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Shear Strength of High-Strength Concrete Beams with Web Reinforcement

by John J. Roller and Henry G. Russell

An experimental investigation of the shear strength of high-strength concrete beams with web reinforcement was conducted. Two series of beam tests were performed, each series consisting of five beams. All beams were designed in accordance with the provisions of ACI 318-83. Primary design variables were concrete compressive strength and the quantity of shear reinforcement. Concrete with compressive strengths of approximately 10,000, 17,000, and 18,000 psi (69, 117, and 124 MPa) was used in the beam specimens. The quantity of shear reinforcement provided in the beams ranged from the minimum amount required by ACI 318-83 to the maximum amount that can be assumed when calculating shear capacity. Actual shear strength of each beam specimen was compared with the shear strength predicted using the provisions of ACI 318-83.

Results of the investigation indicate that for nonprestressed high-strength concrete members subject to shear and flexure only the minimum quantity of shear reinforcement specified in ACI 318-83 needs to increase as the concrete compressive strength increases. ACI Committee 318 recently approved a proposed provision that expresses the minimum quantity of shear reinforcement as a function of the concrete compressive strength. Results from the tests conducted in this investigation confirm the applicability of the new proposed provision for minimum web reinforcement.

Keywords: beams (supports); building codes; compressive strength; **high-strength concretes; shear strength**; structural design; tests; **web reinforcement.**

With the commercial availability of concretes with compressive strengths approaching 20,000 psi (138 MPa), many questions have been raised regarding the applicability of the design provisions stipulated in ACI 318-83, "Building Code Requirements for Reinforced Concrete."[1] Many of the design parameters and equations in ACI 318-83 were derived from results of experimental research programs using concrete with compressive strengths less than 6000 psi (41 MPa). Therefore, it is reasonable to question whether many of the design provisions in ACI 318-83 are applicable or appropriate with higher strength concretes. This paper addresses the shear design provisions of ACI 318-83 for reinforced concrete beams.

The ACI 318-83 code provisions for shear design use the concept that the nominal shear strength V_n of a reinforced concrete member is made up of the sum of two contributing factors. These factors are V_c, the nominal shear strength provided by the concrete, and V_s, the nominal shear strength provided by web reinforcement. Data from shear tests performed on reinforced concrete beams with web reinforcement indicate that the degree of conservatism offered by the code provisions is variable. This conclusion was drawn based on the compilation of results of several investigations performed over the past 40 years.[2-12]

Results from shear tests performed on approximately 150 reinforced concrete beams with web reinforcement are shown in Fig. 1.[2-12] All of the beams reportedly failed in shear. The ratio V_{test}/V_n has been plotted versus concrete compressive strength f'_c. The actual shear strength V_{test} of each beam specimen was compared to the nominal shear strength V_n, which was calculated using actual material properties and the ACI 318-83 code provisions. The nominal shear strength contributed by the concrete V_c and the nominal shear strength contributed by shear reinforcement V_s were computed as

$$V_c = \left(1.9\sqrt{f'_c} + 2500\,\rho_w \frac{V_u d}{M_u}\right) b_w d \qquad \text{[ACI 318-83 Eq. (11-6)]}$$

$$v_s = \frac{A_v f_y d}{s} \qquad \text{[ACI 318-83 Eq. (11-17)]}$$

The horizontal line in Fig. 1 at $V_{test}/V_n = 1$ represents a reference point where the actual shear strength V_{test} equals the shear strength predicted using the code equations V_n. Data points that fall below this line represent beams that had a measured shear strength that

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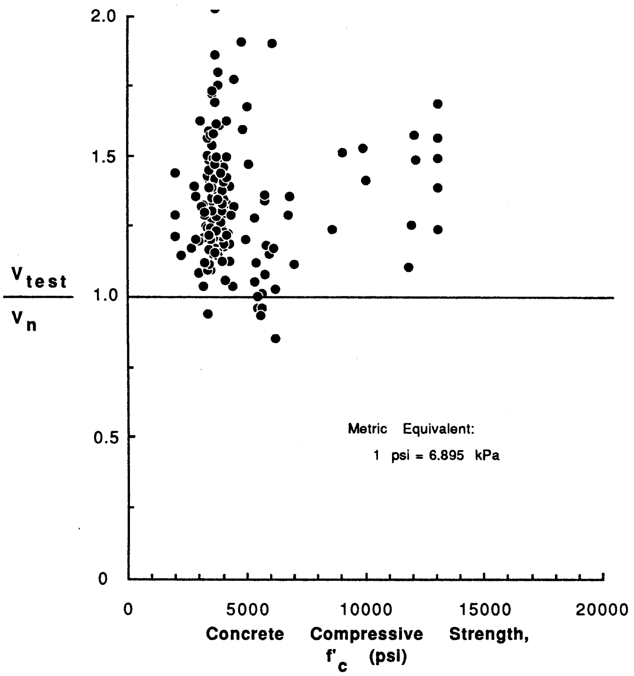


Fig. 1—Test results from beams that failed in shear[2-12]

was less than that predicted by the code design provisions.

