad (6)$$

and $E_c$ is in psi, $L$ is in ft, and $W_c$ is in lb per cu ft.

### Cracked sections

$$f_s = \frac{M}{p j b d^2} = \frac{C_m C_w b D W_c L^2}{p j b d^2} = \frac{C_m C_w W_c L^2}{p j D (d/D)^2}$$

From Eq. (1) and (3):

$$\frac{L}{D} = \frac{C_m}{C_\Delta} \cdot \frac{\Delta}{L} \cdot \frac{E_c}{F} \cdot \frac{n(1-k) p j D}{C_m C_w W_c L^2} \left(\frac{d}{D}\right)^2 \cdot \frac{d}{D}$$

$$\frac{L}{D} = \sqrt{\frac{\Delta}{L} \cdot \frac{E_c}{F} \cdot \frac{12 K_2}{C_\Delta W_c} \cdot \frac{1}{C_w L}} \qquad (5)$$

where

$$K_2 = 12 n (1-k) p j \left(\frac{d}{D}\right)^3 \qquad (7)$$

and $E_c$ is in psi, $L$ is in ft, and $W_c$ is in lb per cu ft.

For metric equivalent, Eq. (5) should be multiplied by 100/12 when $E_c$ is in kgf/cm$^2$, $L$ is in meters, and $W_c$ is in kg/m$^3$.

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In the balloting of the seven members of Subcommittee 1, ACI Committee 435, six voted affirmatively and one negatively. The report was later submitted to ballot of the remainder of the committee. The combined result of the two ballots was that of 18 ballots sent out, 17 were returned with 16 affirmative votes and one negative vote.

## Sinopsis—Résumé—Zusammenfassung


### Deflexiones Permisibles

Se discuten los factores que afectan la deflexión de miembros de concreto reforzado y se enfatiza la importancia de tomarlos en consideración para una estimación precisa de la deflexión. Se incluye una tabla con una lista extensa de casos que requieren limitaciones de la deflexión. Estas están basadas en relaciones $L/\Delta$ y en valores absolutos aplicados a las deflexiones totales o a sus incrementos. Se discuten los parámetros más importantes que afectan las relaciones $L/D$ como un límite indirecto de las deflexiones. Se presentan fórmulas y gráficas para relaciones $L/D$ y se incluyen ejemplos para su uso.

El informe se divide en 5 capítulos: Introducción, Cálculo de las Deflexiones, Deflexiones Permisibles, Relaciones Permisibles Claro/Peralte y Correlación con Estructuras de Edificios Existentes.

### Déformations Permissibles

Les facteurs affectant les déformations des membrures en béton armé en insistant sur l'importance de les prendre tous en considération pour une estimation précise des déformations sont l'objet de cet article. Il inclut un tableau comprenant une liste importante de cas particuliers imposant une limitation des déformations. Ces déformations sont basées sur un rapport $L/\Delta$ et les valeurs absolues appliquées à l'ensemble ou à un incrément des déformations. Les paramétres les plus significatifs affectants les rapport $L/\Delta$ comme limite indirecte de déformations sont discutés. Des formules graphiques pour $L/\Delta$ et des exemples d'utilisation sont présentés. Le rapport est divisé en 5 chapitres: introduction, calcul des déformations permissibles, rapport Portée/Epaisseur permissible et corrélation avec des structures réelles de batiments.

### Zulässige Durchbiegungen

Faktoren, welche die Durchbiegung von Stahlbetonbauteilen beeinflussen, werden diskutiert; die Wichtigkeit der gleichzeitigen Berücksichtigung aller Einflüsse zur genauen Abschätzung der Durchbiegung wird unterstrichen. In einer ausführlichen Tabelle werden jene Fälle zusammengefasst, für die eine Beschränkung der Durchbiegung erforderlich ist. Dazu werden Werte für das Verhältnis Spannweite — Durchbiegung oder absolute Werte angegeben. Die Werte gelten entweder für die gesamte Durchbiegung oder für die Durchbiegung über einen gewissen Zeitraum. Die wesentlichsten Parameter, welche die Gültigkeit des Verhältnisses Spannweite — Balkenhöhe als indirekten Grenzwert für die Durchbiegung beeinflussen, werden diskutiert. Gleichungen und Diagramme für zulässige Werte des Verhältnisses Spannweite — Balkenhöhe werden gegeben und anhand von Beispielen erläutert.

Der Bericht ist in fünf Abschnitte gegliedert: Einführung, Berechnung von Durchbiegungen, zulässige Durchbiegungen, zulässiges Verhältnis Spannweite — Balkenhöhe und der Zusammenhang zwischen Rechenwerten und an Bauwerken beobachteten Durchbiegungen.