STRUCTURAL ENGINEERING

AND

STRUCTURAL MECHANICS

GRADUATE STUDENT RESEARCH REPORT

BEHAVIOR OF A CONTINUOUS PRESTRESSED CONCRETE SLAB WITH DROP PANELS

by

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Fall Semester 19 67 No. 309

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ABSTRACT

The elastic behavior and ultimate strength of a continuous concrete slab prestressed in two directions with drop panels were investigated. The slab was supported at nine points, simulating a flat slab. Prestressing was accomplished by means of unbonded post-tensioned cables. It was loaded uniformly by means of air pressure in plastic bags. Experimental values for deflations and reactions were compared with theoretical values obtained by approximate theories used in present design methods. Observed ultimate strength was compared to that obtained by the plastic hinge theory in conjunction with the approximate beam method as applied to prestressed slabs.

I. INTRODUCTION

1. Goals and Scope

Post-tensioned concrete floor slabs are becoming increasingly popular in new construction. They offer the advantages of thin, crack-free sections which are strong, economical, and aesthetically pleasing. However, because of the statically indeterminant nature of the slabs and the added complications of prestressing, most slab designs are based on approximate analyses.

Though such slabs have performed well in service, there are only a few tests which correlate the actual with predicted slab behavior. In 1956, Lin, Scordelis, and Pister,[1] reported on tests of a prestressed flat slab supported at four corners. These tests showed that correct elastic and yield line theories for slabs gave good indications of the cracking and ultimate loads respectively. In 1957, Lin, Scordelis, and May[2] developed a semi-emperical relationship for the allowable shear stress around column supports of prestressed slabs. Lin, Scordelis, and Itaya[3] analyzed and tested a slab supported on nine columns to investigate the effects of continuity. These tests showed close correlation between elastic theory, beam method,[4] and the actual behavior of the slab. In 1964, Frank Lu of the University of Canterbury, Cristchurch, New Zealand, conducted extensive tests on a 9-panel post-tensioned slab. Part of the results was described in a graduate student report at U. C., Berkeley.[5] These tests again showed close agreement of approximate and correct elastic analyses with the slab behavior.

The current investigation is closely patterned after the Lin, Scordelis, Itaya tests. The slab is of the same size, configuration, and tendon arrangement. The major difference is the inclusion of drop panels over the column supports. In addition, the contribution of the non-prestressed reinforcement over the supports to the ultimate strength of the slab will be considered. The writers feel that such an investigation will be useful because designers are beginning to use drop panels to improve the strength and behavior of long-span slabs.

This investigation has several objectives. The first is to determine if the beam method is applicable to slabs with drop panels. Since the ultimate load is an important design consideration, a second goal is to determine how drop panels affect the failure mode of the slab. Finally, the writers wish to gather sufficient data so that future investigators may compare other approximate or exact methods of analysis with the actual slab behavior.

2. Acknowledgements

The experimental program was conducted at the Structural Research Laboratory of the University of California's Richmond Field Station. The program was supported by faculty and student research grants from the Division of Structural Engineering and Structural Mechanics. The writers are indebted to George Hayler and Louis Trescony for their advice and help in concrete placement, curing and instrumentation respectively. We are thankful for Elwood Brown's help in the initial stages of the experimental work. We are also indebted to Roy Stephen and the staff of the Structural Research Laboratory for advice and aid. We are also thankful for the guidance provided by Professors T. Y. Lin, M. S. Polivka, and A. C. Scordelis.

II. EXPERIMENTAL PROGRAM

1. Test Specimen

The test specimen is designed to simulate an approximately one-third scale model of a four bay floor system. The over-all size is fifteen feet by fifteen feet. The slab is supported at nine points with a seven-foot span between the centers of supports. The slab is three inches thick and the drop panels have a thickness of three additional inches. The corner drop panels were eliminated to simplify forming and construction. (See Figure 1) Since they are not in areas of critical moment or shear, their absence should have little effect on the behavior. The slab was post-tensioned by a total of twenty-four $\frac{1}{4}$-inch diameter tendons with paper-covers spaced at fifteen inches center to center. At the start of testing each tendon had a force of about 7750 pounds. The tendon profile is shown in Figure 3. The drop-panels contain non-prestressed reinforcement in the form of $\frac{1}{4}$-inch diameter undeformed bars. The purpose of this reinforcement is to increase the shear and moment capacity of the panels. A steel plate with a bearing area of six inches by six inches was used at each support. A rocker and roller arrangement permits the necessary rotations and horizontal movements so that no restraints will be introduced at the supports. (See Figure 2.) The center support permits rotations about two axes but no horizontal movement.

2. Fabrication

The slab was formed and poured at ground level. The post-tensioning tendons and non-prestressed reinforcement were precisely held in position by chairs and wires. The concrete was delivered by ready-mix truck from a local supplier. Cylinders and flexure specimens were made at the time of casting. The slab and control specimens were cured for 11 days under wet burlap and plastic sheets, and then left to air-dry until testing. At the age of 18 days the slab was post-tensioned so that tendons were stressed to 170 ksi. At 21 days the prestress was checked and set at 155 ksi for all tendons. At 22 days the slab was lifted out of the forms by attaching cables to eye-bolts which were screwed into nuts cast into the slab at the supports. The slab was temporarily set along-side its original position while the forms were removed and 18-inch diameter by 36-inch high concrete cylinders were positioned to act as pedestals. The supports were then set on these pedestals and set to grade with the use of an engineer's level to simulate the original support elevations for the 9 support points. At 26 days the slab was lifted and set on the supports. The slab corners curled up under its own weight, probably due to the eccentric effects of the non-concordant cables. When the weight of the air bags and testing frame was added on top of the slab, the support points were in contact.

3. Materials

The concrete mix was designed to produce a compressive strength of 5,000 psi at 14 days, the anticipated age of testing. The mix contained 10.25 sacks of Type II cement. The water-cement ratio was 3.8 gals. per sack. The aggregate consisted of Pleasanton sand and gravel having a maximum site of 3/8 inches. Batch proportions by weight based on saturated surface dry conditions were: water, 0.34; cement, 1.00; sand, 1.55; gravel, 1.28. The fineness modulus of the sand was 2.93, gravel, 5.82. The slump was approximately 1 inch and concrete placement took approximately 1 hour. An admixture, Pozzolan 8, was added in the amount of 0.20 lb. per sack of cement. Control specimens were cured in the same manner as the slab. Three 6- by 12- inch test cylinders were tested at 7, 14, and 28 days. Two 6-inch by 6-inch flexural test specimens were tested by third point loading on an 18-inch span at 32 days to determine the modulus of rupture. The secant modulus of elasticity was determined from the average of two cylinders tested at 34 days. The results of these tests are shown in Table 1.

TABLE 1

Age of Concrete	7 d.	14 d.	28 d.	33 d.
Compressive Strength (psi)	5720	6720	7550	7975
Modulus of Rupture				453 psi
Modulus of Elasticity				3.86×10^6 psi

The post-tensioning tendons had a modulus of elasticity of 29.7×10^6 psi. The stress strain diagram is in Figure 7. The non-prestressed reinforcement had a modulus of elasticity of 29.0×10^6 psi and a yield strength of 40,000 psi.

4. Method of Loading

References 1 and 3 reported satisfactory results using an air bag pressing against a reaction frame to load the slab. This method was used in the present investigation since it was quite convenient to tie the reaction frame to the heavy anchor floor slab at the Structural Research Laboratory. Air pressure was measured by the use of a water manometer and a bourdon gage. With this arrangement the pressure could be measured to the nearest 5 psf.

5. Instrumentation

Due to the symmetry of the slab, electrical strain gages were placed on the top and bottom 1/8 of the slab in the arrangement shown in Figure 8. Strains in the tendons were to be determined from the electrical strain gages placed on them near the anchorage ends. Six typical gages were chosen for these measurements; however, the gages failed to function on three of these. The gages were placed on the tendons after the slab was cast; access to them was provided by block-outs. Since these block-outs are small and located at the farthest point from the instrumented quadrant, their effect on the slab behavior and data was assumed to be neglegible.

The support reactions were measured by pressure meters. Each pressure meter consisted of 6-inch diameter by 3/4-inch steel plates with a 1/16-inch oil film between them. The pressure of the oil film was measured by a standard pressure gage and the gages were calibrated prior to use. The center support required three such meters because of the limited range of the available pressure gages.

III. THEORETICAL STUDIES

1. Elastic Analysis

The elastic analysis was based on the beam method.[4] This method consists of considering the entire width of the slab as a beam supported on continuous knife-edges in one direction when the total moment is analyzed in the perpendicular direction. The usual design criterion for post-tensioned slabs is based on allowing no tensile force in the concrete. Based on a force of 7750 lbs. in each tendon, the design load for this slab was 112 psf. Using the same effective tendon force and a modulus of rupture of 453 psi, the load at which cracking started at the top of the center support line was computed to be 337 psf.

The deflection at the center of a bay may also be approximated by the use of the beam method. The process consists of superimposing the deflections of a beam across the column supports and a beam in the opposite direction consisting of a one-foot strip midway between the columns. By this method the deflection was calculated to be 996 w/E. Where w is the uniformly distributed load in psf and E is the modulus of elasticity of concrete in psi, and the deflection is in inches.

The reactions were also determined by use of the beam method. The reactions, expressed as a part of the total load on the slab, W, are 0.435 W for the center support, 0.112 W for the side supports and 0.029 W for the corner supports.

2. Ultimate Load Analysis

Based on the beam method and the plastic hinge theory, the ultimate load was calculated to be 500 psf. According to this analysis, hinges should form over the center support line and 2.45 feet from the side support lines. This load was based on the following assumptions: a rectangular concrete stress block at failure, ultimate concrete strain of 0.0034, maximum tendon stress of 200 ksi at failure. The calculated reactions near the ultimate load were 0.384 W, 0.118 W, and 0.036 W at the center, side, and corner supports respectively. An analysis of the ultimate shear capacity of the slab indicated that shear failure should not be a problem at loads below 650 psf.

IV. EXPERIMENTAL RESULTS

1. Loading

Measurements were first taken in approximately 50 psf increments up to a load of 200 psf. Pressure was then released from the air bags. Next the load was increased in 100 psf increments to a load of 300 psf, at which point it was increased by 50 psf increments until the first visible cracking at the bottom of the slab occurred at 434 psf. Load was increased to 500 psf when most of the bottom cracks developed. The pressure was then released until the cracks almost closed at 145 psf. Load was then increased until the slab failed in flexure at 580 psf. The slab was still able to sustain additional load until it failed in combined shear and flexure at 612 psf. The crack patterns on the bottom and top of the slab appear in Figures 14 and 15 respectively. In Figure 14, the numbers indicate the sequence in which the cracks appeared at the bottom. This crack pattern is almost the same as that predicted by the ultimate load analysis.

The slab failure was interesting in many ways. The western half of the slab (See Figure 14.) exhibited the most severe cracking and deflections. The maximum deflection measured at the north-south crack on that half of the slab was about 3 1/4 inches. After the slab failed and the live load was released, this deflection was reduced to about 1 1/4 inches. This unsymmetrical failure might have been the result of a slight leak observed in the air bag on the northeast quadrant.

A second interesting occurrence was the failure of a button on the jacking end of the third tendon from the northern edge of the slab. When the tendon failed at a load of approximately 600 psf, the anchorage end of the tendon pierced a 3/4 inch sheet of plywood and did not stop until approximately 2 feet of the tendon was projecting out of the concrete. An examination of the button indicated a cup-cone type failure with the tip of the cone about 1/4 button radius within the button. Such a fracture indicates failure was caused by a stress concentration around the button rather than a complete tensile failure of the tendon itself. Such a failure not only demonstrates the safety hazards involved in the testing of unbonded tendons, it also indicates a potential source of weakness in the type of tendons used in this test.

A combined shear and flexure failure around the center support was the source of the ultimate failure of the slab. Cracks occurred on the top of the slab at a location approximately above the edge of the drop panel. The cracks were at an angle of about 25° from the horizontal.

The predicted failure load was 500 psf; however, the slab actually failed in flexure at 580 psf. The broken tendon indicates the maximum tendon stress may have exceeded the assumed value of 200 ksi. If we assume a maximum tendon stress of 230 ksi at failure, the calculated ultimate load becomes 562 psf. This value is within 3 1/2 per cent of the test value. Such a deviation is well within the inaccuracies inherent in experimental work, the assumption of a rectangular stress block for concrete, and the beam method of analysis.

Table II compares the behavior of the present slab with that of the slab in Reference 3. The only significant modification in the present slab is the addition of drop panels and the slightly higher tendon stress (about 15 per cent) in both the elastic and ultimate range. This table shows that in this case, drop panels increased the over-all average depth by about 15 per cent, whereas, the cracking and ultimate loads were increased about 25 and 40 per cent respectively.

2. Strain Gages

The most unfortunate occurrence during this study was the loss of strain gage data. The automatic recording apparatus which the writers planned to use could not be repaired in time for the test. At the completion of the testing, the alternate, manual recording device was found to be malfunctioning, and as a result, all strain measurements were useless.

3. Deflections

Up to a load of 400 psf, the center deflections varied linearly with the load. (See Figure 9.) Because of this linear variation it is easy to compare the calculated and measured deflections at a given load of 100 psf. Using the measured value of E in the expression for the deflection gives 0.026 inches at the center of the bay. The measured value was 0.042 inches. This difference reflects the

①	②	③	④	⑤	⑥	⑦	⑧	⑨	⑩
		PREVIOUS VALUES			PRESENT VALUES				
NO.	PARAMETER	PREDICTED	ACTUAL	ACTUAL / PREDICTED (%)	PREDICTED	ACTUAL	ACTUAL / PREVIOUS (%)	COL. ⑦ / COL. ④ %	APPX. POSSIBLE % OF ACTUAL PRESENT TO PREVIOUS VALUES IF PARAMETERS V, VI, VII WERE SAME AS BEFORE
I	DESIGN LOAD	76 psf	—	—	112 psf	—	—	—	—
II	CRACKING LOAD	194 psf	330 psf	174	337 psf	434 psf	129	132	125 %
III	ULTIMATE LOAD	347 psf	362 psf	104	500 psf	580 psf	116.0	160	142 %
IV	DEFL. @ FIRST CRACKING	.17"	.2"	115	.30"	.40"	135	200	—
V	MAX. DEFL. BEFORE FAILURE	—	2"	—	—	3.25"	—	163	—
VI	EFF. PRES./TENDON	—	6840 lbs.	—	—	7750 lbs	—	113	—
VII	MAX. STRESS [illegible]	—	200 ksi	—	—	235 ksi	—	117.5	—
VIII	CONC. STR.	—	6000 psi	—	—	8000 psi	—	133	—

TABLE II

approximate nature of the beam method. Due to the large number of uncontrolled factors in a design situation, such an error is acceptable if the designer realizes the qualitative nature of the estimate.

The deflected shapes for various cross-sections at 100 and 200 psf are shown in Figures 10 and 11. These shapes may prove to be of use for comparison with values found by an exact elastic analysis.

4. Reactions

The measured reactions at different supports for various loads are shown in Figure 12. These values tend to come close to the predicted values in the elastic range. The variations may be due to some slight unsymmetry in the specimen. Due to the plastic load redistribution, the reactions in the ultimate range tended to be much closer to the predicted values. (See Figure 13.)

V. SUMMARY AND CONCLUSIONS

The behavior of a 15 x 15 foot prestressed concrete slab, 3 inches thick, supported at 9 points with drop panels an additional 3 inches thick around the supports was studied experimentally and by approximate theoretical analysis. Loading conditions were that of a uniformly distributed load over the entire slab.

Using the beam method of analysis the design live load based on no tension in the concrete was computed to be 112 psf. The live load for cracking based on a modulus of rupture for concrete of 453 psi, was 337 psf. The ultimate live load based on a maximum tendon stress of 200 ksi was 500 psf. The actual behavior proved to be ideal for practical applications of this type of slab. Due to the loss of strain gage data, there was no way of determining when the first crack occurred at the top surface. However, cracking at the bottom surface was first observed at a live load of 434 psf. The maximum deflection at that time was about 0.40 inches. The slab failed in flexure at 580 psf and complete failure of combined shear and flexure occurred at a live load of 612 psf. Just prior to complete failure, the maximum deflection was 3 1/4 inches.

Based on this study the following conclusions were made:

1. The beam method of analysis may be used to satisfactorily predict the elastic and ultimate behavior of a post-tensioned concrete slab with drop panels.

2. For practical applications, much more significance should be given to the ultimate load than to the cracking load since the slab can sustain much additional load and deflection between cracking and complete failure.

3. Comparing the behavior of the present slab with that described in Reference 3, the only significant modification being the addition of drop panels, the cracking and ultimate load carrying capacity of the slab were increased significantly.

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5. Gupta, B. S., "An Analysis of a Continuous Flat Slab Prestressed in Two Directions", Graduate Student Research Report, Div. of S. E. S. M., University of California, Berkeley, 1966.