It can be seen in Fig. 1 that the conservativeness offered by the ACI code design provisions for shear is variable. A great deal of this variability is to be expected due to the numerous variables that have an effect on shear strength. Many of the beams represented in Fig. 1 do not meet the design requirements of the ACI code and therefore may not be representative of practical design conditions. These beams were either over-reinforced with longitudinal steel by code standards, had less than the minimum required amount of shear reinforcement, or had shear reinforcement spacing greater than that specified in the code. In Fig. 2, only those beams that meet all the design requirements of the current code are included. This figure indicates the limited amount of test data for specimens that are representative of the current code provisions and, more importantly, the lack of data at concrete strengths greater than approximately 7000 psi (48 MPa).

RESEARCH SIGNIFICANCE

The research described in this paper was initiated as a result of a proposed revision to the ACI 318 Code.

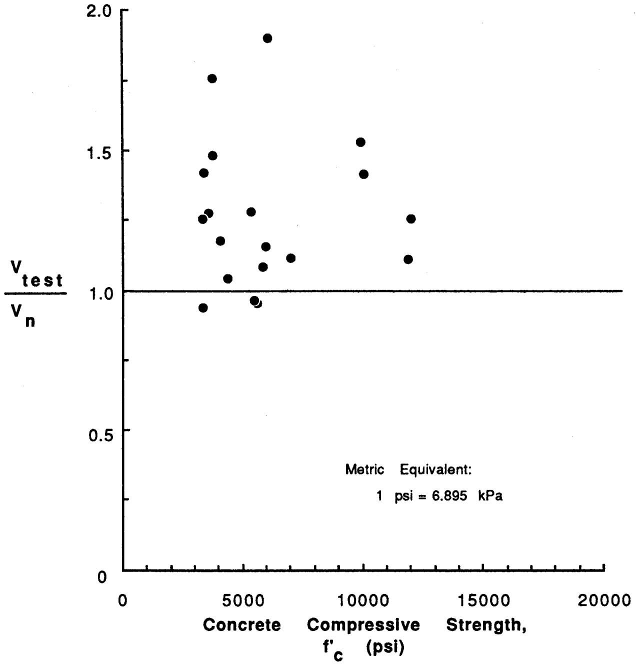


Fig. 2—Results from beams that failed in shear and met all design requirements of ACI 318-83[2-12]

The proposed revision was to limit the value of $\sqrt{f'_c}$ that could be used in the design of members subjected to shear and torsion. The limitation was proposed as a result of a lack of test data about the shear strength of high-strength concrete beams. During the course of the investigation, the proposed revision was modified to allow highervalues of $\sqrt{f'_c}$ for reinforced concrete beams with web reinforcement, provided that the minimum required amount of web reinforcement was increased.[13] Results from beams tested as part of this research were used to confirm the revised code provision.

EXPERIMENTAL PROGRAM

Specimen details

The investigation consisted of two different test series with each series including five beam specimens. Each beam specimen was designed in accordance with the provisions of the ACI 318-83 code. Beam specimen designs were intended to be both representative of the ACI 318-83 code provisions as well as practical design conditions. Each beam specimen was intentionally designed to have a calculated nominal bending moment capacity M_n that was 5 to 40 percent greater than the calculated moment required to cause a shear failure.

For the first test series, five beams of equal concrete compressive strength but with different quantities of shear reinforcement were constructed. These beam specimens had a rectangular cross section with 14-in. (356-mm) width and an effective depth d of 22 in. (559 mm). The concrete compressive strength used in the beams of the first test series was approximately 17,000 psi (117 MPa). The quantity of shear reinforcement provided in the beam specimens varied from the minimum amount required by ACI 318-83 ($A_v = 50\ b_w s/f_y$) to the maximum amount that can be assumed when calculating shear capacity ($A^v = 8\ \sqrt{f'_c}\, b_w s/f_y$). The maximum amount of shear reinforcement that can be assumed was calculated using ACI 318-83 Eq. (11-17) and the provision in Section 11.5.6.8. Specific details pertaining to each of the five Series 1 beams (Specimens 1 through 5) are given in Table 1. A cross section of each beam specimen is shown in Fig. 3.

For the second test series, five beams of rectangular cross section were also constructed. These beams were 18 in. (457 mm) wide and had an effective depth d of 30 in. (762 mm). Two of the five beams had a concrete compressive strength of approximately 10,000 psi (69 MPa). The remaining three beams had a concrete compressive strength of approximately 18,000 psi (124 MPa). The quantity of shear reinforcement provided in the beams of the second test series varied from the minimum amount required by ACI 318-83 ($A_v = 50\ b_w s/f_y$) to approximately three times the minimum required amount. Specific design details pertaining to each of the five Series 2 beams (Specimens 6 through 10) are given in Table 1. A sketch of each beam cross section is given in Fig. 4.

