ACI Stability Resistance Factor for RC Columns

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Abstract: Load and resistance factors are used to reduce the probability of overloading, understrength, or both to acceptable levels. The ACI Building Code specifies the use of resistance factors for beam-columns in the following ways: (1) Resistance factors are applied to cross-section strength; and (2) resistance factors are applied to critical buckling strength used in moment magnification due to slenderness effects. Probabilistic examination of ACI resistance factors as used in (1) was reported in an earlier study and is available in the literature. This paper examines the ACI resistance factors as employed in (2) and suggests a value for design office use for computing moment magnification of concrete slender columns.

Introduction

The actual strength of a reinforced concrete (RC) member varies from calculated nominal strength due to variations in material strengths and dimensions of the member, as well as due to uncertainties inherent in the equations used to compute member strength. Similarly, actual loads that act on a member differ from calculated nominal loads due to variations in constituent material densities, as well as uncertainties inherent in applied loads. These variabilities in strength and loading are included in member design through safety provisions of the structural codes.

In 1963, the American Concrete Institute (ACI) Building Code (2) introduced ultimate strength design in which load factors and resistance factors were specified for design of reinforced concrete members. The 1963 load factors were slightly modified for use in the 1971, 1977, and 1983 ACI Building Codes (3,4,5). The ACI load factors and resistance factors have been criticized, because they were based on a simple, nonprobabilistic analysis.

For the past ten years, research has been done in North America to introduce reliability-based load and resistance factor design (LRFD) for concrete structures. This procedure statistically combines variations in loads and strength to calculate load and resistance factors. A reliability-based LRFD procedure was used in this study to examine the resistance factors contained in the current ACI Building Code (5) for use in the stability design of slender reinforced concrete tied columns. The results are limited to columns bent in single curvature and located in nonsway frames subjected to short time loads.

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In an earlier study, resistance factors for reinforced concrete and prestressed concrete beams in flexure and shear and short-tied columns designed in accordance with the 1983 ACI Building Code were examined (8). Ref. 8, however, did not include slender columns. The research reported herein is intended to fill this gap. Both of these studies were specifically aimed at design office use. Thus, resistance factors were examined for representative structural members based on relative occurrences of different types of loads that act on concrete buildings.

A new set of load factors has been recently included in the American National Standards Institute (ANSI) Standard A58.1-1982 (1). The resistance factors corresponding to the ANSI load factors have been derived by MacGregor (7). However, the ANSI load factors have not yet been adopted in the ACI Building Code. The resistance factors reported herein and in Ref. 8 can be directly used in design offices, since these are compatible with current ACI load factors. Furthermore, these resistance factors along with the current ACI load factors will provide a probabilistic basis for comparison when the ANSI load factors and related resistance factors are considered for inclusion in the ACI Building Code.

Throughout this paper, resistance factors applied to cross-section strength are referred to as the cross-section resistance factors ϕ_c, whereas those applied to critical buckling strength for computing moment magnification of slender columns are referred to as the stability resistance factors ϕ_s. This definition and terminology do not conform to the current ACI Code (5), but are used in this paper to simplify the presentation and discussion of the research reported in the paper.

Definition of Failure and Safety Index

Consider a large number of members, each designed to have the same nominal resistance and assumed to be subjected to the same specified service loads. Since loads are variable, the density of lifetime maximum load effect U for all the members can be represented by a probability distribution. Similarly, due to variations in constituent material strengths, geometry, and design simplifications, the density of resistance R for all the members can be represented by another probability distribution. Combinations of U and R in which $R < U$ represent failure.

A function $Y = R/U$ representing the ratio of resistance to load effect can be obtained with a mean value of $\bar{Y}$ and a standard deviation of σ_Y. The probability distribution of this function depends on those for R and U. The failure condition can then be represented by Y (or R/U) < 1.0. The probability of failure is the ratio of the area of the part of the curve in which $Y < 1.0$ to the total area under the curve. The safety index β is a multiple of standard deviations by which $\bar{Y}$ exceeds the failure level (or $Y = 1.0$). If the type of probability distribution of Y is known, the probability of failure can be calculated from β. Thus, β is taken as a measure of structural reliability.