FIGURES

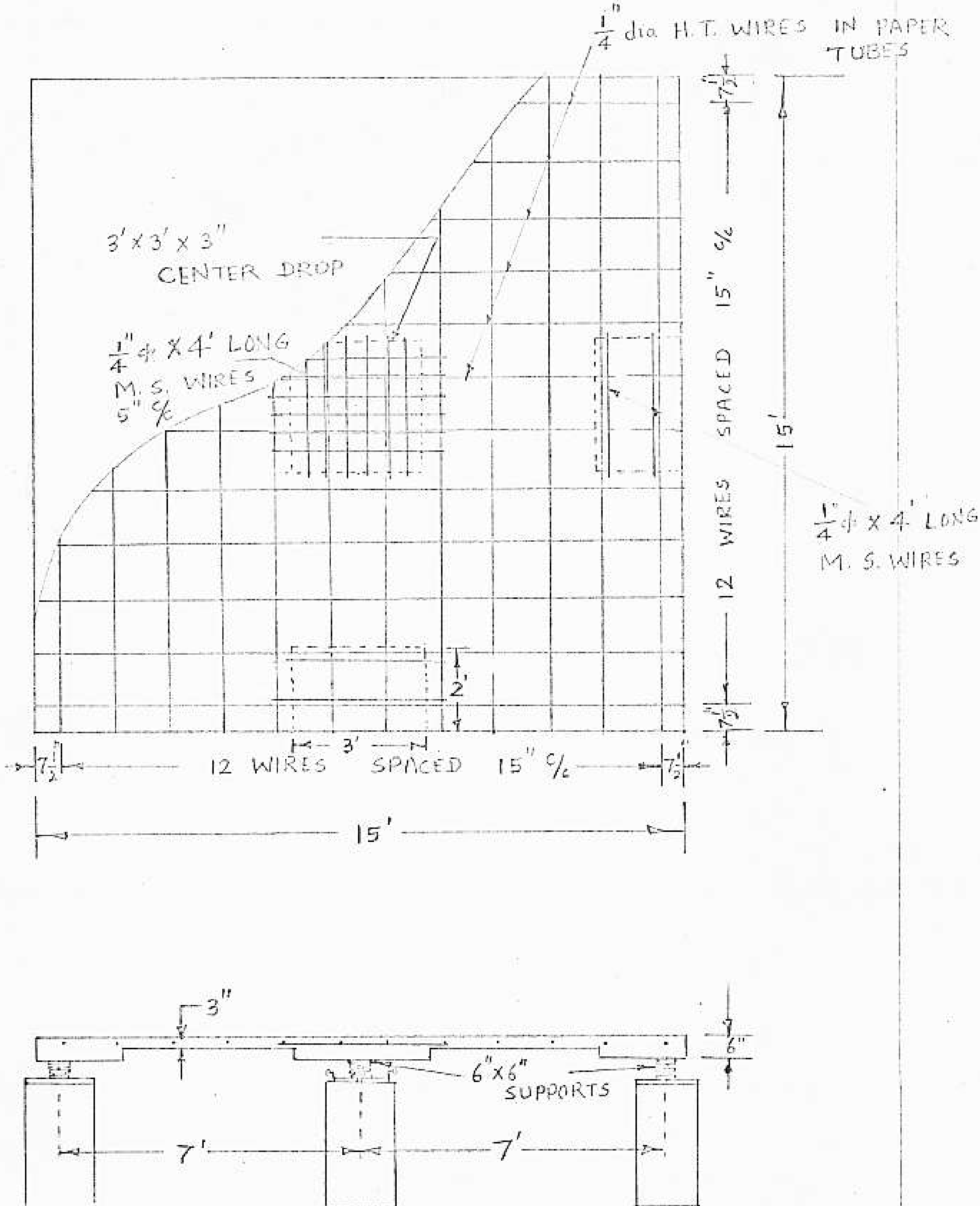


FIGURE 1. PLAN AND ELEVATION OF SLAB SHOWING STEEL ARRANGEMENT

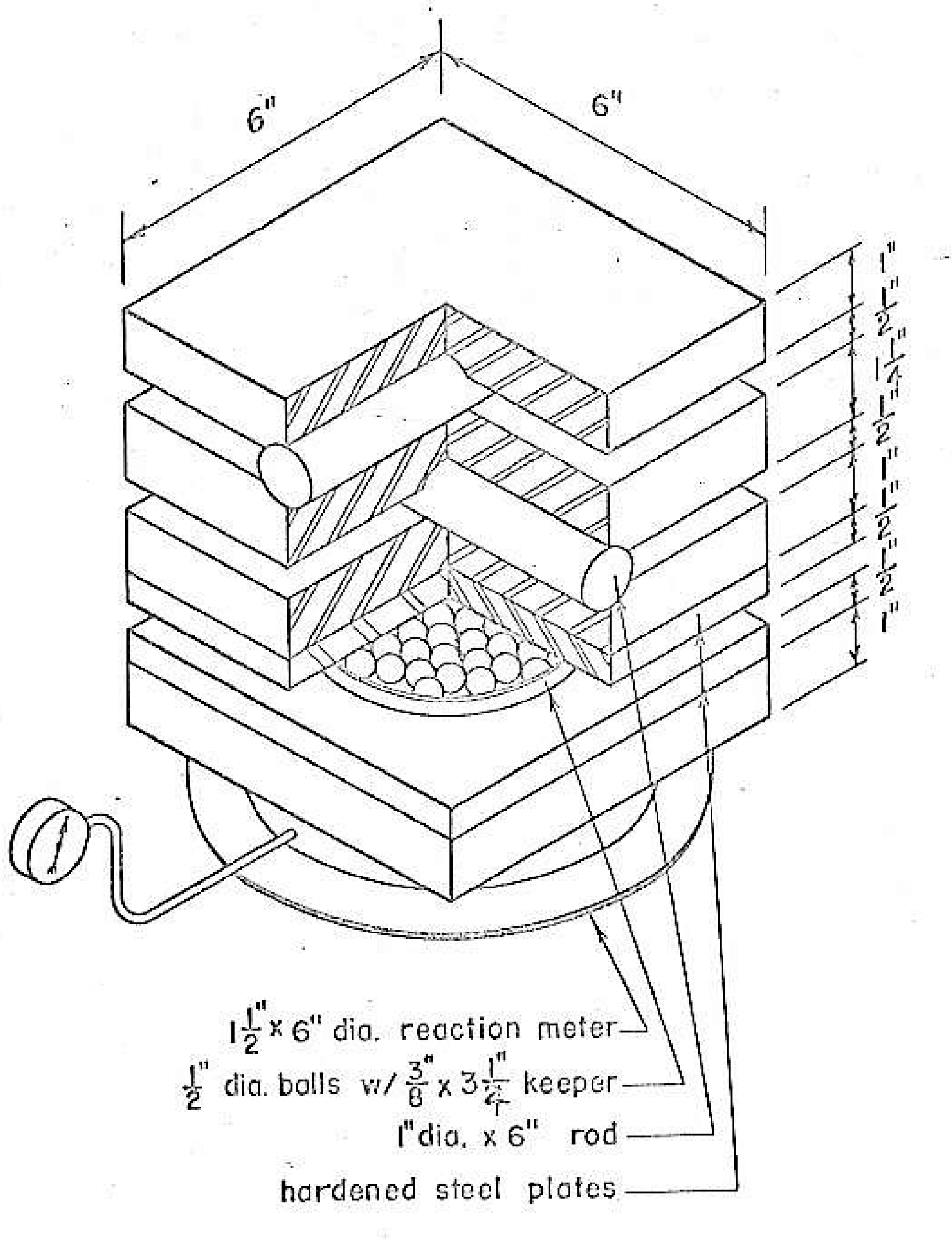


FIGURE 2. DETAIL of SUPPORT ASSEMBLY

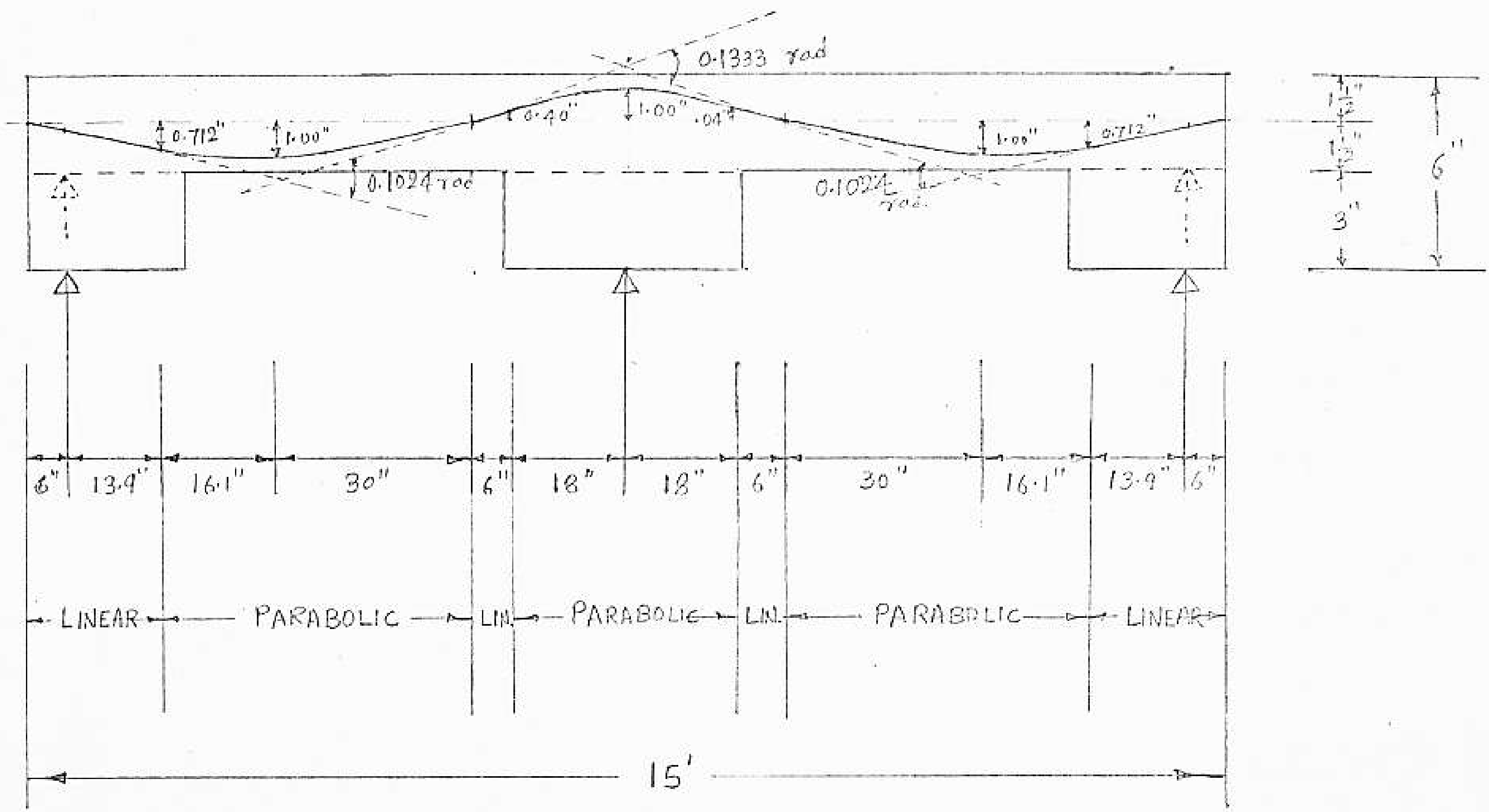


FIG 3 TYPICAL CABLE PROFILE

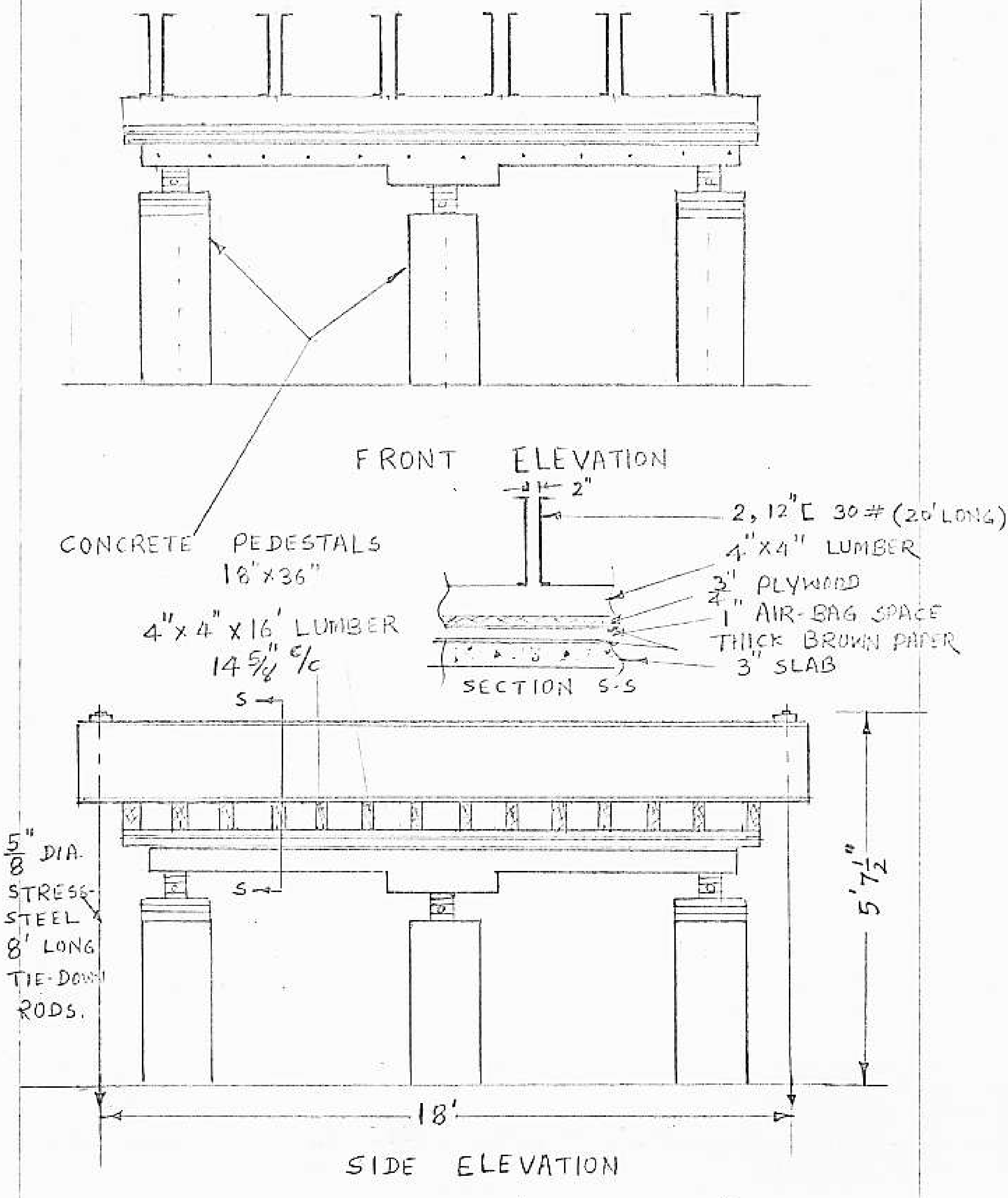


FIGURE 4. LOADING FRAME

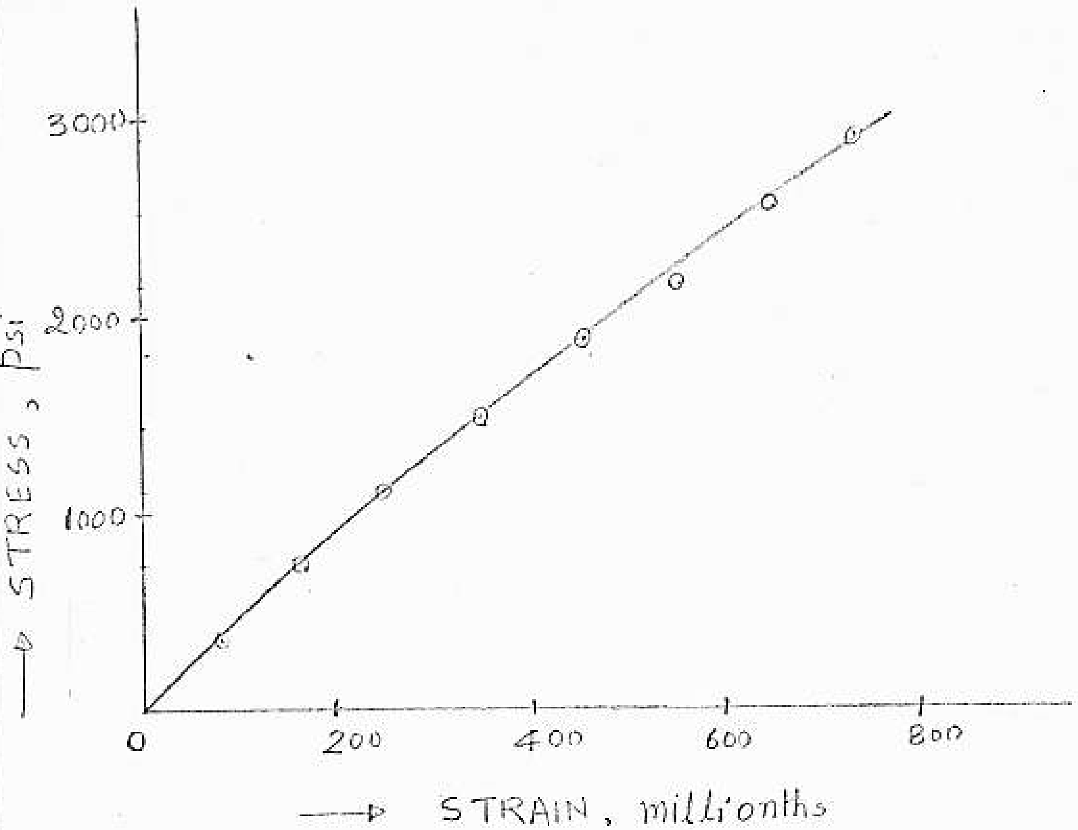


FIG 5. TYPICAL STRESS-STRAIN CURVE FOR 6"x12" CONCRETE CYLINDER AT AGE 34 DAYS.