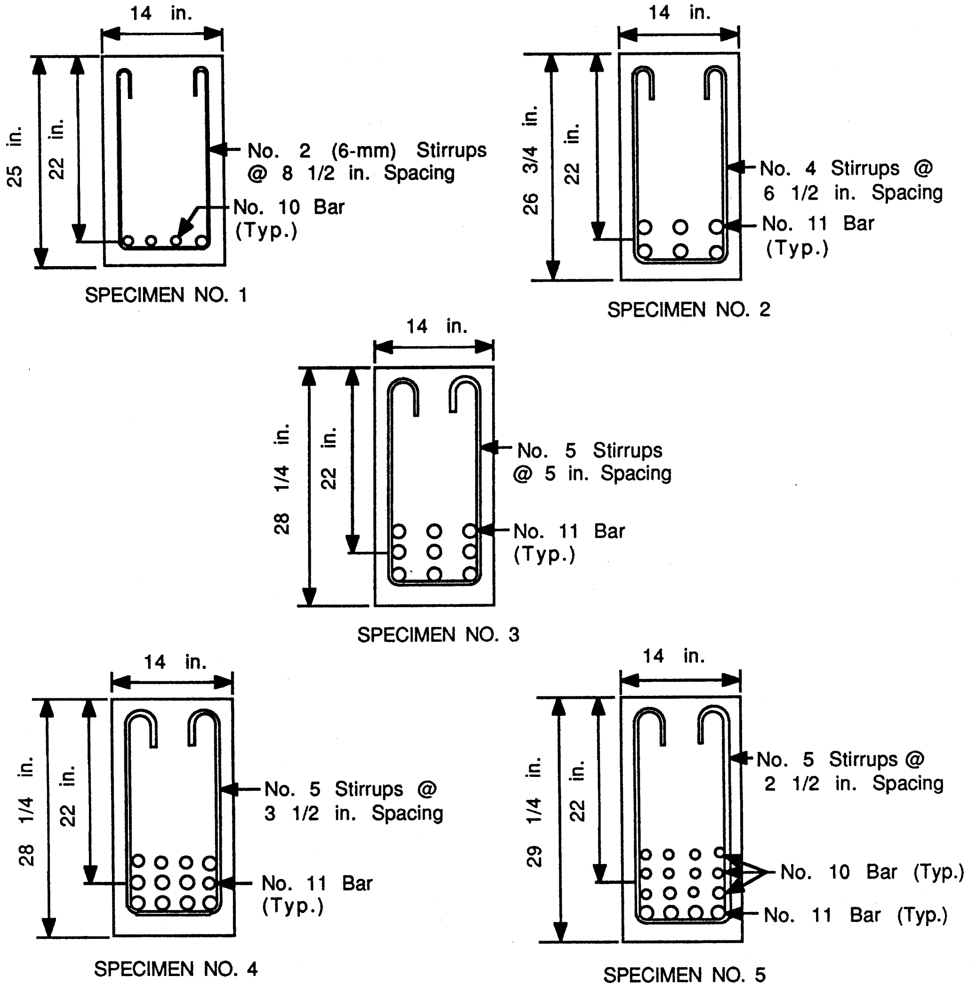
Materials

Two different concrete mixes were used to cast the ten beam specimens. One mix had a specified 28-day compressive strength of 14,000 psi (97 MPa), with potential 90-day strength in excess of 17,000 psi (117

Table 1 — Beam specimen details

Specimen No.	Concrete compressive strength f'_c, psi	ρ/ρ_b[*]	$V_c/b_w d$,[†] psi	$V_s/b_w d$,[‡] psi
		Test Series 1		
1	17,420	0.21	267 ($2.03\sqrt{f'_c}$)	44 ($0.3\sqrt{f'_c}$)
2	17,420	0.34	281 ($2.13\sqrt{f'_c}$)	286 ($2.2\sqrt{f'_c}$)
3	17,420	0.51	296 ($2.25\sqrt{f'_c}$)	588 ($4.5\sqrt{f'_c}$)
4	17,420	0.68	312 ($2.36\sqrt{f'_c}$)	840 ($6.4\sqrt{f'_c}$)
5	17,420	0.85	321 ($2.43\sqrt{f'_c}$)	1176 ($8.9\sqrt{f'_c}$)
		Test Series 2		
6	10,500	0.36	209 ($2.04\sqrt{f'_c}$)	53 ($0.5\sqrt{f'_c}$)
7	10,500	0.41	210 ($2.05\sqrt{f'_c}$)	102 ($1.0\sqrt{f'_c}$)
8	18,170	0.24	272 ($2.02\sqrt{f'_c}$)	53 ($0.4\sqrt{f'_c}$)
9	18,170	0.30	276 ($2.05\sqrt{f'_c}$)	102 ($0.8\sqrt{f'_c}$)
10	18,170	0.34	280 ($2.08\sqrt{f'_c}$)	150 ($1.1\sqrt{f'_c}$)

[*] Based on actual material properties.
[†] Based on actual material properties and Eq. (11-6) of ACI 318-83.
[†] Based on actual material properties and Eq. (11-17) of ACI 318-83.
1 psi = 6.895 kPa.