Reliability Analysis of Slender Columns

A risk analysis computer program originally developed at the University of Michigan (10) was used in this study. The program is based

TABLE 1.—Nominal Properties of Columns Studied[a]

Column designation (1)	Slenderness ratio (l/h) (2)	Longitudinal reinforcement ratio (ρ_g) (3)
4004	10	0.022
4005	20	0.022
4006	30	0.022
4017	20	0.012
4029	20	0.033

[a]All columns were rectangular tied colums with cross-section dimensions of 12 × 12 in. (305 × 305 mm), specified concrete strength of 5,000 psi (34.5 MPa), specified reinforcement yield strength of 60,000 psi (414 MPa), and clear concrete cover to column ties of 1.5 in. (38 mm). Each column was studied for end eccentricity ratios (e/h) of 0.1, 0.3, 0.5, 0.7, 1.0, and 1.5. Three different column tributary areas were investigated: 400 sq ft, 1,000 sq ft, and 10,000 sq ft (37 m^2, 93 m^2, and 929 m^2).

on a first-order, second-moment probabilistic analysis and was used to compute the safety indices β presented later in this paper.

The acceptable probability of failure was established in terms of the safety index using calibration studies of the 1983 ACI Building Code. Based on probability distributions of strengths and loads available in the literature, the computer program described in Ref. 10, and the stability resistance factors ϕ_s ranging from 0.7–1.0, the β-ϕ_s curves were generated for several slender columns. These curves were then used to determine ϕ_s required for the target β's computed from calibration studies. The columns studied were considered typical of reinforced concrete buildings and are described in Table 1.

The design format was based on specified loads U_n multiplied by load factors α that are greater than 1.0, and on nominal resistances R_n multiplied by resistance factors ϕ of less than 1.0, so that the factored resistance was greater than or equal to the effect of factored loads as shown in the following:

$$\phi R_n \geq \Sigma \alpha U_n \quad (1)$$

The factored loads were taken as the maximum value from the set of loads specified in the 1983 ACI Building Code (5):

$$\Sigma \alpha U_n = [1.4D + 1.7\,(L + S)] \geq 0.75\,[1.4D + 1.7\,(L + S) + 1.7W] \quad (2)$$

in which D, L, S, and W = the specified dead, live, snow, and wind loads, respectively. The effects of earthquake and deformation loads were not included. The factored resistances ϕR_n were calculated in accordance with the provisions of the 1983 ACI Building Code (5).

The ACI Building Code requires the use of ϕ factors for columns in two different ways: (1) Resistance factors designated as ϕ_c in this paper are applied to the strength of column cross sections; and (2) resistance factors designated as ϕ_s in this paper are applied to critical buckling strength used in computation of moment magnification for slender columns through the expression (5)

$$M_c = \left[\frac{C_m}{1 - \left(\frac{P_u}{\phi_s P_c}\right)}\right] M_2 \geq M_2 \quad (3)$$

in which M_2 = factored moment applied to the ends of a slender column; P_u = factored axial load acting on the column ends; P_c = critical buckling strength of the column; and C_m = equivalent uniform moment diagram factor. In this study, C_m was taken equal to 1.0 since the columns studied were assumed to be subjected to single curvature bending in nonsway frames with equal moments applied at both ends, so that

$$M_c = \left[\frac{1}{1 - \left(\frac{P_u}{\phi_s P_c}\right)}\right] M_2 \geq M_2 \quad (4)$$

The resistance factors for the cross-section strength of reinforced concrete tied columns have been reported in Ref. 8. The reported values are: ϕ_c = 0.7 for the nominal axial load capacity greater than or equal to the nominal balanced load capacity; and ϕ_c varies linearly from 0.7–0.9 as the nominal axial load capacity decreases from the balanced value to zero. These values of ϕ_c are the same as those specified in ACI Standard 318-83 (5) and were included in the computation of the factored cross-section strengths. The factored cross-section strengths thus calculated were then used in the determination of ϕ_s required for stability considerations. Hence, the stability resistance factors presented later in this paper are related only to the load factors, the cross-section resistance factors, and the design expressions of the 1983 ACI Building Code (5).