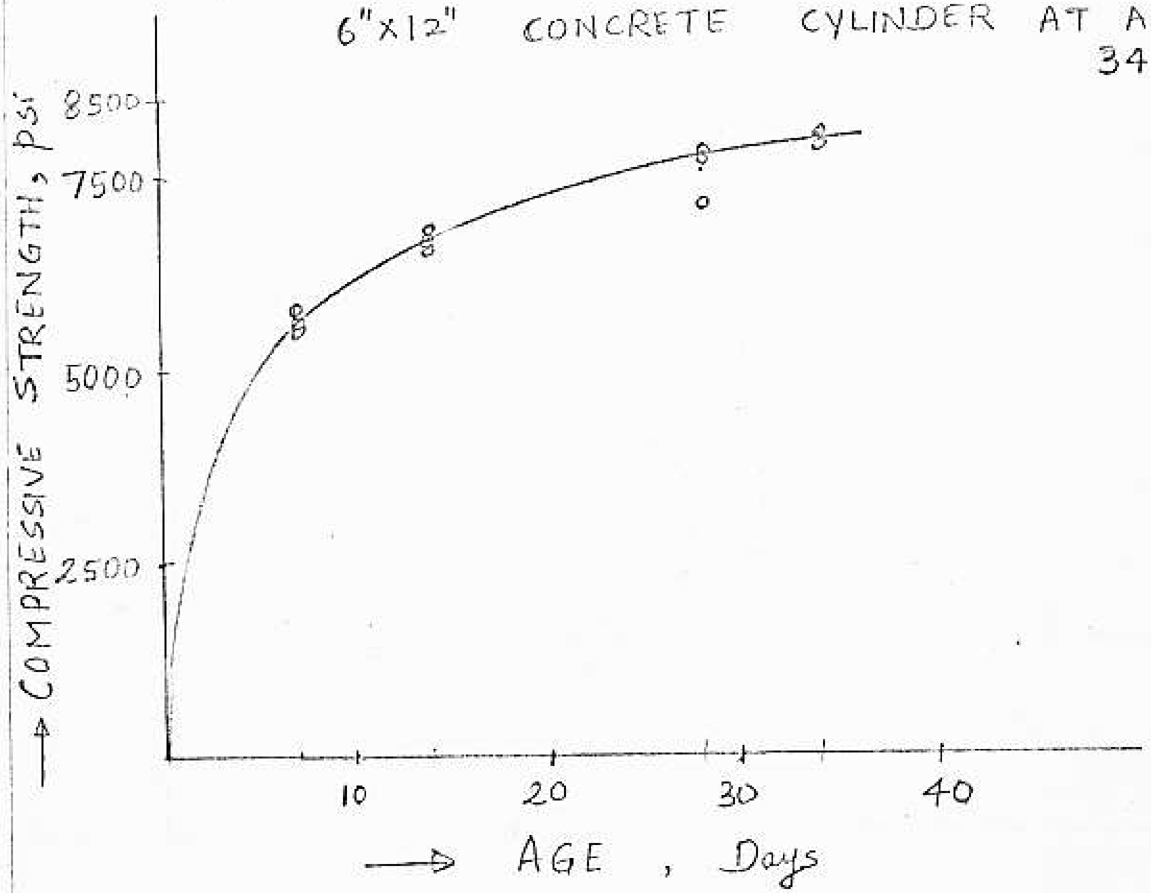


FIG 6. AGE-COMPRESSIVE STRENGTH RELATIONSHIP FOR 6"x12" CONCRETE CYLI.

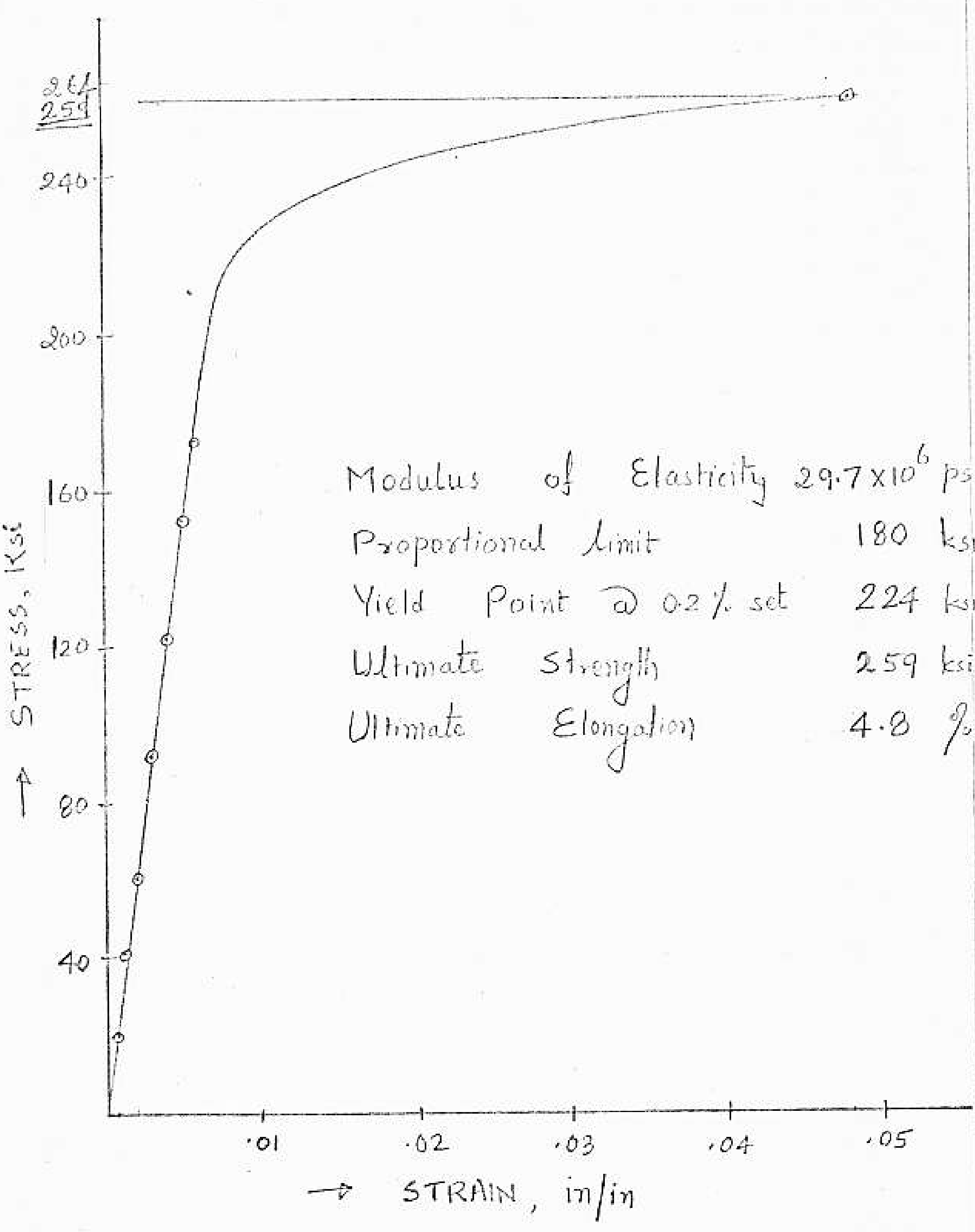


FIG 7. TYPICAL STRESS-STRAIN CURVE FOR $\frac{1}{4}$" DIA. PRE-STRESSING STEEL.

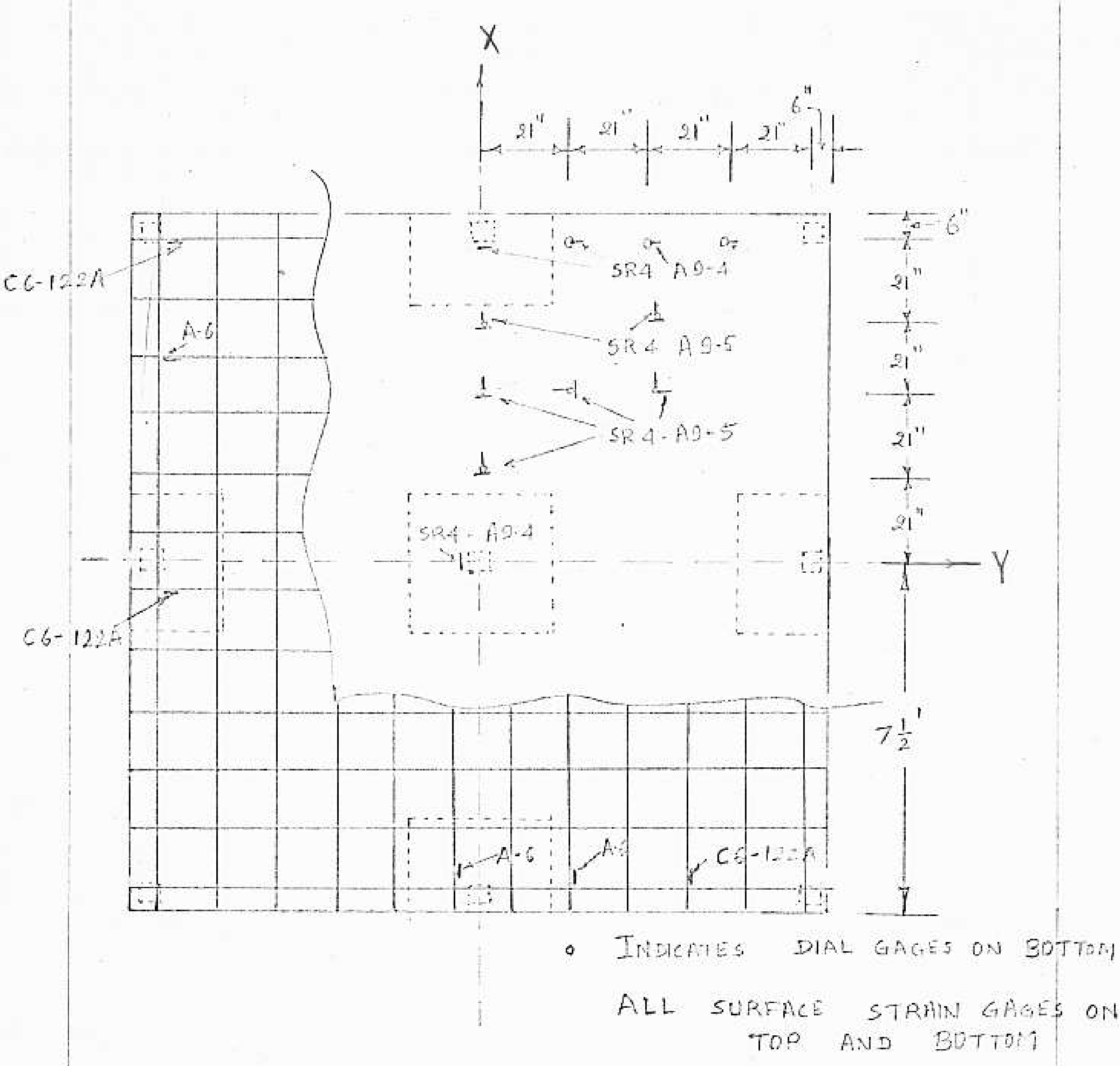


FIG. 8. LOCATION OF GAGES

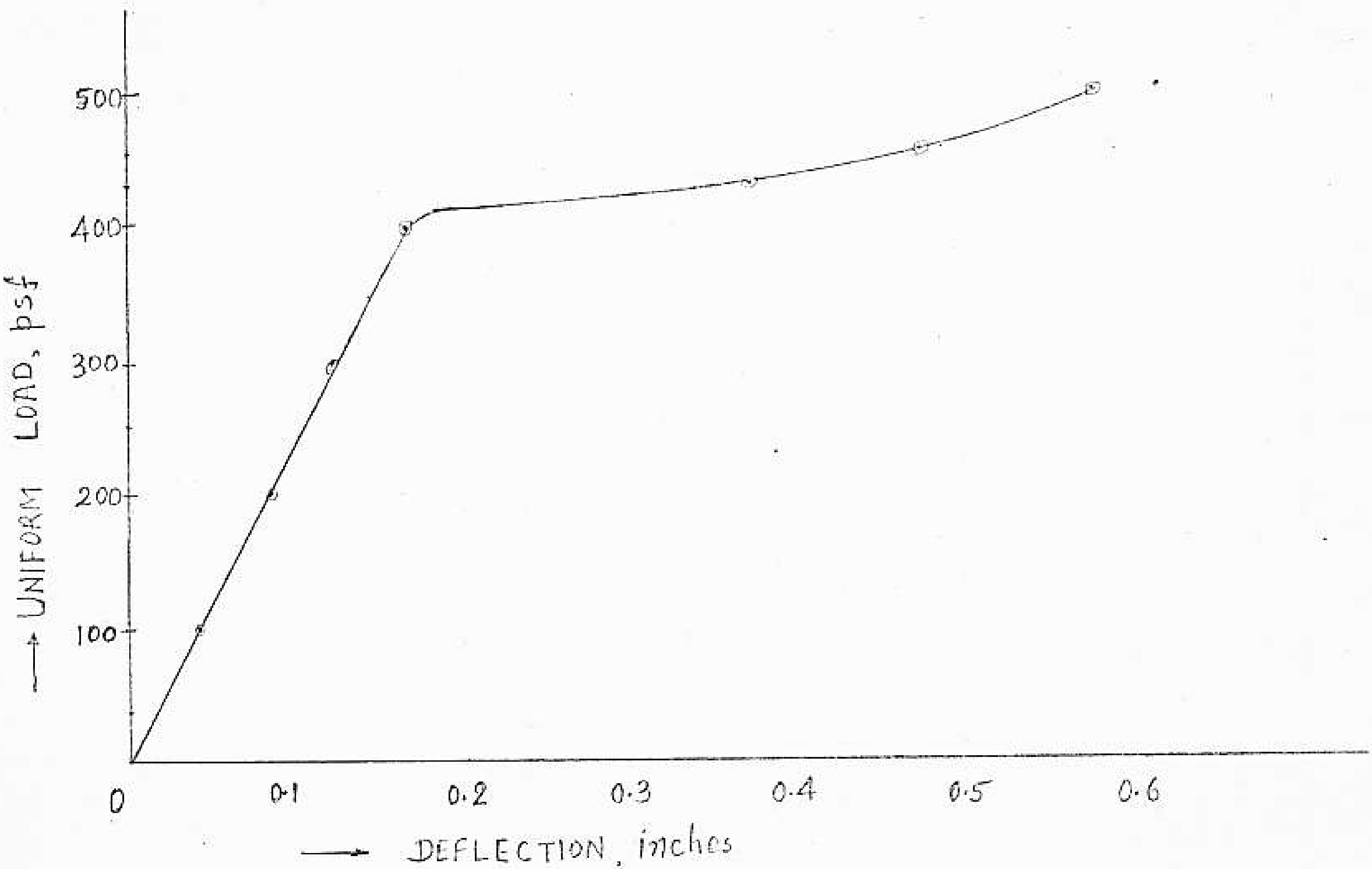


FIGURE ~~8~~ 9. UNIFORM LOAD-DEFLECTION CURVE FOR PANEL-CENTER.

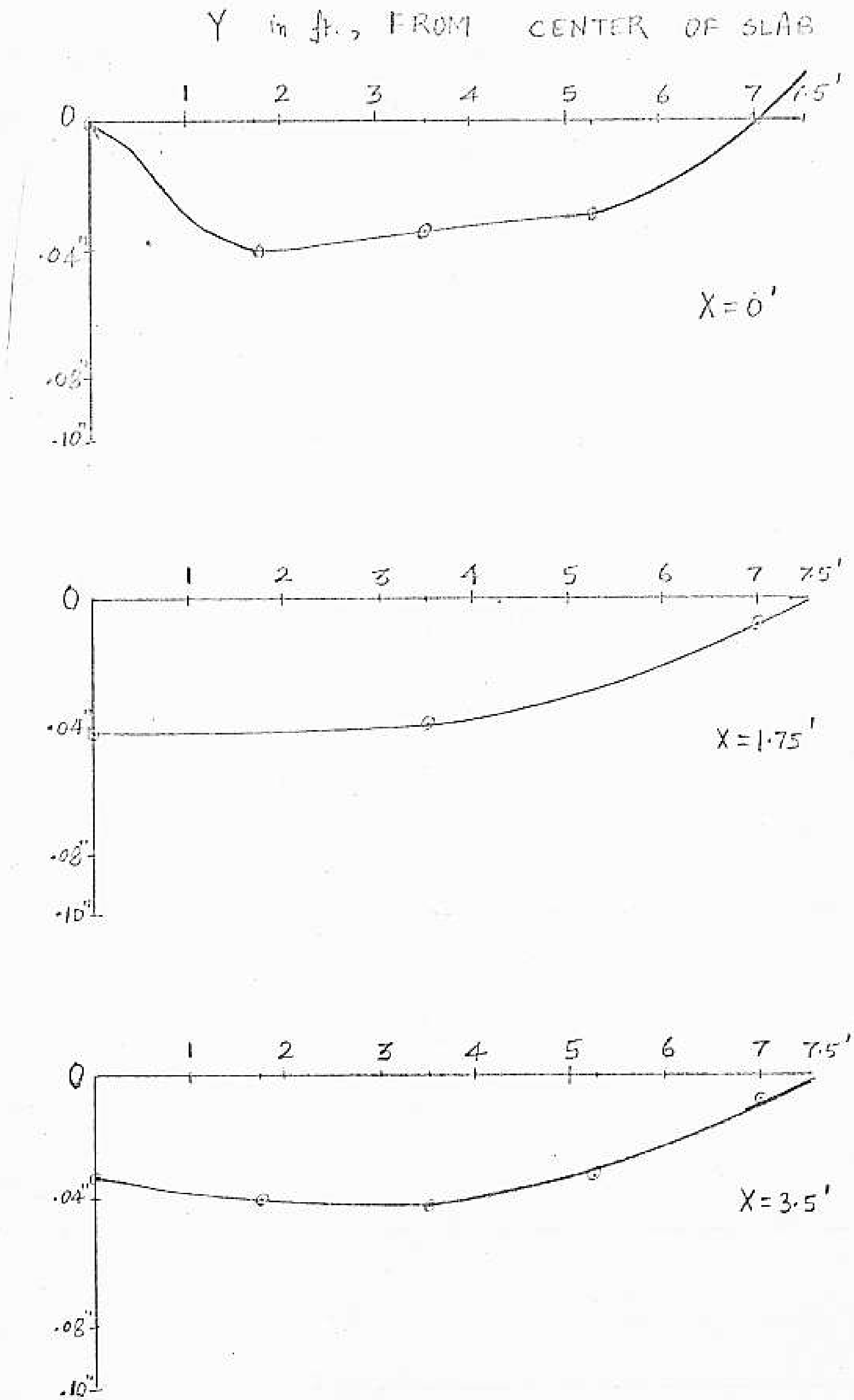


FIG 10. DEFLECTED SHAPES FOR UNIFORM LOAD OF 100 psf.