Metric Equivalent : 1 in. = 25.4 mm

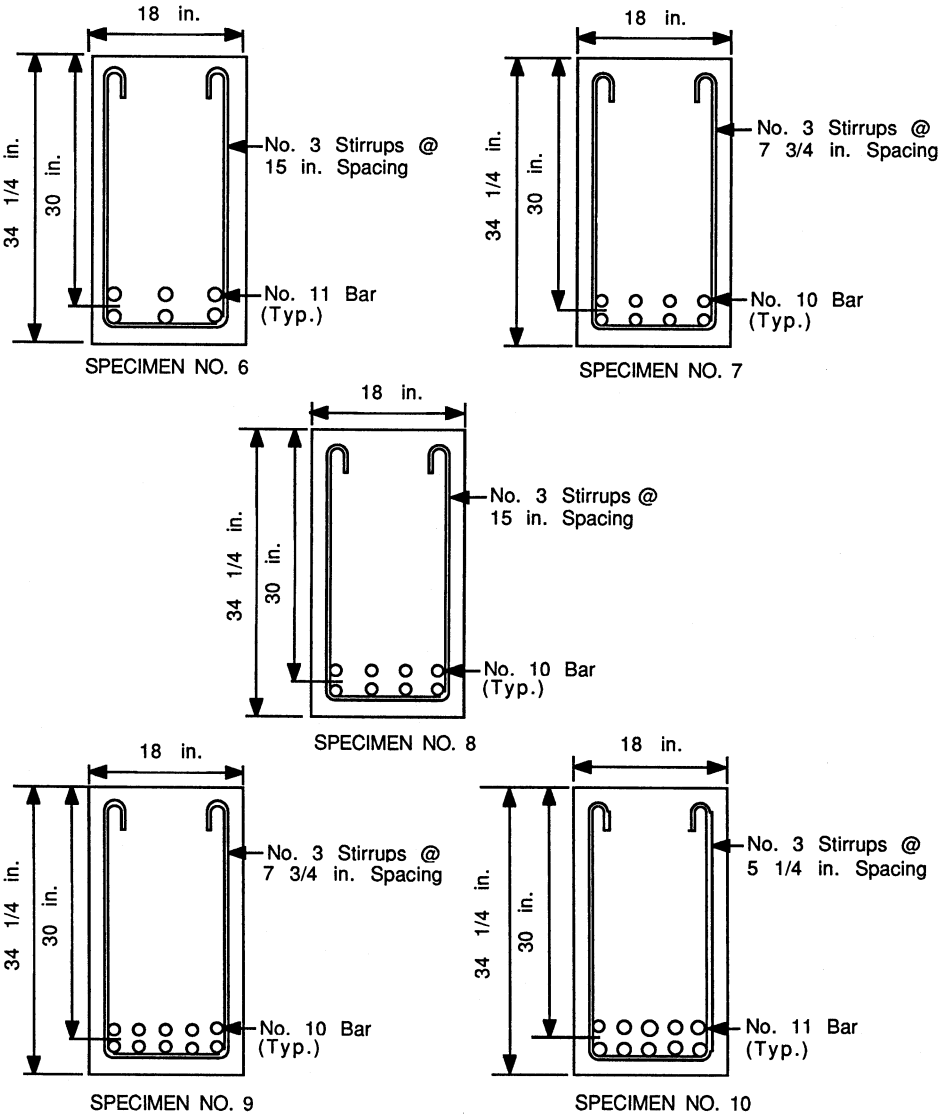
Fig. 3—Beam specimen cross sections—Test Series 1

MPa). The other mix has a specified 28-day compressive strength of 9000 psi (62 MPa) with potential 90-day strength in excess of 10,000 psi (69 MPa). Both concrete mixes were provided by a local ready-mix supplier.

The concrete mix with the specified 28-day compressive strength of 14,000 psi (97 MPa) was made using a selected ASTM Type I portland cement. Fly ash (Class C) and silica fume were included in the mix to improve the properties of the concrete in both the fresh and hardened state. The water-cementitious ratio for this mix was approximately 0.26. The maximum size of the coarse aggregate was ½ in. (12.7 mm). A high-range water reducer (superplasticizer), ASTM C 494 Type F, and water-reducing retarder, ASTM C 494 Type D, were added to the mix to improve workability. Details regarding the various mix constituents and quantities are given in Table 2. Upon delivery, this concrete had a slump of approximately 10 in. (254 mm), as determined using test methods described in ASTM C 143.

The concrete mix with the specified 28-day compressive strength of 9000 psi (62 MPa) was also made using a selected ASTM Type I portland cement. Fly ash (Class C) was included in this mix to improve workability and enhance the long-term compressive strength of the concrete. The water-cementitious ratio for this mix was approximately 0.31. The maximum size of the coarse aggregate was ½ in. (12.7 mm). A water-reducing retarder, ASTM C 494 Type D, was added to the mix to improve workability. Details regarding the various mix constituents and quantities are given in Table 2. Upon delivery, this concrete had a slump of approximately 3 in. (76 mm), as determined using test methods described in ASTM C 143.

Steel reinforcement used in each beam specimen consisted of hot-rolled deformed bars. With the exception of the shear stirrups in beam specimen No. 1 of the first test series, all steel reinforcing bars conformed to ASTM A 615, Grade 60. The 6 mm diameter (0.24 in.) reinforcing bars used for stirrups in beam specimen No.