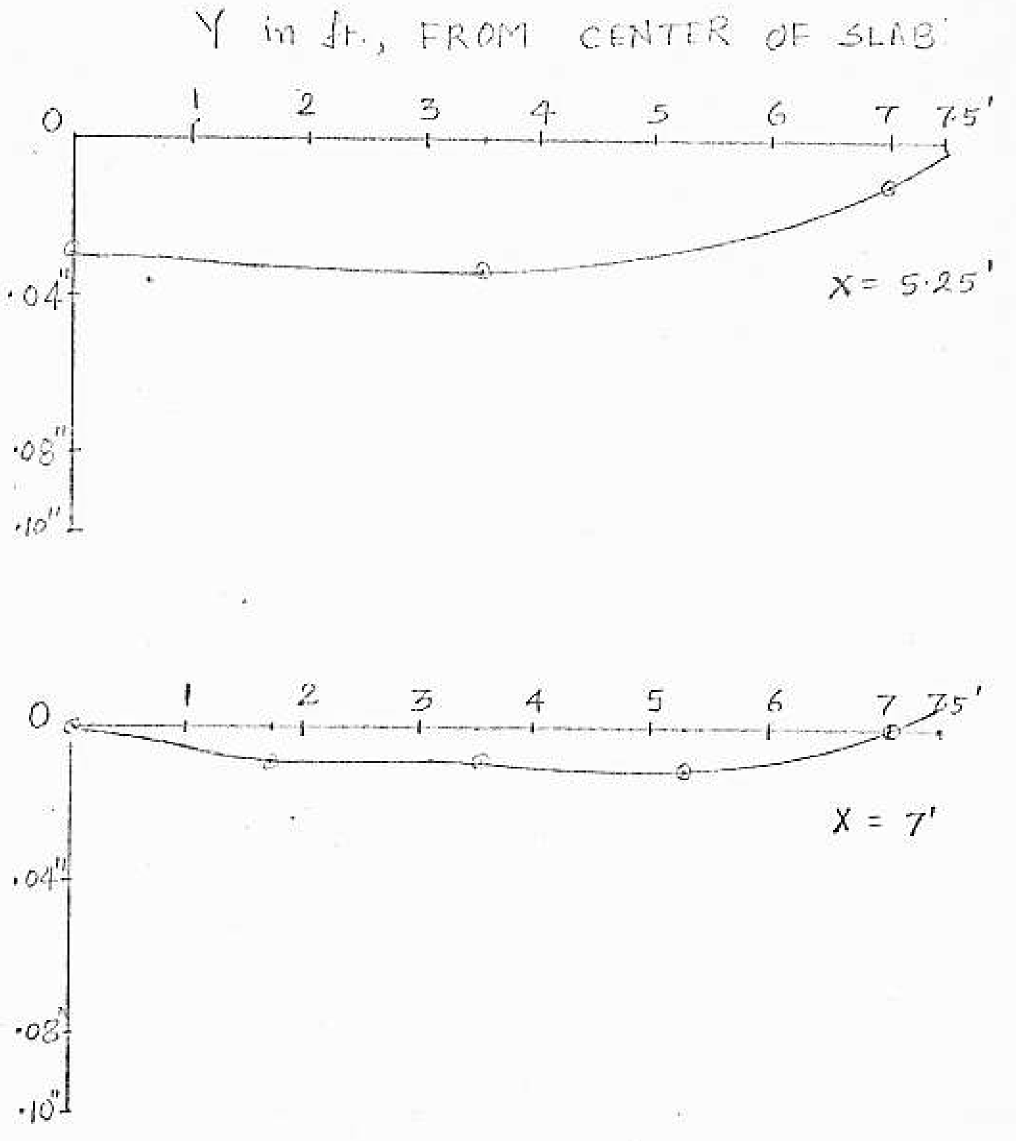


FIGURE 10. (CONT.) DEFLECTED SHAPES FOR UNIFORM LOAD OF 100 psf.

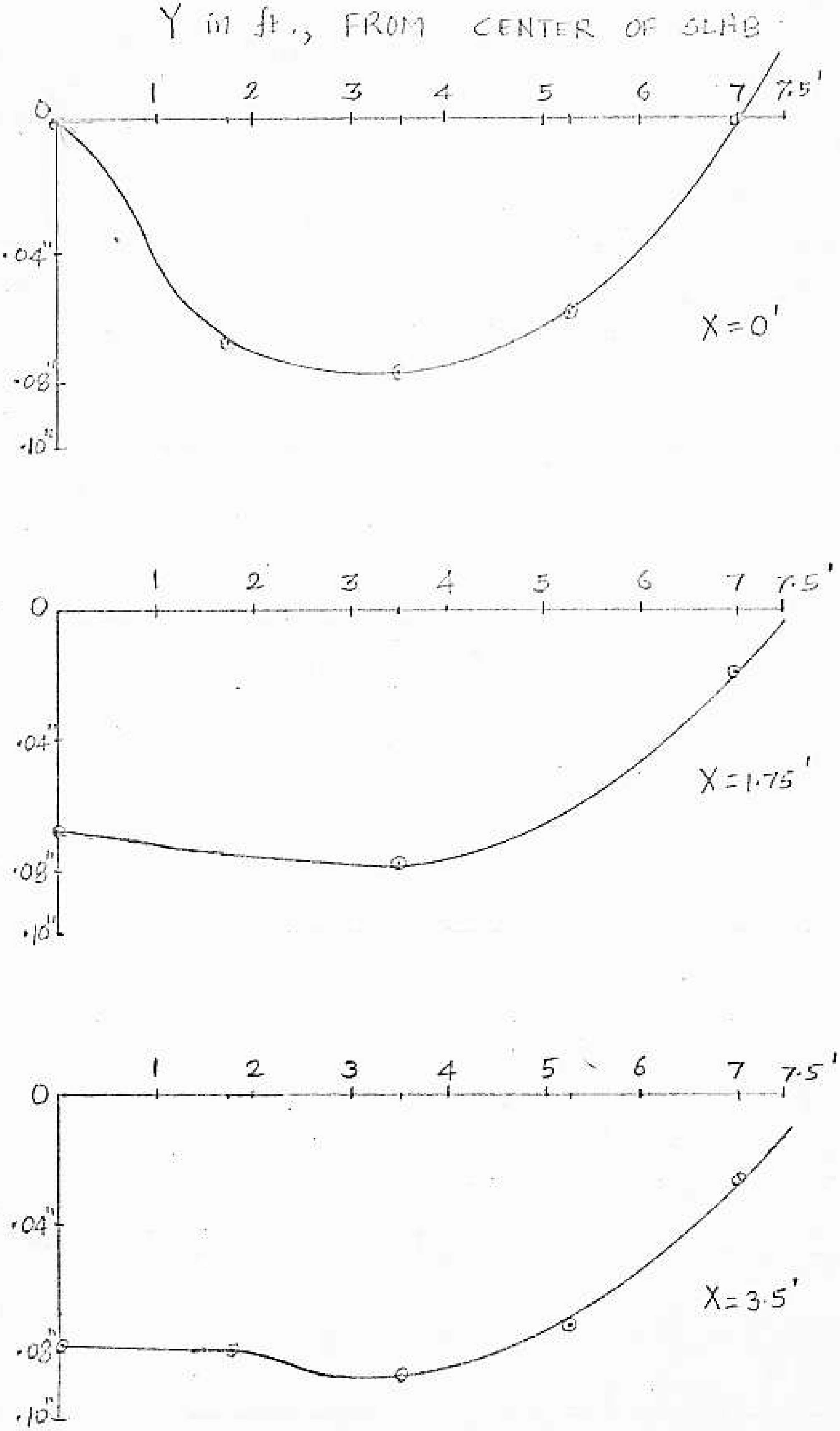


FIG 10 11. DEFLECTED SHAPES FOR UNIFORM LOAD OF 200 psf.

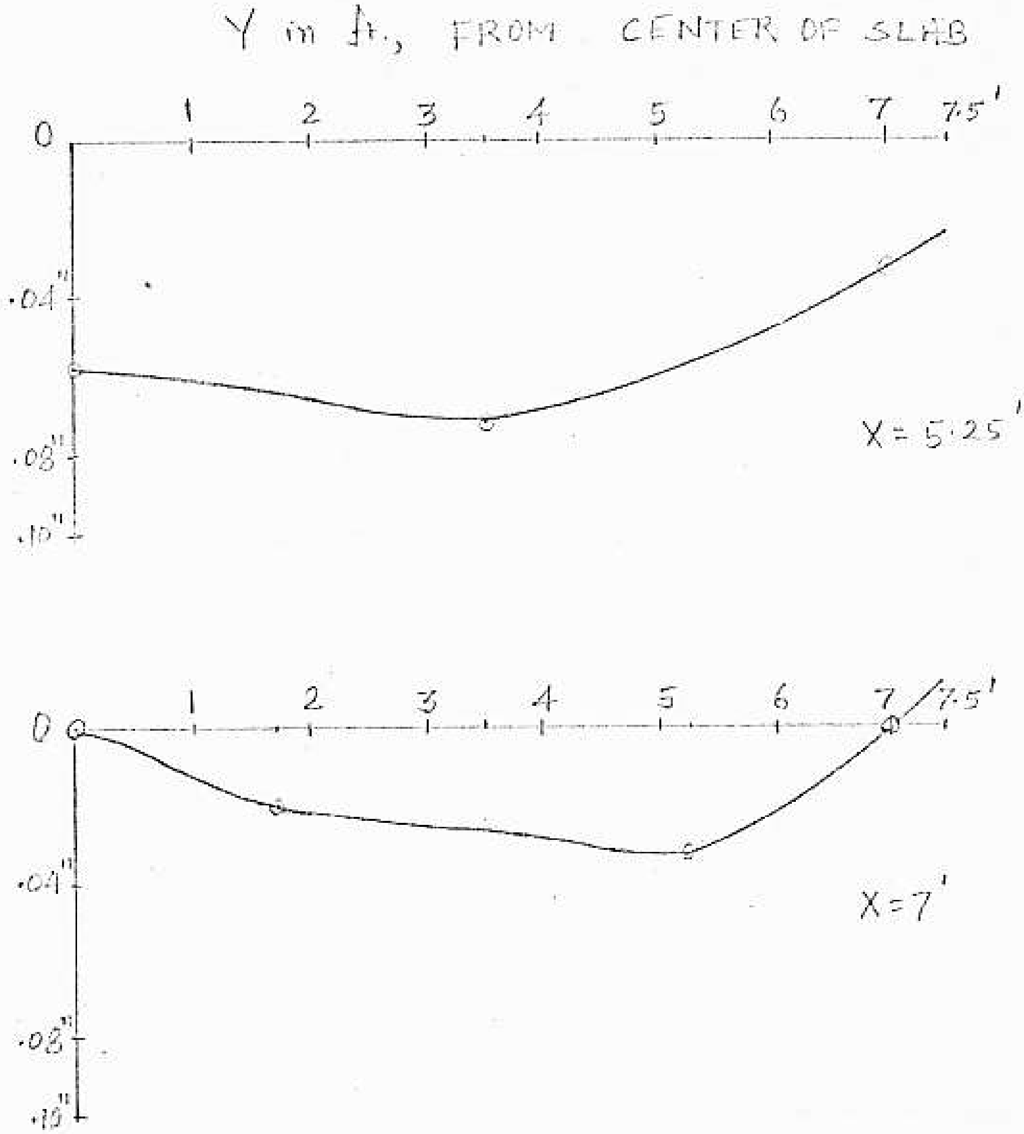


FIGURE 10.(CONT.) DEFLECTED SHAPES FOR UNIFORM LOAD OF 200 psf.