SPECIMEN NO. 6

SPECIMEN NO. 7

SPECIMEN NO. 8

SPECIMEN NO. 9

SPECIMEN NO. 10

Metric Equivalent : 1 in. = 25.4 mm

Fig. 4—Beam specimen cross-sections—Test Series 2

Table 2 — Details of high-strength concrete mixes (per yd^3)

Mix constituents	f'_c = 14000 psi at 28 days	f'_c = 9000 psi at 28 days
Cement	800 lb	823 lb
Class C fly ash	12.5 percent of cement weight	12 percent of cement weight
Silica fume	19 percent of cement weight	—
Coarse aggregate	1700 lb	1740 lb
Sand	1120 lb	1140 lb
Water	273 lb	290 lb
High-range water reducer	As needed for desired slump	—
Water-reducing retarder	16 oz	33 oz

1 psi = 6.895 kPa; 1 lb (mass) = 0.454 kg (mass); 1 oz = 29.57 cc.

Table 3 — Physical material properties of concrete

Beam specimens	Concrete age, days	Concrete compressive strength f'_c,* psi	Splitting tensile strength f_{sp},* psi
1 through 5	90	17,420	650
6 and 7	105	10,500	611
8 through 10	98	18,170	817

*Based on the average of three cylinder tests.
1 ksi = 1000 psi = 6.895 MPa.

1 were obtained from Sweden and had properties similar to those of ASTM A 615, Grade 60 domestic reinforcing bars.

Material properties

During casting of the beam specimens, 6 x 12 in. (152 x 305 mm) concrete cylinders were taken for material property tests. Test cylinders were cast in accordance with ASTM C 31. Test cylinders and beam specimens were cured under identical conditions. Just prior to conducting tests on the beams, material property tests were conducted on concrete test cylinders to determine concrete compressive strength (ASTM C 39) and splitting tensile strength (ASTM C 496). Physical material properties associated with each of the beam specimens are given in Table 3.

Five different reinforcing bar sizes were used for the beam specimens in Test Series 1. Three different reinforcing bar sizes were used for the Test Series 2 beam specimens. All the bars of a given size for each series came from the same material heat. Tension tests were conducted on full-size bar samples in accordance with ASTM A 370 to determine yield strength, ultimate strength, and total elongation. Physical properties of the reinforcing steel are given in Table 4.

Test details

Beam specimens were tested in flexure using a single concentrated load at midspan. The shear span-to-depth a/d ratios used for the first and second test series were 2.5 and 3.0, respectively. Both ends of the beams were free to rotate and translate under load. Details of the test setup for the first and second test series are given in Fig. 5 and 6, respectively. Fig. 7 is a photograph of the test setup for the first test series.

Table 4 — Properties of reinforcing steel

Bar size	Yield strength, psi	Tensile strength, psi	Elongation,* percent
Test Series 1			
6 mm	58,980	75,210	19.7
#4	65,000	95,500	17.9
#5	66,400	107,020	15.1
#10	68,500	107,480	15.8
#11	62,500	100,130	18.7
Test Series 2			
#3	64,550	83,640	18.7
#10	70,080	113,780	14.4
#11	67,310	101,280	19.1

*Elongation measured over a 8-in. (203-mm) gage length.
1 psi = 6.895 kPa; 1 in. = 25.4 mm.

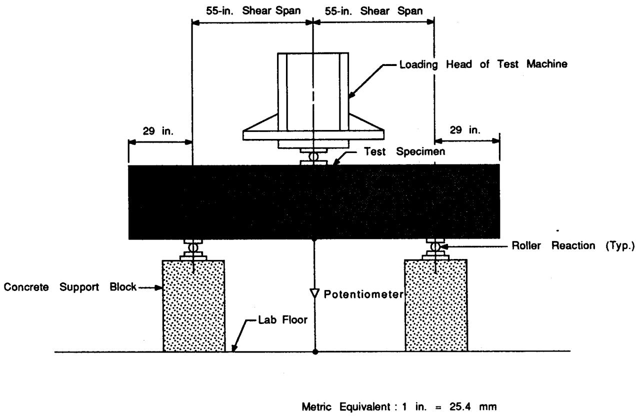


Fig. 5—Test setup, Series 1

Load was applied to the beam specimens using a 1,000,000 lb capacity (4448 kN) compression testing machine. Beam deflection under load was measured using a single linear potentiometer located at midspan.

Each beam specimen was instrumented with strain gages on the longitudinal reinforcing bars as well as on the shear reinforcement. In each specimen, strain gages were placed at midspan on the two outermost longitudinal bars of the bottom layer of reinforcement. Selected stirrups along the shear span were also instrumented with strain gages that were placed at approximately mid-depth.