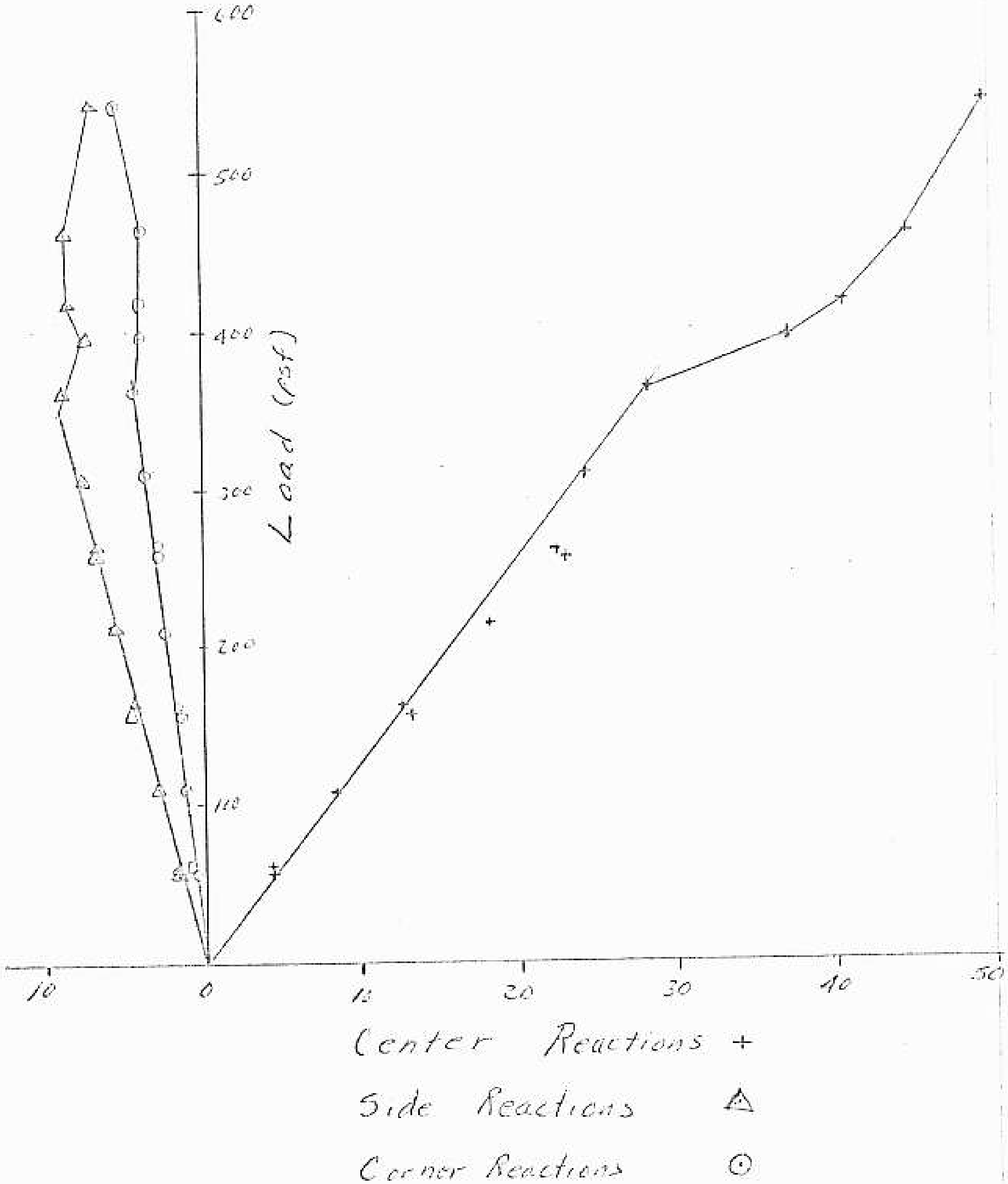


FIGURE 12 MEASURED REACTIONS

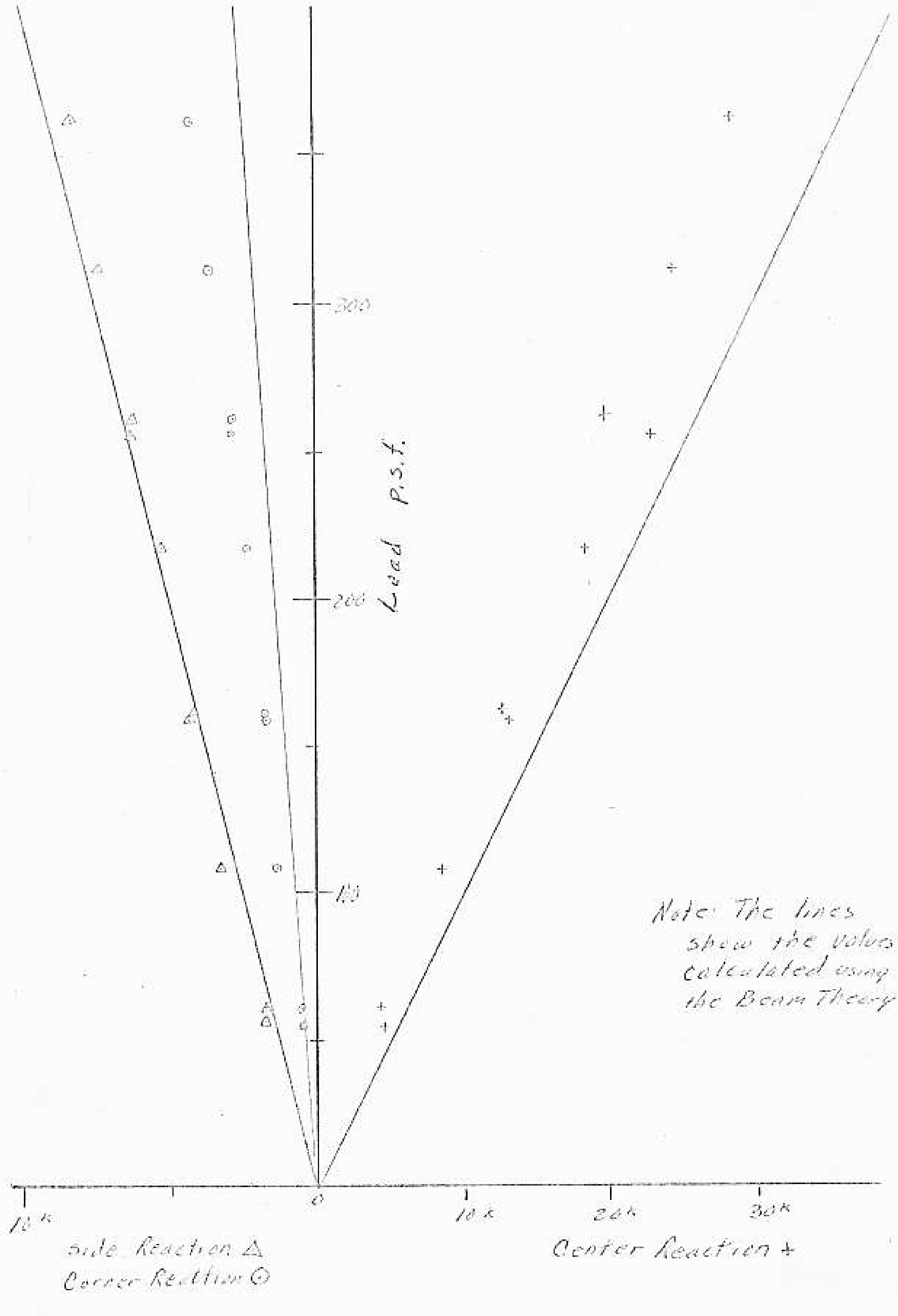
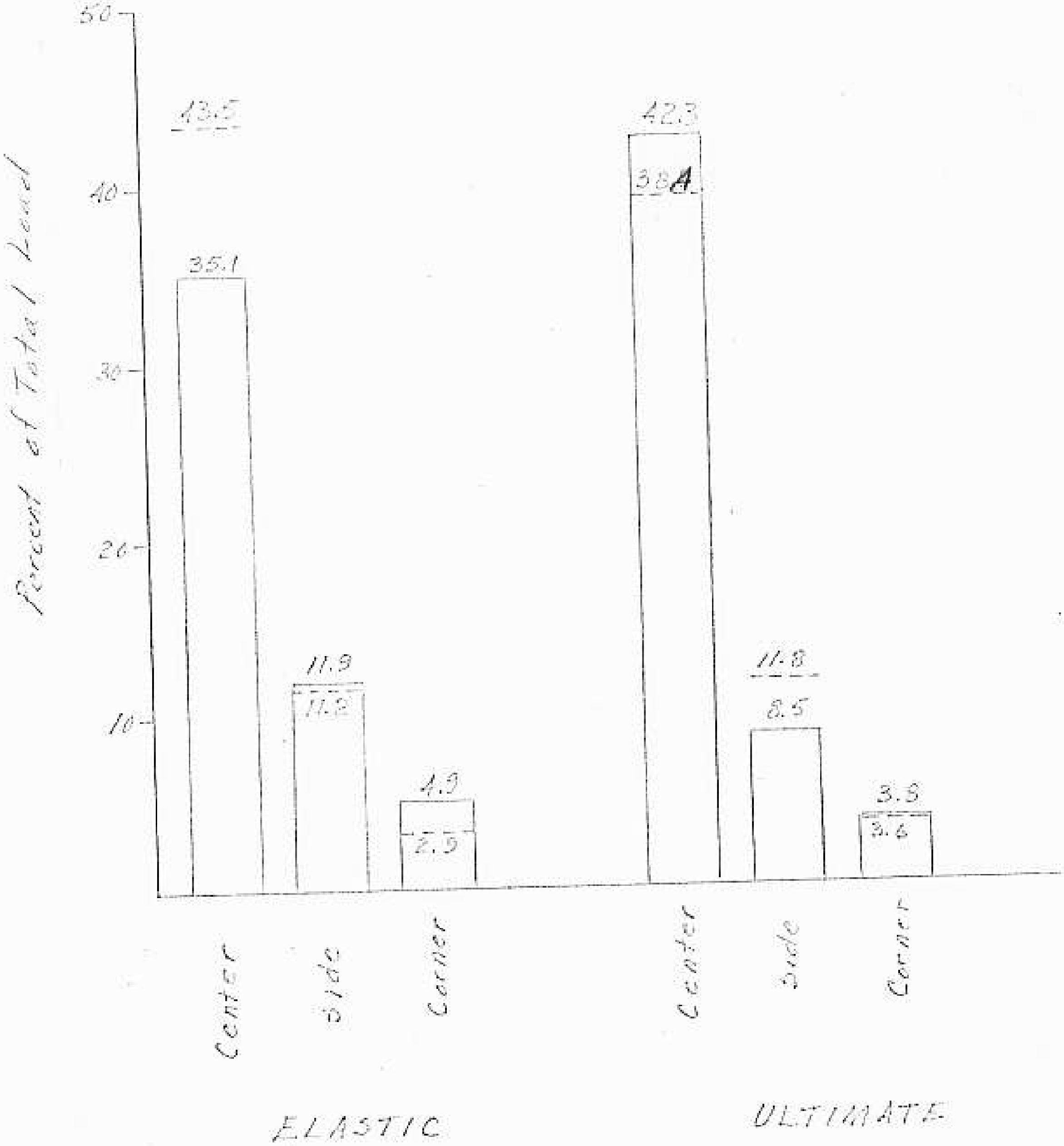


FIGURE 12.a MEASURED REACTIONS AND CALCULATED VALUES



Note: Dotted lines show values calculated by Beam Theory

FIGURE 13 LOAD DISTRIBUTION

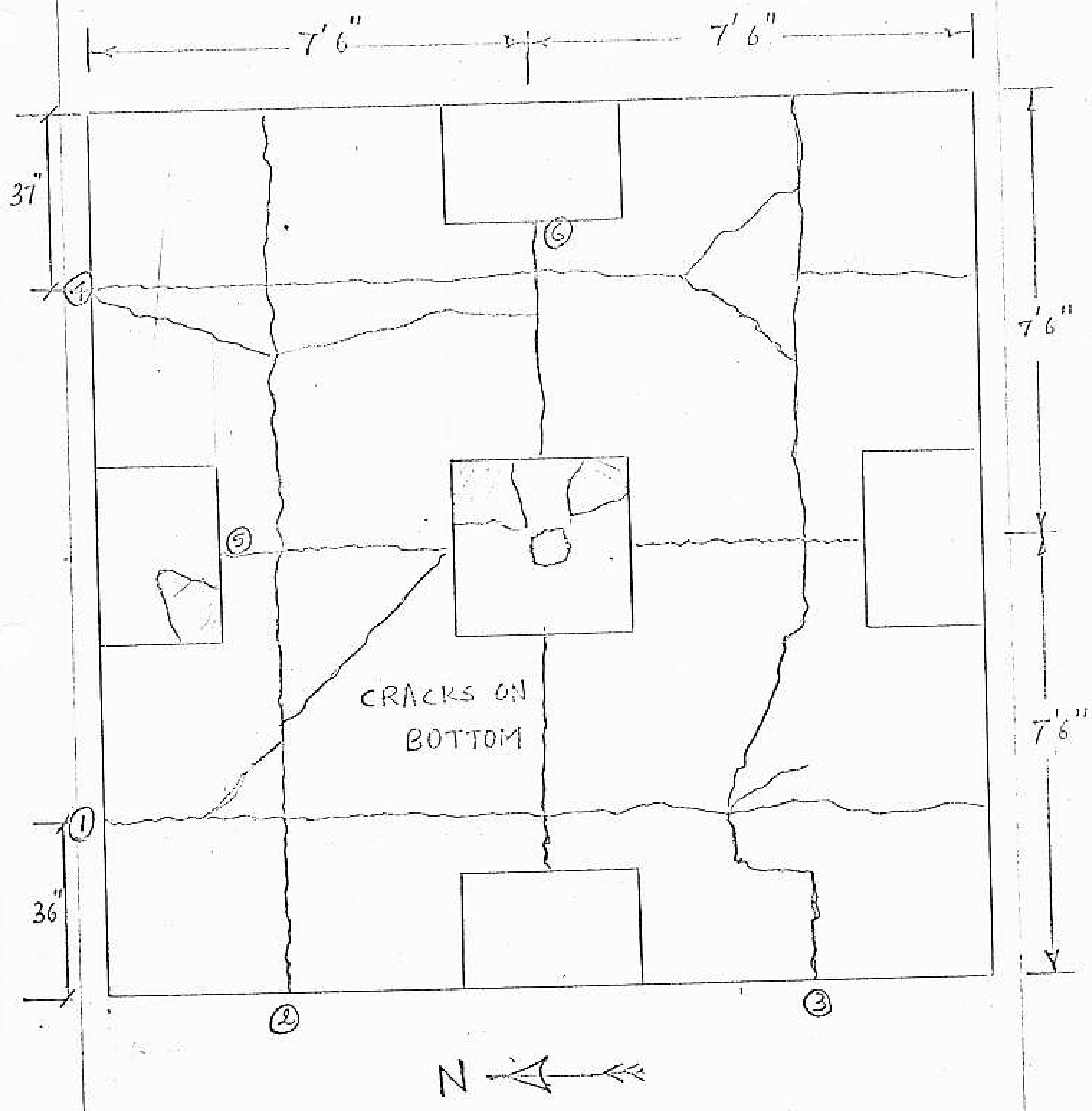


FIG 14. CRACK PATTERN at FAILURE (q_{LL} = 612 psf) for UNIFORM LOAD on ENTIRE SLAB.

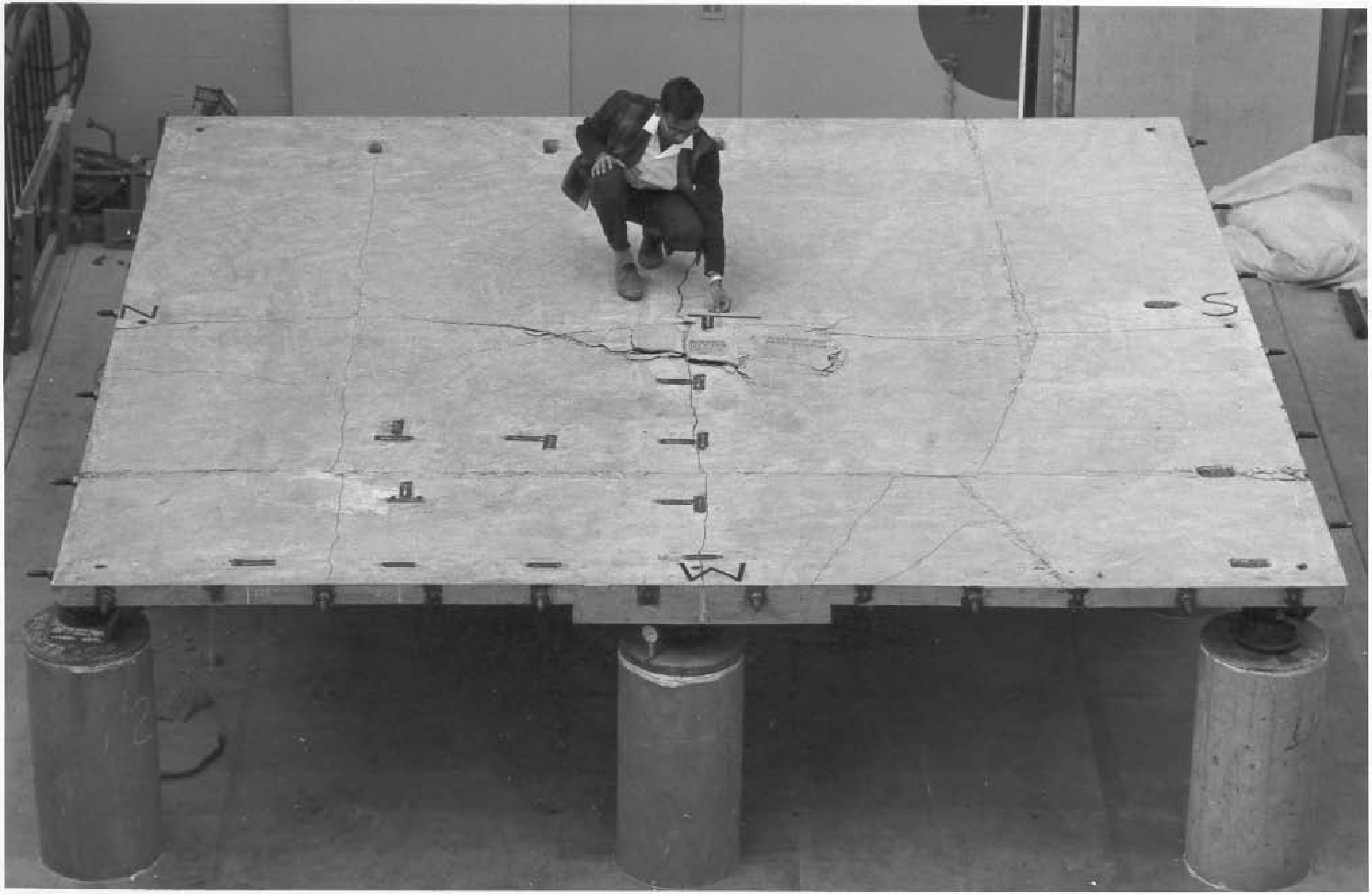
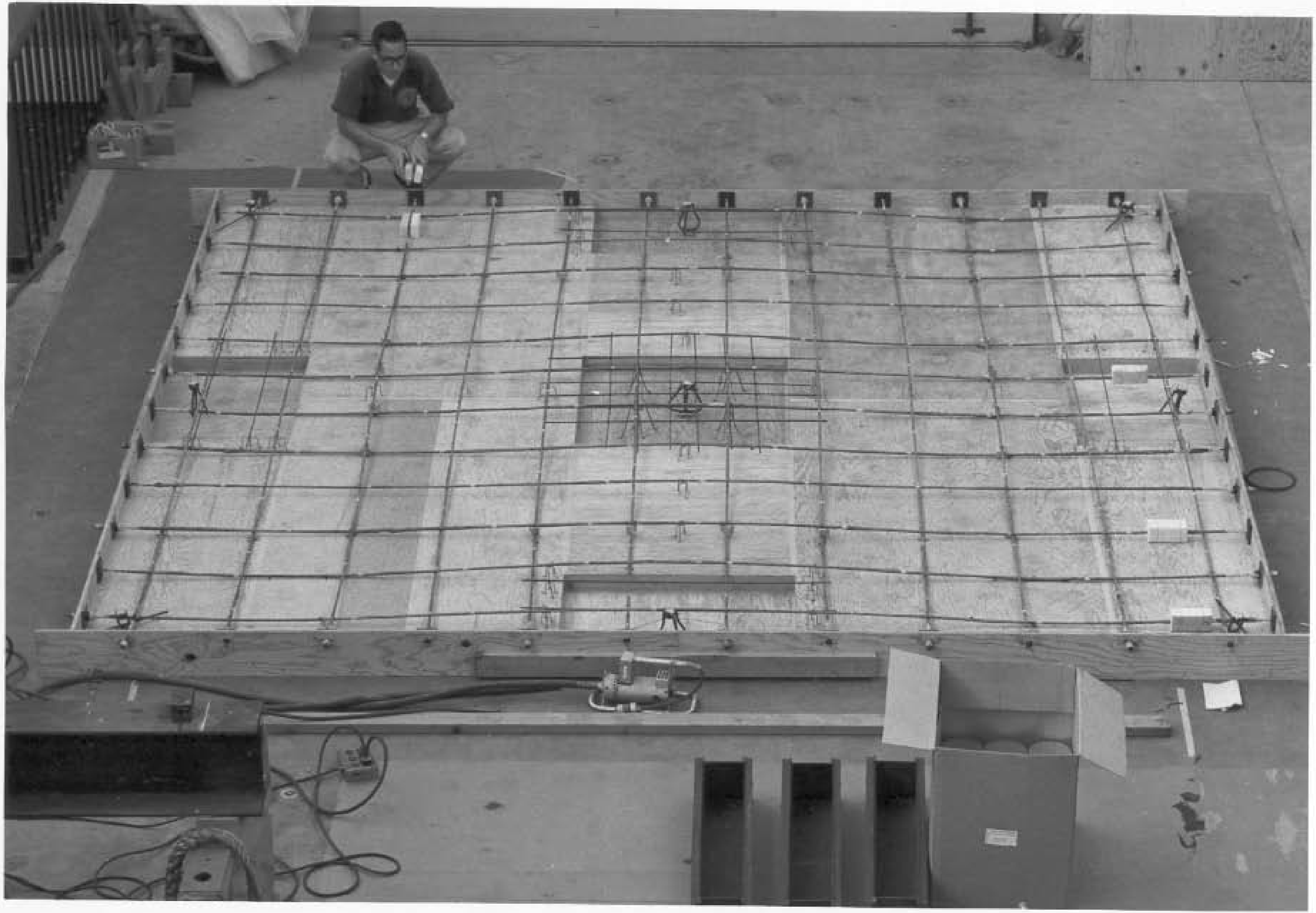
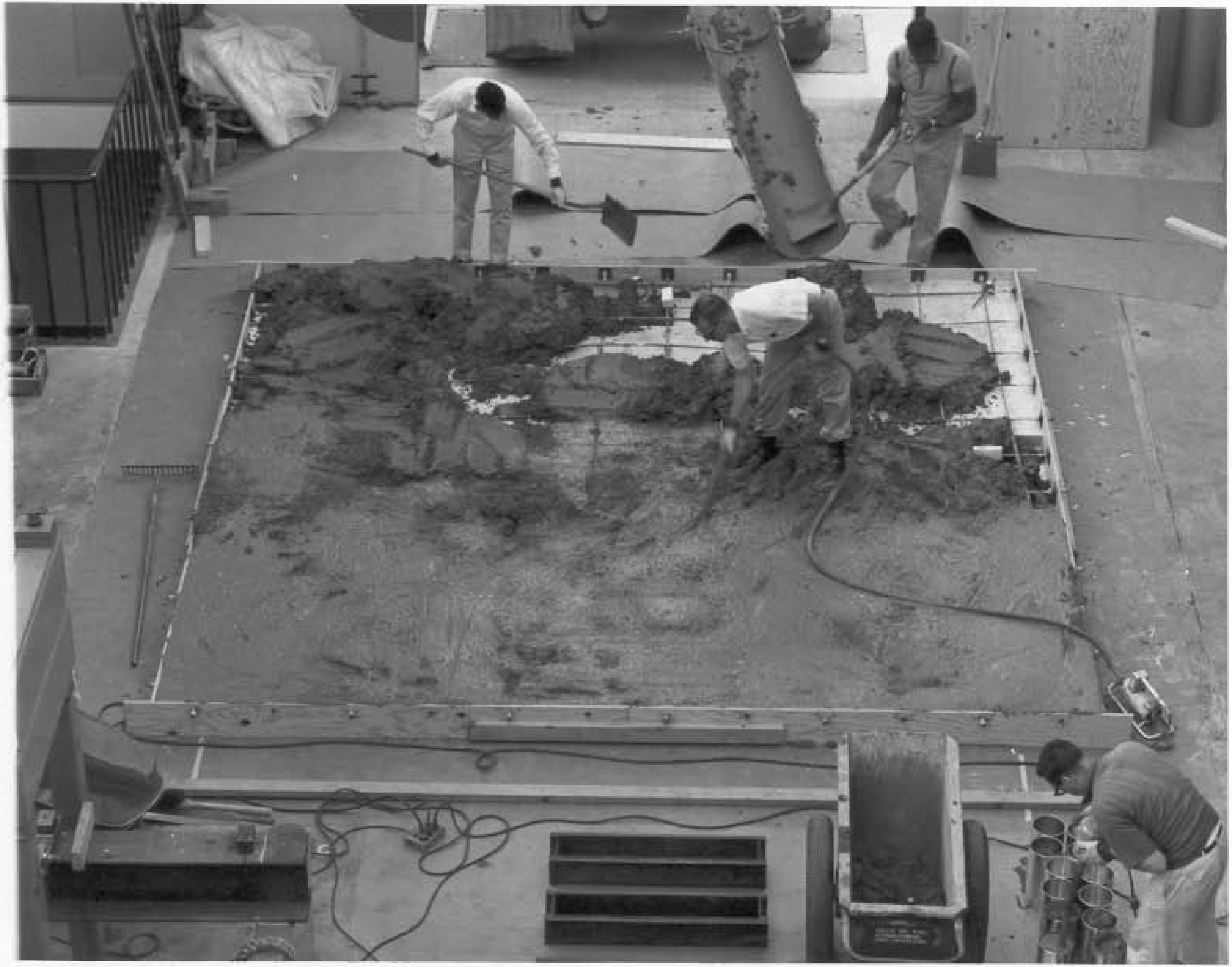


FIGURE 15: CRACK PATTERN ON TOP SURFACE AT FAILURE

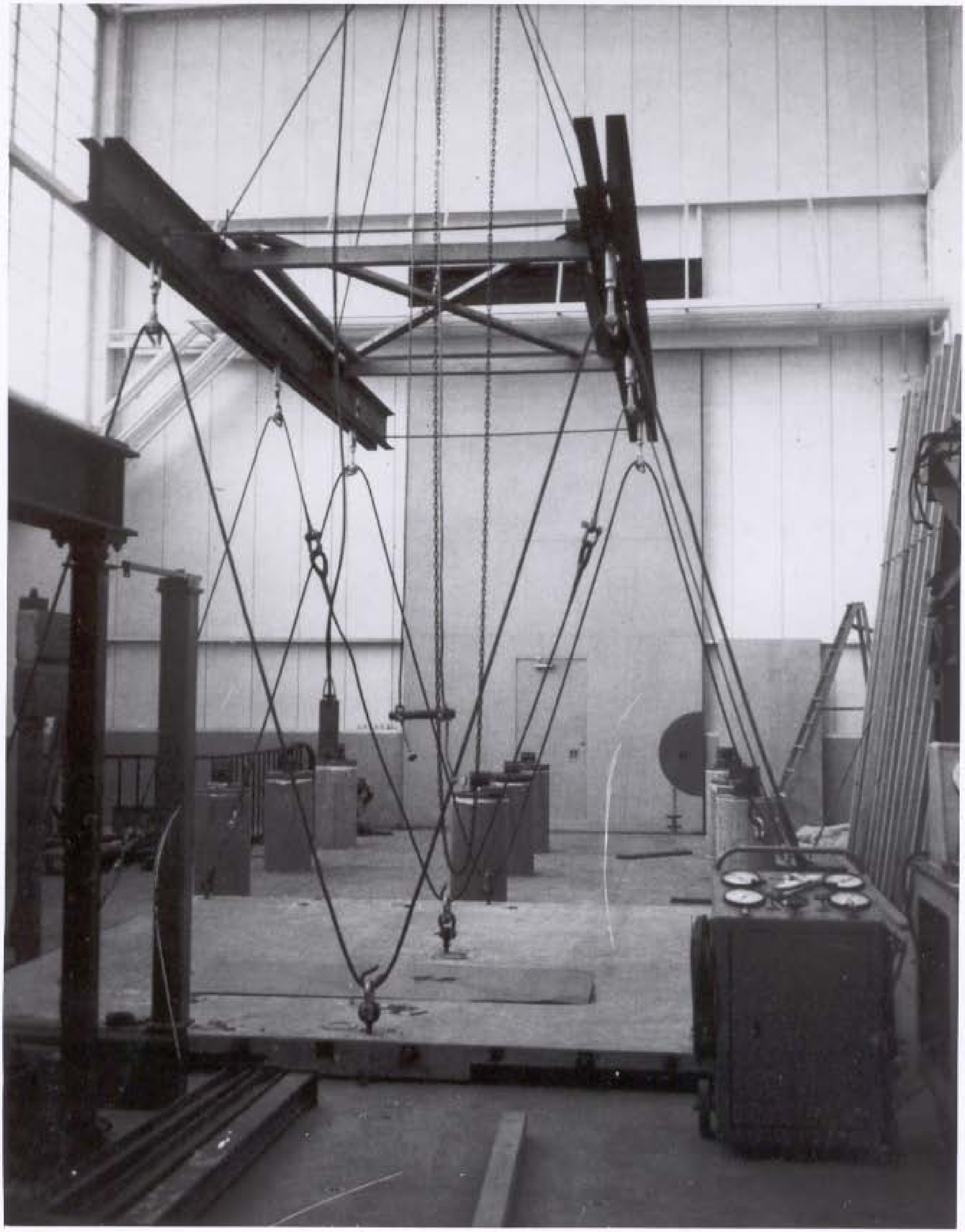
PHOTOGRAPHS



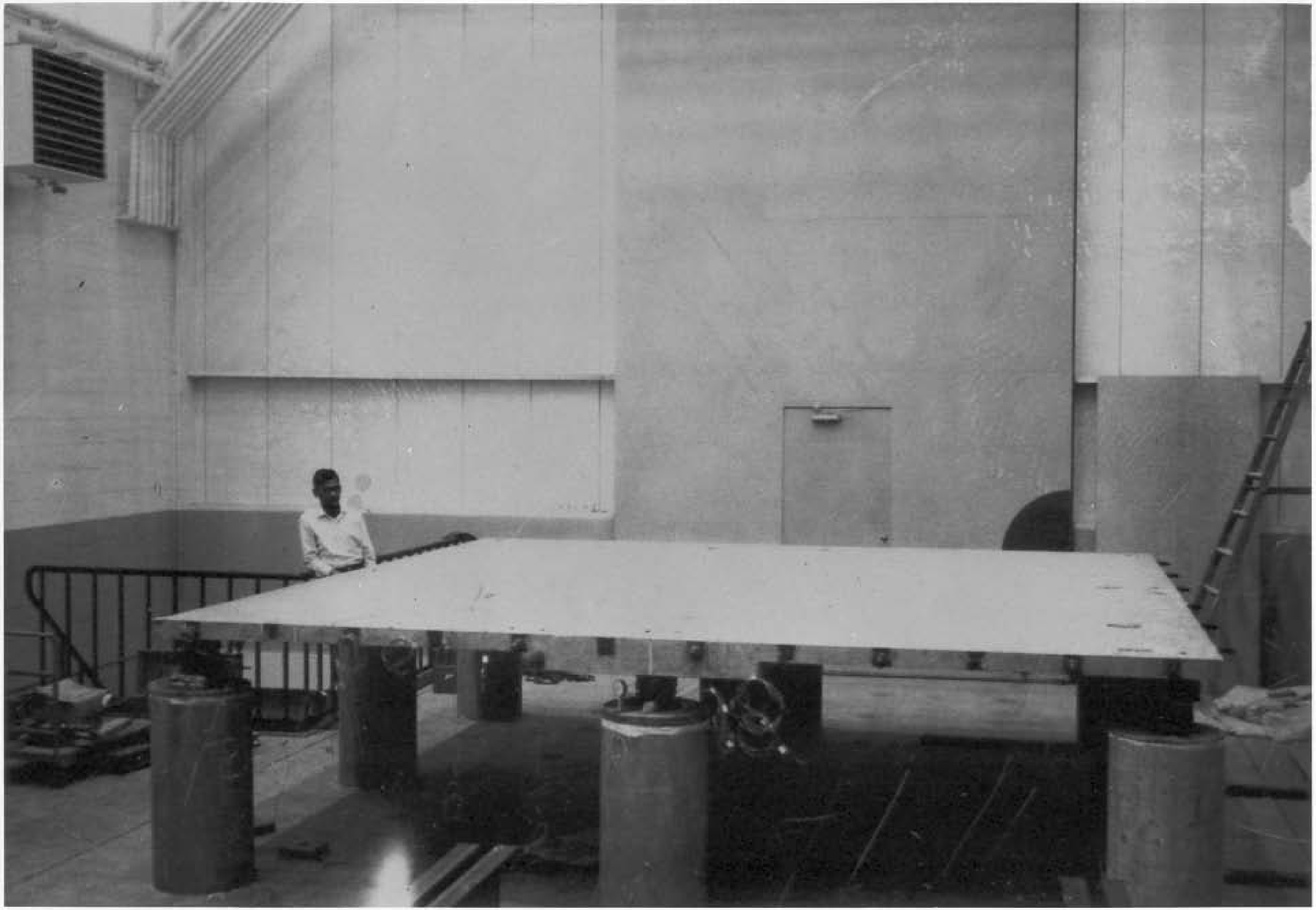
PHOTOGRAPH 1: FORMS AND STEEL ARRANGEMENT