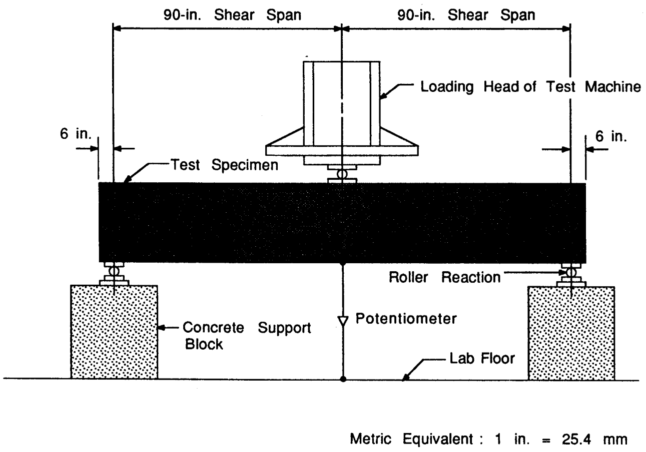


Fig. 6—Test setup, Series 2

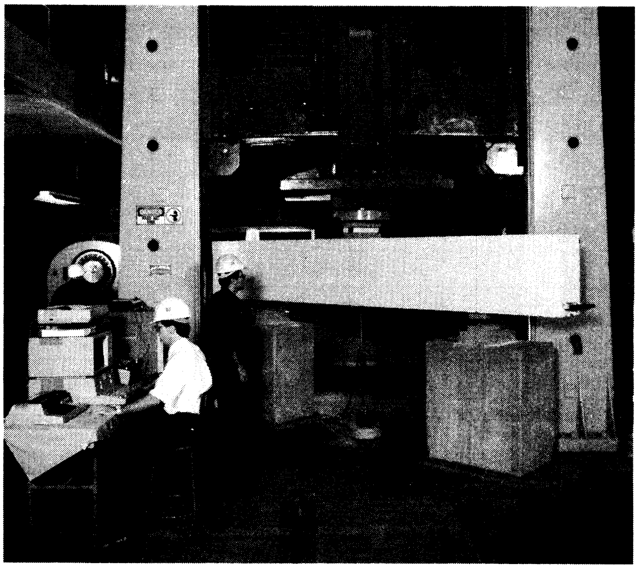


Fig. 7—Test setup for Series 1

Test procedure

Each of the beam specimens was tested to failure under flexural loading. Beam load, deflection, and reinforcing steel strains were monitored throughout each test. Load was applied to the beams in increments ranging from 5 to 30 kips (22 to 133 kN), depending on the estimated strength of the beam.

Test results

Major results from the beam tests are given in Table 5. Table 5 includes the shear strength of each beam specimen, as predicted using ACI 318-83 code equations and actual material properties, and the measured shear strength of each beam specimen determined from the maximum load. All beam specimens failed in shear.

Table 5 — Specimen test results

Specimen No.	Calculated shear strength,* psi	Measured shear strength, psi	Failure mode
Test Series 1			
1	311 (2.4$\sqrt{f'_c}$)	217 (1.6$\sqrt{f'_c}$)	Diagonal tension
2	567 (4.3$\sqrt{f'_c}$)	801 (6.1$\sqrt{f'_c}$)	Shear compression
3	884 (6.7$\sqrt{f'_c}$)	1208 (9.2$\sqrt{f'_c}$)	Shear compression
4	1152 (8.7$\sqrt{f'_c}$)	1416 (10.7$\sqrt{f'_c}$)	Shear compression
5	1497 (11.3$\sqrt{f'_c}$)	1631 (12.4$\sqrt{f'_c}$)	Shear compression
Test Series 2			
6	262 (2.6$\sqrt{f'_c}$)	277 (2.7$\sqrt{f'_c}$)	Shear compression
7	312 (3.1$\sqrt{f'_c}$)	328 (3.2$\sqrt{f'_c}$)	Shear compression
8	325 (2.4$\sqrt{f'_c}$)	201 (1.5$\sqrt{f'_c}$)	Diagonal tension
9	378 (2.8$\sqrt{f'_c}$)	312 (2.3$\sqrt{f'_c}$)	Shear compression
10	430 (3.2$\sqrt{f'_c}$)	488 (3.6$\sqrt{f'_c}$)	Shear compression

*Based on actual material properties and Eq. (11-6) and (11-17) of ACI 318-83.
1 psi = 6.895 kPa.

DISCUSSION OF TEST RESULTS

General behavior

Cracking of each specimen progressed as follows. Flexural cracks at midspan developed during the early stages of loading. Additional flexural cracks developed along the shear span as the load increased. These cracks gradually became inclined as they propagated above the longitudinal reinforcement. The first diagonal crack that was not initiated by a flexural crack generally occurred at a shear load that was less than V_c, as calculated using the provisions of the ACI 318-83 code. However, the difference between the calculated V_c and the actual load corresponding to the occurrence of the first diagonal crack gradually decreased as the quantity of web reinforcement increased. For beam Specimen 5, which contained the maximum amount of web reinforcement that can be assumed when calculating shear capacity, the calculated V_c correlated well with the shear load that initiated the first diagonal crack.

As indicated in Table 5, three out of the ten beam specimens failed at a strength that was less than the calculated nominal shear strength V_n predicted using ACI 318-83 code provisions. Two of these three beams (Specimens 1 and 8) contained approximately the minimum required amount of web reinforcement. The third beam (Specimen 9) contained approximately twice the minimum amount of shear reinforcement required by the ACI 318-83 code.

The ratio of the measured shear strength to the calculated strength V_{test}/V_n is plotted versus the calculated nominal shear strength provided by shear reinforcement $V_s/b_w d$ for each beam specimen in Fig. 8 and 9. As indicated in these figures, for beam specimens with concrete compressive strengths greater than 17,000 psi (117 MPa), the ACI 318-83 code equations for predicting shear strength become unconservative as the value of $V_s/b_w d$ approaches the minimum required amount by ACI 318-83 of 50 psi (345 kPa).