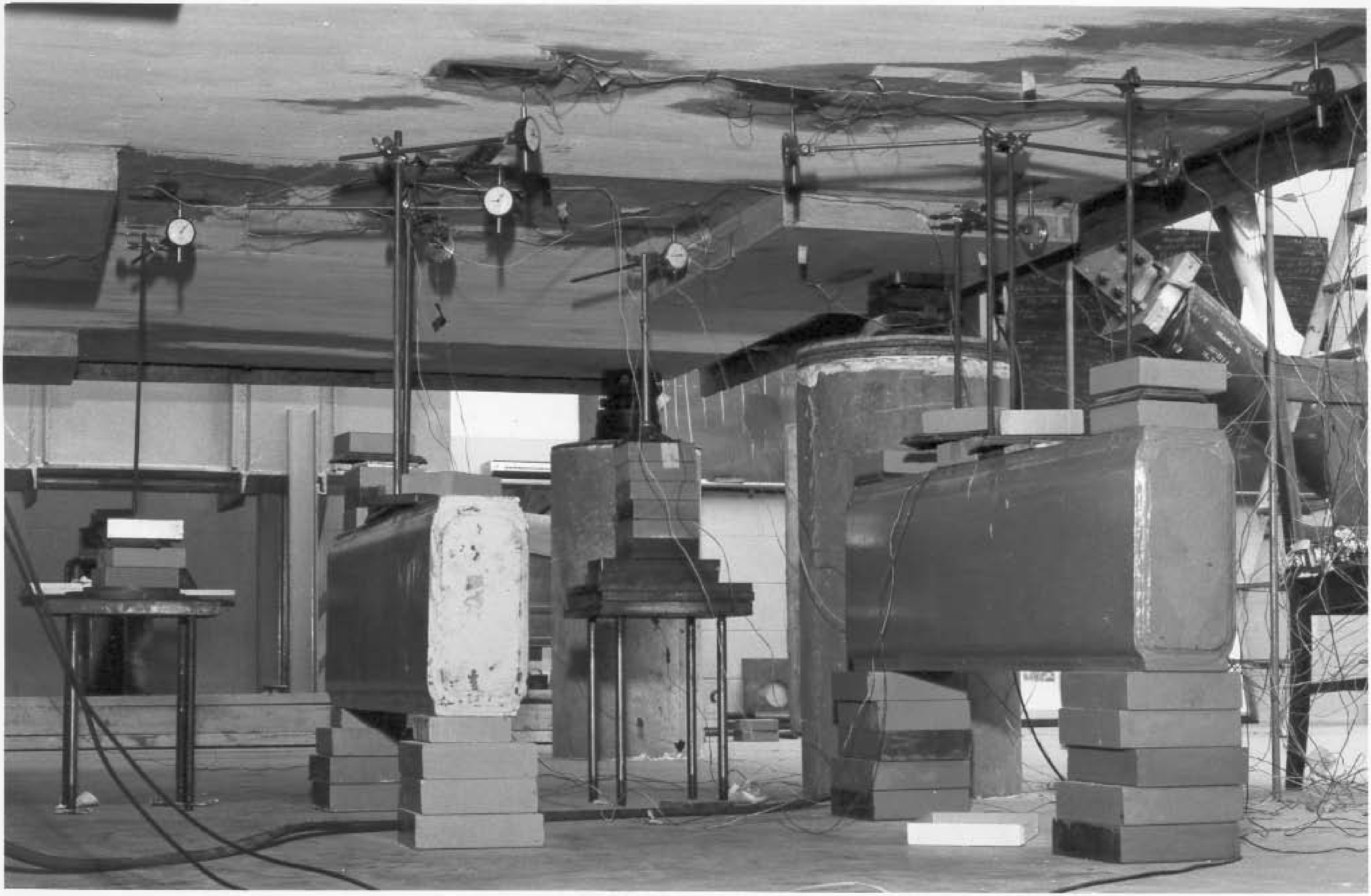
PHOTOGRAPH 2: PLACING CONCRETE



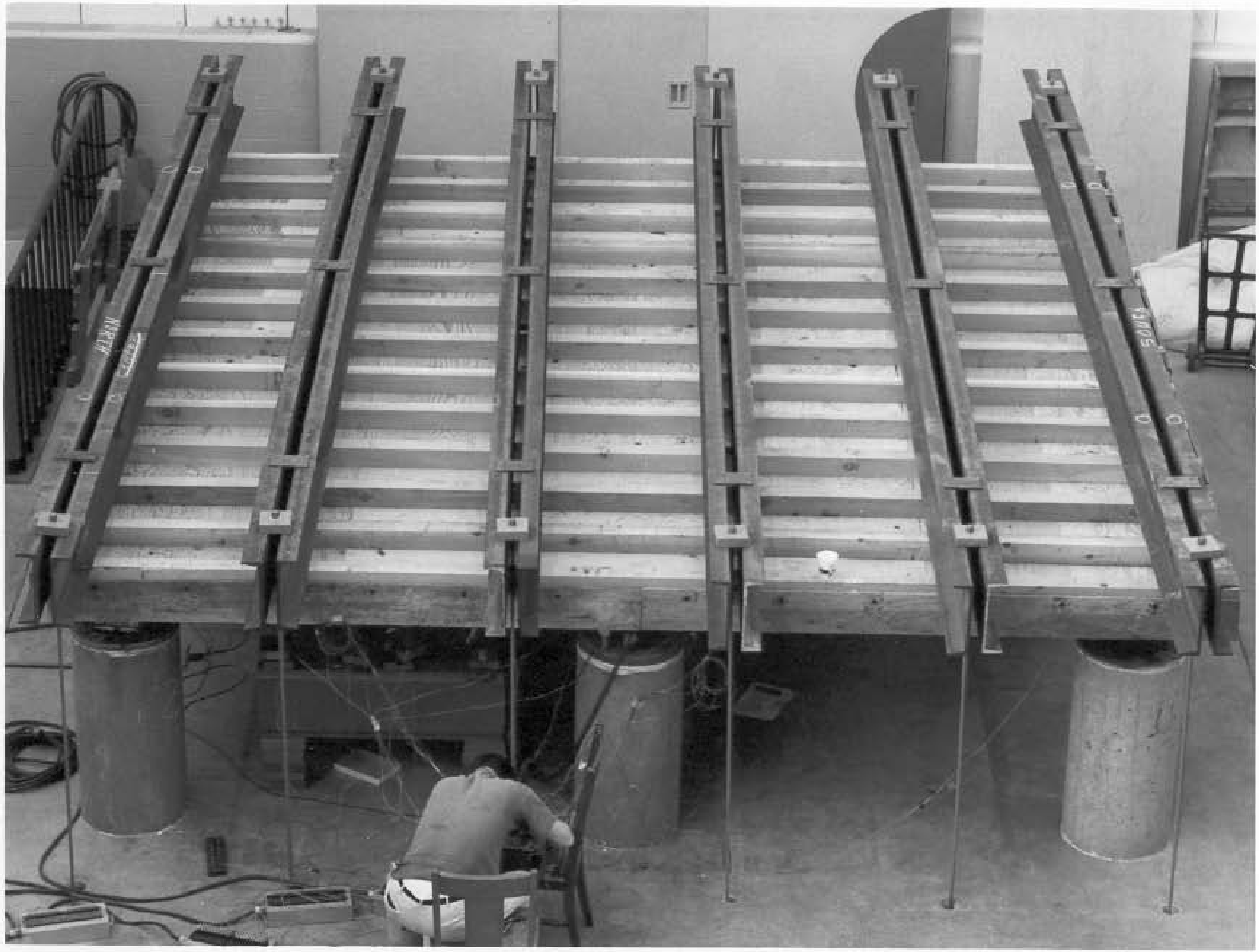
PHOTOGRAPH 3: LIFTING DEVICE



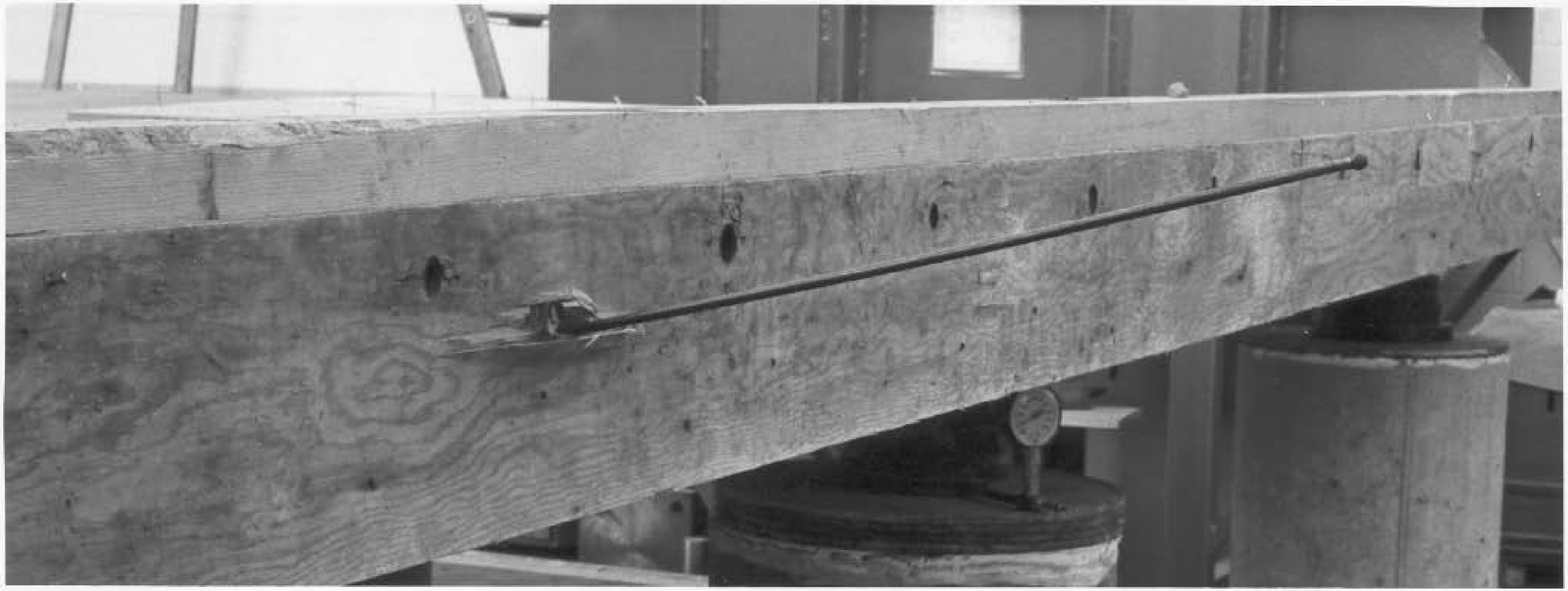
PHOTOGRAPH 4: SLAB IN PLACE ON PEDESTALS



PHOTOGRAPH 5: INSTRUMENTATION



PHOTOGRAPH 6: SLAB WITH LOADING FRAME JUST PRIOR TO TEST



PHOTOGRAPH 7: ANCHORAGE END OF BROKEN TENDON

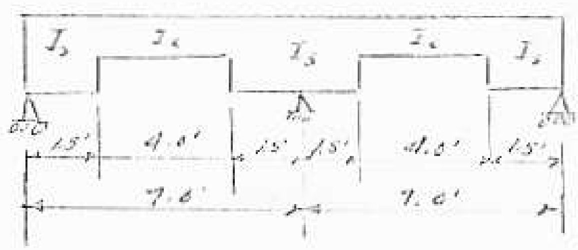
PHOTOGRAPH 8: CENTER SUPPORT AFTER FAILURE

CALCULATIONS

Elastic Analysis

Using the Beam Method of analysis we need only analyze on direction because the slab is symetrical. The 6 inch over-hang will be neglected as part of the span but will be counted on for moment resistance.

Consider the following beam



Section Near Supports

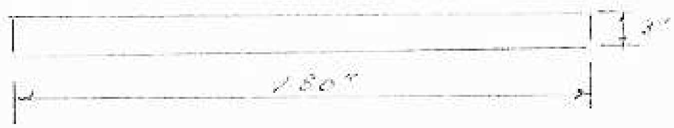
$A_c = 792 \text{ in}^2$

$I_s = 2225 \text{ in}^4$

$(S_s)_{Top} = 905 \text{ in}^3$

$(S_s)_{bottom} = 630 \text{ in}^3$

Section Near Center Span



$$A_c = 590 \text{ in}^2$$

$$I_0 = 120 \text{ in}^4$$

$$S_0 = 270 \text{ in}^3$$

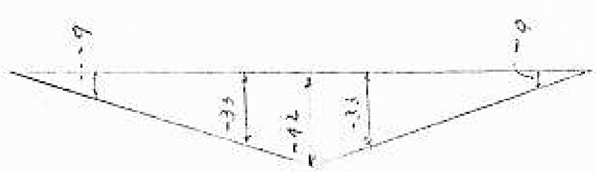
to simplify analysis let $I_0 = 1.0$
$I_s = 5.3$

Analyze the section for the statically indeterminant moments and shears

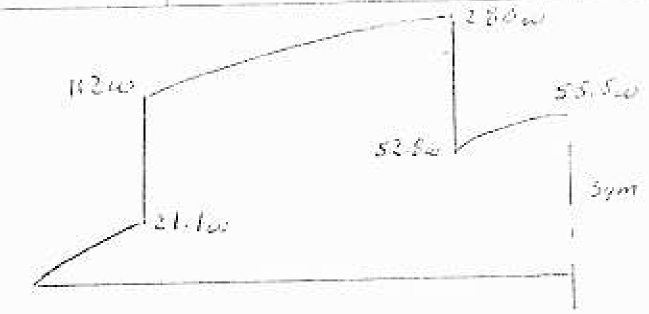
R = 0

M_0 (in-lb)

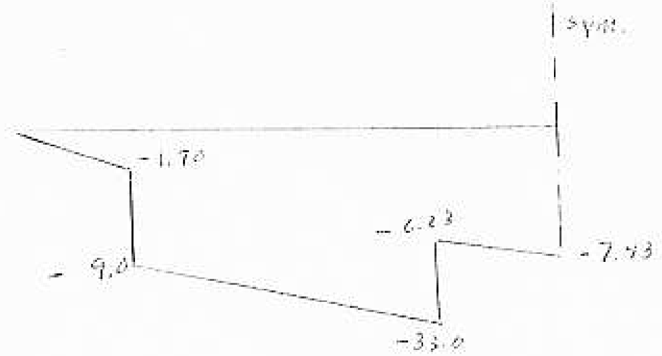
R = 1 lb



M_1 (in-lb)



$\frac{M_o}{I}$



$\frac{M_1}{I}$

$$\delta_{10} = -R_c \ \delta_{11}$$

where

$$\delta_{10} = \int_0^L \frac{M_o}{I} (M_1)\, dl$$

$$\delta_{11} = \int_0^L \frac{M_1}{I} (M_1)\, dl$$

$$\delta_{10} = 2\left\{ \tfrac{1}{6}\left[18\{0 - 4(4.5)(10.8) - (21.1)9\} + 18\{(112)(-9) - 4(230)(21) - 33(280)\} + 15\{52.8(-33) - 4(54.8)(37.5) - 55.5(42)\} \right]\right\} w$$

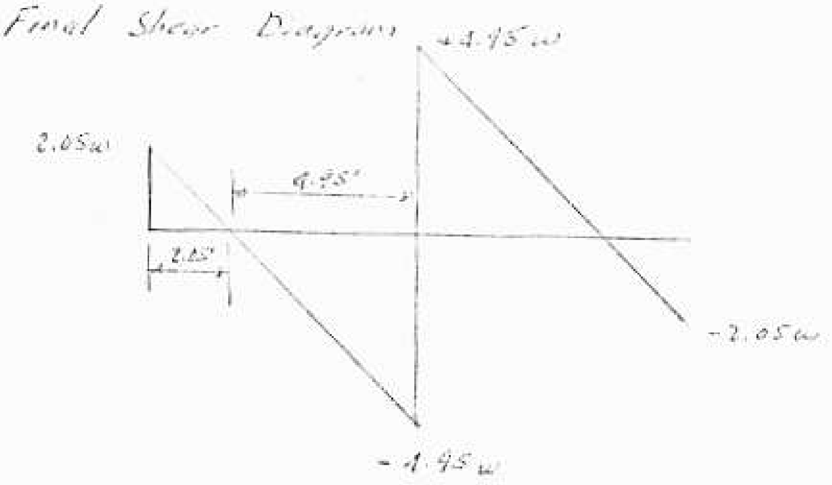
$$\delta_{10} = -561,000\ w$$

$$\delta_{11} = 2\left\{ \tfrac{1}{6}\left[18\{0 + 4(.85)(4.5) + (1.7)(9)\} + 18\{(9)(9) + 4(21)(21) + (33)(33)\} + 15\{(6.23)(33) + 4(37.5)(7.06) + (7.43)(42)\} \right]\right\}$$

$$\delta_{11} = 57,700$$

$$R_c = \frac{561,000 w}{56,700} = 9.9 w$$

Final Shear Diagram



$(M_{max})_{pos} = \frac{1}{2}(2.05)^2(12)\,w = 25.2\,w$ in-lb

$(M_{max})_{neg} = \left\{\frac{1}{2}\left[(4.95)^2(12)\right] - 25.2\right\}w = 122\,w$ in-lb

Design Load

Defining the design load as the maximum uniformly distributed load which causes no tension in the concrete, we may proceed as follows:

Stresses due to prestressing alone

the average stress in each tendon was 7,750 lbs

so $P = (7{,}750 \frac{lb}{tendon})(12 \text{ tendons}) = 94{,}000$ lbs

Section over supports

$f_{top} = -\frac{P}{A} - \frac{Pe}{(S)_{top}} = -\frac{94{,}000}{392} - \frac{94{,}000(1.96)}{905} = -324$ psi

$f_{bottom} = \frac{-94{,}000}{392} + \frac{94{,}000(1.96)}{636} = +172$ psi

Section near midspan

$f_{top} = -\frac{94{,}000}{590} + \frac{94{,}000(1.0)}{270} = +158$ psi

$f_{bottom} = -\frac{94{,}000}{590} - \frac{(94{,}000)(1.0)}{270} = -508$ psi

Check design Load for positive moment controlling

$$f_{bottom} + \frac{M}{S} = 0$$

$$M = -(f_{bottom})S = -(-508)(270) = 137,000 \text{ in-lb}$$

$$(M_{max})_{pos} = 25.2\,w \text{ in-lb}$$

$$w = \frac{137,000}{(25.2)(15 ft.)} = 363 \frac{lb}{ft^2}$$

check for negative moment controlling

$$M = -(f_{top})S_{top} = -(-324)(905) = 293,000 \text{ in-lb}$$

$$(M_{max})_{neg} = 122\,w \text{ in-lb}$$

$$w = \frac{293,000}{(122)(15 ft)} = 160 \text{ psf} \leftarrow \text{controls}$$

$$DL = 48 \text{ psf}$$

$$\text{Design Load} = 112 \text{ psf} \leftarrow$$ Design Load

Cracking Load

the negative moment section again controls

the average value of the modulus of rupture

was 453 psi. so

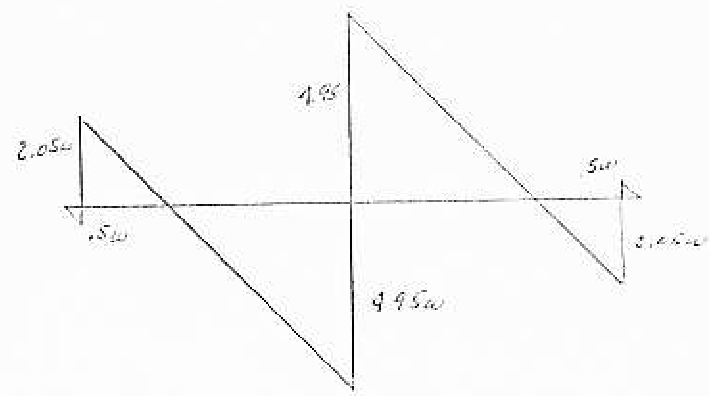
$$f_t + \frac{M}{S_{top}} = 453$$

$$M = S(453 - (f_t)) = 905[453 - (-324)] = 704,000 \text{ in-lb}$$

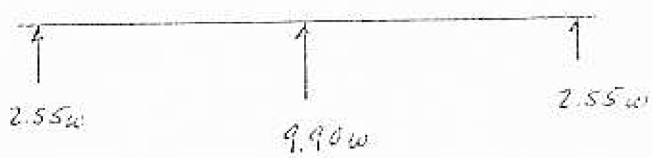
$$w_{cr.} = \frac{704,000}{122(15 ft)} - 48 = 337 \text{ psf} \leftarrow$$ Cracking Load

Reactions - Elastic

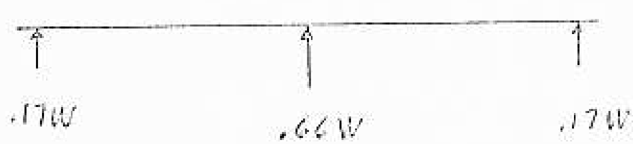
Shear Diagram considering the effects of the 6 inch over-hang



Reactions



Reactions as a part of the total Load W=w(15)