Beam Specimens 6 and 8 had concrete compressive strengths of 10,500 psi (72 MPa) and 18,200 psi (126 MPa), respectively. These two specimens incorporated identical shear reinforcement details and cross-sectional dimensions. Both specimens were tested using a shear span that was three times the effective depth (a/d = 3). The ratios of V_{test}/V_n for Specimens 6 and 8 were 1.06 and 0.59, respectively.

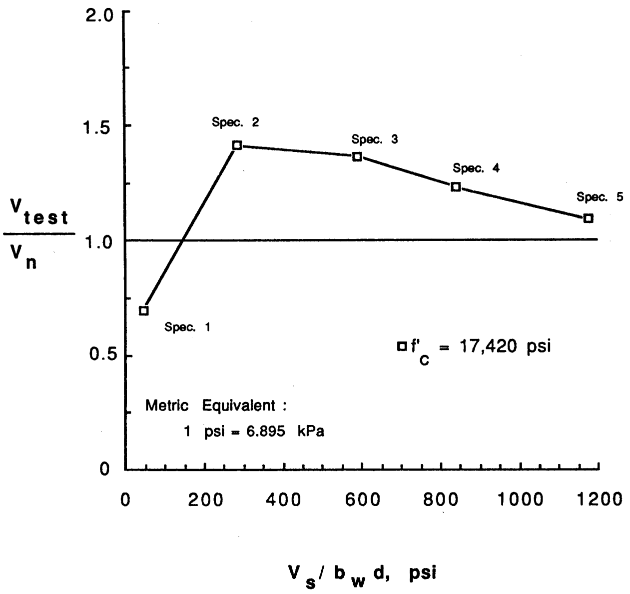


Fig. 8—V_{test}/V_n *versus* V_s/b_wd*—Test Series 1*

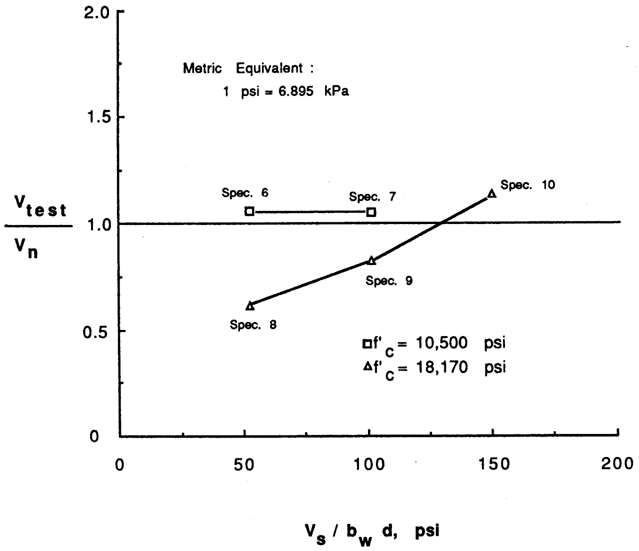


Fig. 9—V_{test}/V_n *versus* V_s/b_wd*—Test Series 2*

Similarly, beam Specimens 7 and 9 had approximately twice the minimum amount of web reinforcement required by the ACI 318-83 code. These two beams also had identical shear reinforcement details, cross-sectional dimensions, and shear spans. For beam specimen 7, which had a concrete compressive strength of approximately 10,500 psi (72 MPa), the actual shear strength exceeded the nominal shear strength predicted using the code equations. However, for beam Specimen 9, which had a concrete strength of approximately 18,000 psi (124 MPa), the actual shear strength was only 83 percent of that predicted by the code equations. These findings suggest that the minimum required amount of web reinforcement should be related to the concrete compressive strength.

Beams with minimum web reinforcement

According to ACI 318-83, where shear reinforcement is required for nonprestressed members subject to shear and flexure only, the minimum nominal shear stress provided by shear reinforcement V_s/b_wd shall be 50 psi (345 kPa). Three out of the ten beam specimens tested (Specimens 1, 6, and 8) contained approximately the minimum amount of web reinforcement required by ACI 318-83. Two out of these three beams failed in shear at a strength that was not only less than the calculated V_n, but also less than the calculated V_c. This indicates that the current code provisions may overestimate the V_c term for high-strength concrete beams containing the minimum required amount of web reinforcement. This finding also correlates well with the observation regarding the occurrence of the first diagonal crack mentioned earlier.

As indicated in Fig. 8 and 9, beam specimens that had significantly more web reinforcement than the minimum amount required in ACI 318-83 failed at a load that was higher than the value predicted by the code equations ($V_{test}/V_n > 1$). This indicates that at some amount of web reinforcement the code equations start to become conservative again, regardless of the apparent deficiency in the V_c term. It appears that this minimum amount of web reinforcement is dependent upon the compressive strength of the concrete.