For the slab

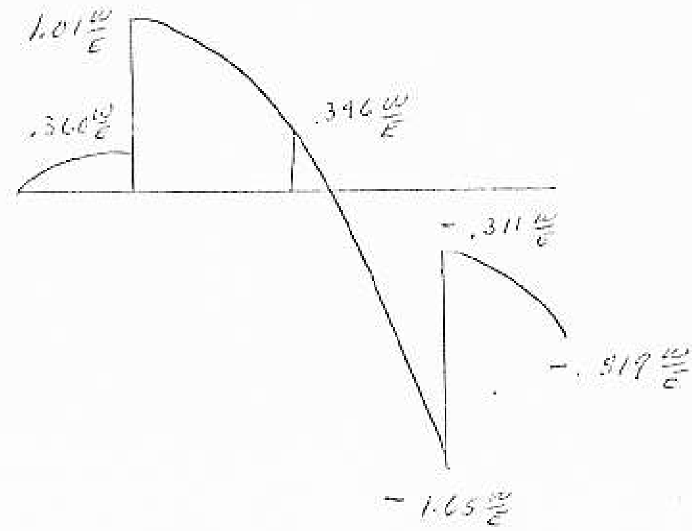
Center Support (.66)(.66)W = .435 W

Side Support (.66)(.17)w = .112 W

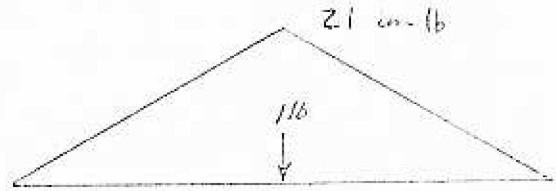
Corner Support (.17)(.17)w = .029 W

Deflection Estimate

$\frac{M}{EI}$ diagram for 1 span — w in psf, E in psi



Moment Diagram for virtual structure



$$\Delta_{center} = \int_0^L m \frac{M}{EI}\, dl$$

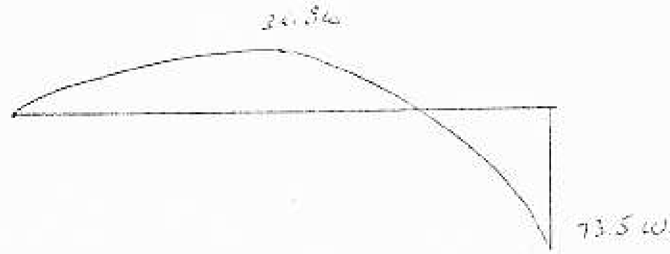
$$= \frac{1}{6}\Big\{ 18\,[\,0 + 4(4.5)(.27) + (9.0)(.36)\,]$$
$$+ 24\,[\,(1.01)(9.0) + 4(15.0)(.845) + (21.0)(.346)\,]$$
$$+ 24\,[\,(21.0)(.346) - 4(15.0)(.153) - (9.0)(1.65)\,]$$
$$+ 18\,[\,-(9.0)(.311) - 4(4.5)(.441) - 0\,]\Big\}\frac{w}{E}$$

$$\Delta_{center} = 194\,\frac{w}{E}$$

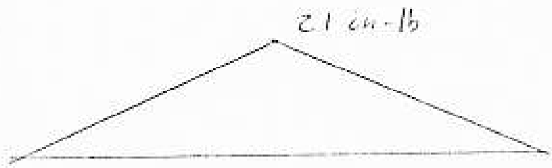
This is the ~~moment~~ deflection midway between the center and side support

To estimate the deflection at the center of the panel assume a 1 foot strip and determine the deflection. Add this deflection to Δ_{center}

Moment Diagram



Virtual Moment Diagram



$$\delta_p = \frac{1}{EI}\int_0^L m M \, dl \qquad I = \frac{bh^3}{12} = \frac{12(3)^3}{12} = 27 \text{ in}^4$$

$$= \left(\frac{1}{27}\right)\frac{1}{6}\Big\{42\left[0 + 4(10.5)(27.6) + (21.0)(36.8)\right]$$
$$+ 42\left[(21.0)(36.8) + 4(10.5)(8.2) + 0\right]\Big\}\frac{\omega}{E}$$

$$\delta_p = 802\,\frac{\omega}{E}$$

Total deflection at the center of a panel is

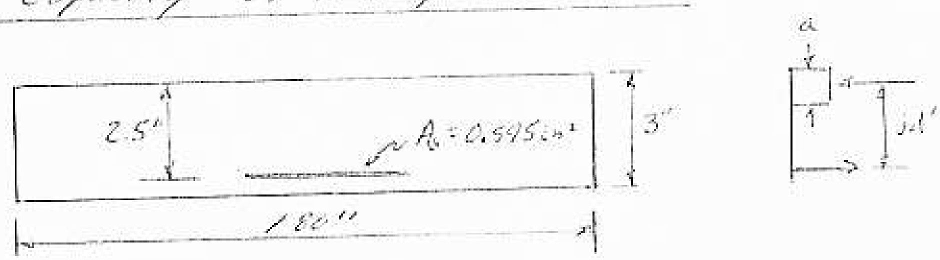
$$(802 + 194)\frac{\omega}{E} = 996\,\frac{\omega}{E} \text{ inches}$$

Ultimate Load Analysis

Assumptions:

1. Rectangular stress block for concrete with ultimate stress equal to $0.85 f'_c$
2. Concrete Strain at failure $\epsilon_{cu} = 0.0034 \frac{in}{in}$
3. Maximum stress in tendons $f_{su} = 200$ ksi
4. Residual stress in non-prestressed reinforcement $f'_{so} = -10$ ksi.

Moment Capacity of Mid-span Section



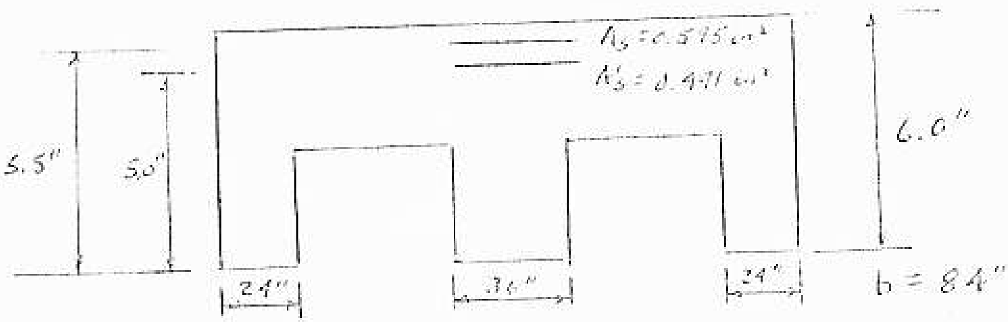
$$F_{su} = (200\text{ ksi})(0.595\text{ in}^2) = 119\text{ kips}$$

$$a = \frac{119\text{ k}}{(0.85)(7.975\text{ ksi})(180\text{ in})} = 0.098\text{ in}$$

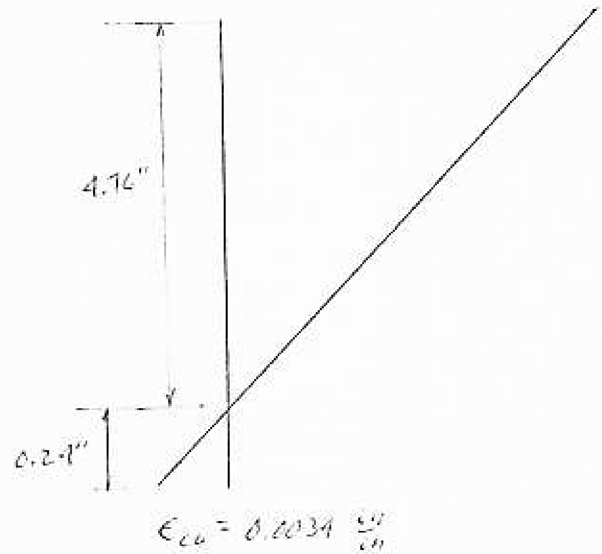
$$jd' = 2.50 - \tfrac{1}{2}(0.098) = 2.45\text{ in}$$

$$M_c = (2.45\text{ in})(119\text{ kips}) = \underline{292\text{ k-in}}$$

Moment Capacity at Support



try $a = 0.24"$



$\Delta \epsilon_{s'} = \frac{4.76}{.24}(.0034) = 0.067$

$\Delta \sigma_{s'} = (0.067)(29 \times 10^3 \text{ksi}) = 1950 \text{ksi}$

$\therefore \sigma_s = 40 \text{ksi}$ (yield strength)

$F_s' = (40 \text{ksi})(0.491 \text{ in}^2) = 19.6 \text{k}$

$F_{su} = (200 \text{ksi})(0.595 \text{ in}^2) = 119 \text{k}$

Steel Force 138.6 kips

ok

$F_{cu} = (0.85)(7.975 \text{ksi})(0.24 \text{in})(84 \text{in}) = 136 \text{ kips}$

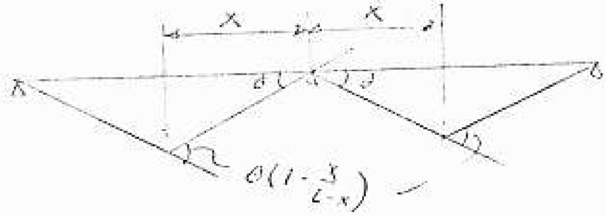
$M_s = (19.6 \text{k})[4.76 + \frac{1}{2}(.24)] + 119 \text{k}[5.50 - \frac{1}{2}(.24)]$

$= (19.6 \text{k})(4.88 \text{ in}) + (119 \text{k})(5.38 \text{ in})$

$M_s = 95.7 + 640 = 736 \text{ k-in}$

Analyze by Plastic Hinge Method

$M_s = 2.52\, M_c$



Internal work must equal external work

$$2 M_c\, \theta\left(1+\frac{x}{L-x}\right) + 2.52 M_c \theta = 2\left(\tfrac{1}{2} L \theta x\, w_u\right)$$

$$w_u = 2\frac{M_c}{L}\left[\frac{1}{x} + \frac{1}{L-x} + \frac{2.52}{x}\right]$$

we want the minimum w_u so

$$\frac{\partial w_u}{\partial x} = 2\frac{M_c}{L}\left[\frac{-1}{x^2} + \frac{1}{(L-x)^2} - \frac{2.52}{x^2}\right] = 0$$

this simplifies to

$$x^2 - 2.79 L x + 1.40 L^2 = 0$$

For $L = 7.0$ ft the solution is

$$x = 4.55 \text{ ft.}$$

This is close enough to consider that the section properties at mid span still apply

$$w_u = 2\frac{(2.92 \text{ k-in})}{(7 \text{ ft})(12 \tfrac{in}{ft})}\left[\frac{3.52}{4.55} + \frac{1}{2.45}\right] = 8.21 \text{ k/ft}$$

$$w_{ult} = \frac{(8.21 \text{ k/ft})(1000 \tfrac{lb}{k})}{15.6 \text{ ft}} - 18 \text{ psf} = \underline{500 \text{ psf}}$$ ← Ultimate Load

(18 psf: D.L.)

Reactions - Ultimate

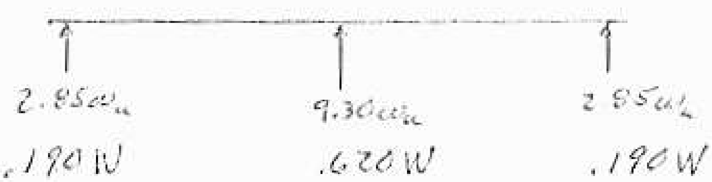
First examine 1 span

Reactions for a simple beam
w_u in #/ft

Effects of end moment $M_s = \frac{(736\ in)}{(12\ in/ft)(500\ psi)(15\ ft)}$

$= 8.20\, w_u$

Totals for the entire beam



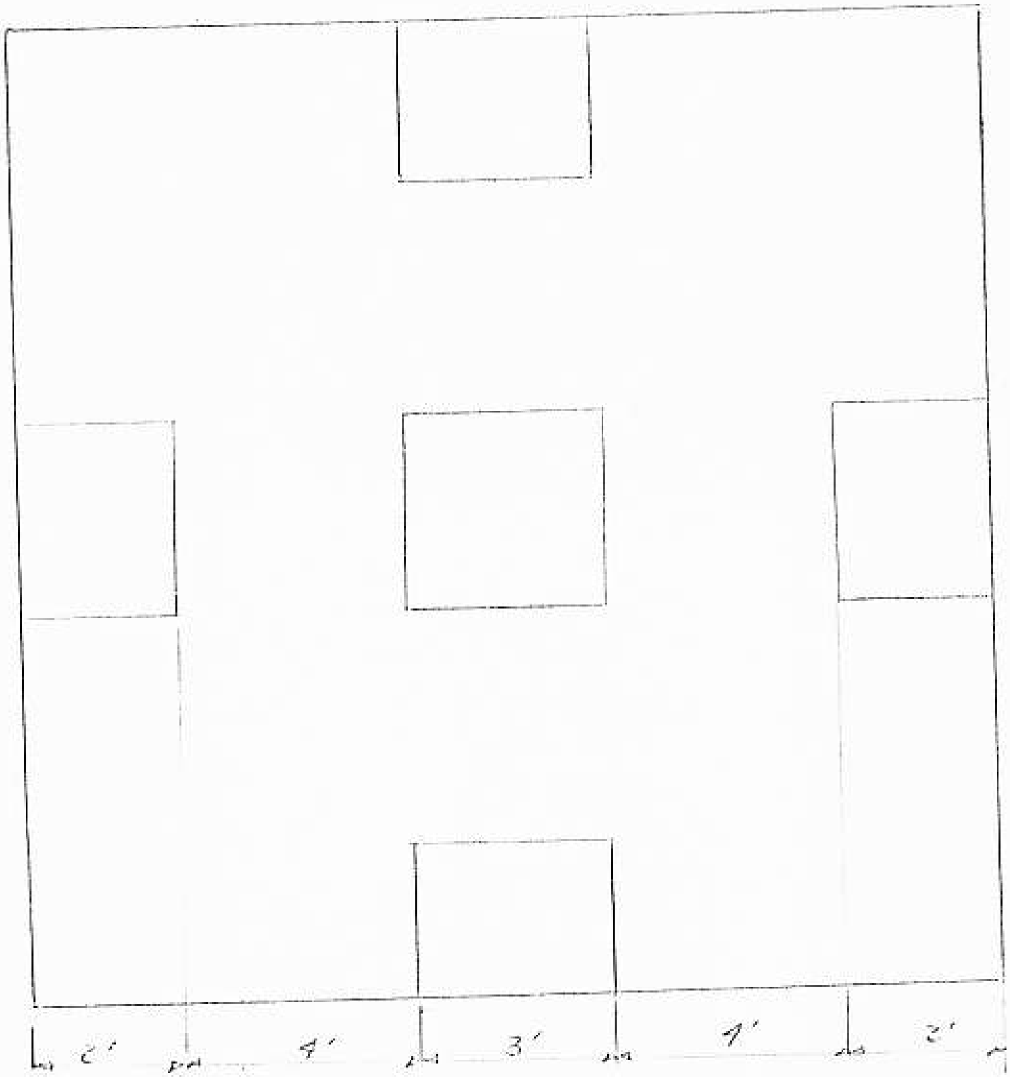
For the slab

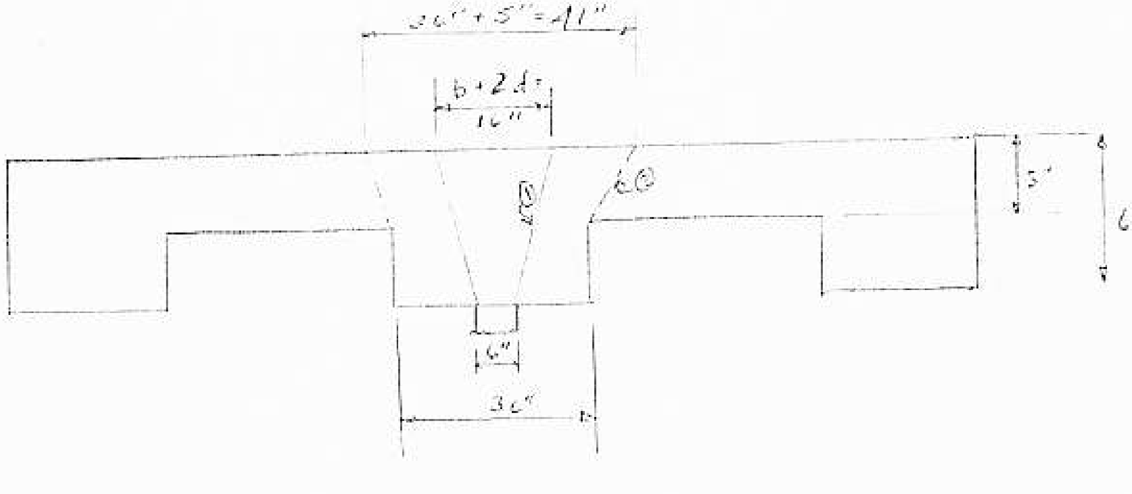
Center Support $(.620)(.620)W = .384\,W$

Side Support $(.620)(.190)W = .118\,W$

Corner Support $(.190)(.190\,W = .036\,W$

Punching Shear





Check the ultimate shear capacity by three methods. Considering the possibility the ultimate load may reach 650 psf the load on the center support will be

$$V = (650 psf)(15 ft)(15 ft)(.384) = 56 \text{ kips}$$

ACI Code

Pattern 1

$$v_1 = \frac{V}{bjd} = \frac{56 \times 10^3 lbs}{(69 in)(7/8)(5 in)} = 200 psi$$

Pattern 2

$$v_2 = \frac{56 \times 10^3 lbs}{4(91 in)(7/8)(2.5 in)} = 155 psi$$

Allowable punching shear = $0.04 f'_c$

$$= 0.04(7,975) = 319 psi \qquad \underline{\underline{OK}}$$

Elstner-Hognestad Method

$$v_2 = 333 + \frac{.046}{\phi_o} f'_c$$

$$\phi_o = \frac{P_{shear}}{P_{flex}}$$

$$P_{flex} = 6.627 M_u = 6.627 \times \frac{(2.5 in)^2 (7,975 psi)}{3}$$

$$P_{flex} = 6.627 \times 16.7 \text{ kips}$$

$$\phi_o = \frac{56 \text{ kips}}{6.627 \times 16.7 \text{ kips}} = 0.506$$

$$v_2 = 333 + \frac{.046}{.506}(7,975)$$

$$= 333 + 725 = 1058 \text{ psi allowable}$$ ok

Whitney Method

$$v_{allow} = 100 + 0.75 \frac{M_u}{d^2} \sqrt{\frac{d}{d_s}}$$

$$= 100 + 0.75 \frac{16,700}{(2.5)^2} \sqrt{\frac{2.5}{84}}$$

$$= 100 + 346 = 446 \text{ psi} > 200 \text{ psi}$$ ok