ACI Committee 318 recently published a proposed new code provision that expresses the minimum required amount of web reinforcement as a function of the concrete compressive strength.[13] This proposed provision is worded:

> Add a new Section 11.1.2 and renumber old Sections 11.1.2 and 11.1.3 to 11.1.1 and 11.1.4, respectively.
>
> **11.1.2** — The values of $\sqrt{f'_c}$ used in this chapter shall not exceed 100 psi except as allowed in Section 11.1.2.1.
>
> **11.1.2.1** — Values of $\sqrt{f'_c}$ greater than 100 psi shall be permitted in computing V_c, V_{ci}, and v_{cw} for reinforced or prestressed concrete beams and concrete joist construction having minimum web reinforcement equal to f'_c /5000 times, but not more than three times the amounts required by Sections 11.5.5.3, 11.5.5.4 or 11.5.5.5.

The proposed new code provision only affects members with concrete compressive strengths greater than 10,000 psi (69 MPa). When concrete compressive strength exceeds 10,000 psi (69 MPa) and web reinforcement is required ($V_n > V_c/2$), the proposed provision will require the minimum nominal shear stress provided by shear reinforcement V_s/b_wd to be at least $0.01 f'_c$, but not greater than 150 psi (1034 kPa). Beam Specimens 1, 6, 8, and 9 do not satisfy the proposed code provision and, therefore, would not be representative designs. As indicated in Table 1, beam Specimen 5 ended up being over-reinforced when actual material properties of the longitudinal reinforcement were considered; therefore, it would not be a representative design. However, beam Specimens 2, 3, 4, 7, and 10 satisfy all requirements of ACI 318-83 as well as the proposed code provision. In Fig. 10, the results from

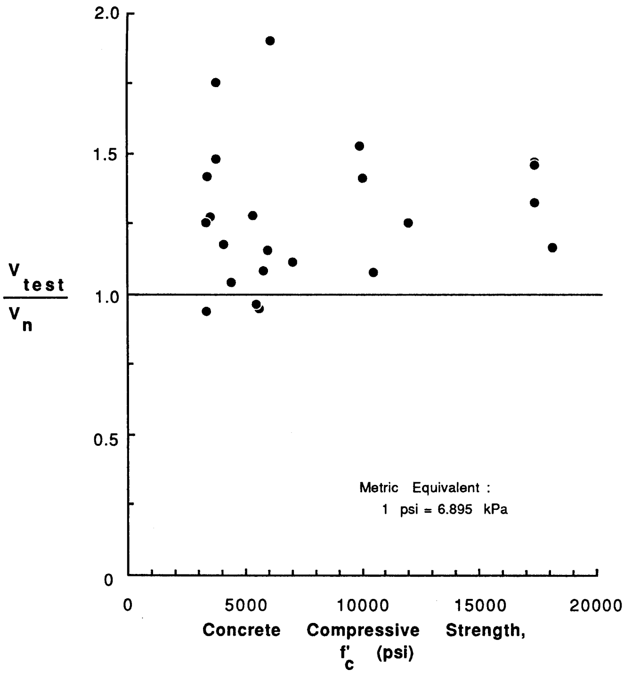


Fig. 10—Results from beams that failed in shear and met all design requirements of both ACI 318-83 and the new proposed code provision for minimum web reinforcement

these five beams have been plotted along with those beams from Fig. 2 that meet the requirements of the new proposed provision for minimum web reinforcement. Based on these results, it appears that the proposed provision for minimum web reinforcement is applicable.

CONCLUSIONS

The purpose of this investigation was to evaluate the shear-strength performance of high-strength concrete beams with web reinforcement relative to the current ACI 318-83 design provisions for shear. The following conclusions may be drawn based on the results of shear tests conducted on ten beam specimens with web reinforcement.

1. For nonprestressed members subject to shear and flexure only, the current ACI 318-83 code provisions overestimate the nominal shear strength provided by the concrete V_c when concrete compressive strength is greater than 17,000 psi (117 MPa).
2. The minimum quantity of shear reinforcement specified in ACI 318-83 code Eq. (11-14) needs to be increased as the concrete compressive strength increases to compensate for the evident lack of conservatism in the V_c term at high concrete compressive strength levels.
3. The proposed revision to ACI 318-83 requiring an increase in the minimum amount of web reinforcement for concretes with compressive strengths in excess of 10,000 psi (69 MPa) is appropriate.

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NOTATION

a = shear span, distance between concentrated load and face of support, in. (mm)
A_s = area of longitudinal reinforcement, in.^{2} (mm^2)
A_v = area of shear reinforcement within a distance s, in.^{2} (mm^2)
b_w = beam web width, in. (mm)
d = effective depth of beam, in. (mm)
f'_c = compressive strength of concrete, psi (MPa)
f_{sp} = splitting tensile strength of concrete, psi (MPa)
M_n = nominal moment at a given section
M_u = factored moment at a given section
s = spacing of shear reinforcement, in. (mm)
V_c = nominal shear strength provided by concrete, lb (kgf)
V_n = total nominal shear strength $V_c + V_s$, lb (kgf)
V_s = nominal shear strength from shear reinforcement, lb (kgf)
V_{test} = shear strength as determined by test, lb (kgf)
V_u = factored shear force at a given section, lb (kgf)
ρ_w = ratio of nonprestressed tension reinforcement = A_s/b_wd
ρ_b = reinforcement ratio producing balanced strain conditions

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