Description of Strength Used in Reliability Analysis

The resistance statistics used in the reliability analysis were generated by using a Monte Carlo technique (6,9), which required a theoretical model involving a deterministic procedure to express the ultimate resistance. The theoretical analysis used was intended to closely model the resistance of reinforced concrete slender columns subjected to combined axial force and flexural moment and was based on a more elaborate procedure than normal design computations. This analysis involved the following steps:

1. A set of column properties was selected, and the factored strength of the column ϕR_n was calculated in accordance with ACI 318-83 Code (5). In this calculation, the cross-section resistance factors ϕ_c as specified in Refs. 5 and 8 were included in the factored resistance of the column. The stability resistance factor ϕ_s in Eq. 4 was taken equal to 1.0, 0.9, 0.8, and 0.7. Thus, four values of ϕR_n, one for each of the ϕ_s-values, were calculated for each of the six end eccentricity ratios (e/h) studied. The e/h ratios studied were 0.1, 0.3, 0.5, 0.7, 1.0, and 1.5. The computations for several ϕ_s-values were necessary in order to develop β-ϕ_s curves presented later in this paper.

TABLE 2.—Resistance Statistics for Typical Slender Columns[a]

End eccentricity ratio (e/h) (1)	Column 4004 $\bar{R}/\phi R_n$ (V_R) (2)	Column 4005 $\bar{R}/\phi R_n$ (V_R) (3)	Column 4006 $\bar{R}/\phi R_n$ (V_R) (4)	Column 4017 $\bar{R}/\phi R_n$ (V_R) (5)	Column 4029 $\bar{R}/\phi R_n$ (V_R) (6)
(a) Using ϕ_s = 1.0					
0.7	1.240 (0.110)	1.110 (0.120)	1.030 (0.110)	1.040 (0.100)	1.120 (0.100)
0.3	1.270 (0.125)	1.090 (0.130)	1.020 (0.125)	1.075 (0.130)	1.120 (0.125)
0.1	1.315 (0.145)	1.290 (0.155)	1.390 (0.155)	1.425 (0.165)	1.250 (0.160)
(b) Using ϕ_s = 0.9					
0.7	1.250 (0.110)	1.130 (0.120)	1.075 (0.110)	1.070 (0.100)	1.150 (0.100)
0.3	1.285 (0.125)	1.135 (0.130)	1.090 (0.125)	1.130 (0.130)	1.155 (0.125)
0.1	1.330 (0.145)	1.375 (0.155)	1.510 (0.155)	1.525 (0.165)	1.320 (0.160)
(c) Using ϕ_s = 0.8					
0.7	1.265 (0.110)	1.175 (0.120)	1.135 (0.110)	1.105 (0.100)	1.180 (0.100)
0.3	1.305 (0.125)	1.195 (0.130)	1.185 (0.125)	1.200 (0.130)	1.225 (0.125)
0.1	1.350 (0.145)	1.465 (0.155)	1.670 (0.155)	1.660 (0.165)	1.400 (0.160)
(d) Using ϕ_s = 0.7					
0.7	1.280 (0.110)	1.220 (0.120)	1.205 (0.110)	1.150 (0.100)	1.225 (0.100)
0.3	1.330 (0.125)	1.265 (0.130)	1.280 (0.125)	1.285 (0.130)	1.285 (0.125)
0.1	1.370 (0.145)	1.610 (0.155)	1.875 (0.155)	1.815 (0.165)	1.525 (0.160)

[a]The probability distributions were assumed to follow the lognormal curve. The resistance factors for the cross-section strength ϕ_c, specified in Refs. 5 and 8, were included in the strength description of slender columns. The slender column resistance factor (ϕ_s in Eq. 4) was taken equal to 1.0, 0.9, 0.8, and 0.7 in computation of resistance statistics shown above.

2. Using a random number generator, values of each of the variables affecting column strength (such as compressive strength of concrete, yield strength of reinforcing bars, cross-section dimensions, and concrete cover) were selected from statistical distributions of those variables. Column length and end eccentricity were taken as deterministic values.
3. The properties selected in step 2 were used to generate a series of moment-curvature curves for various axial load levels. The maximum moment from each of these curves and the corresponding axial load were

taken as a point on the cross-section interaction diagram for a column with the properties selected in step 2. A curve-fitting routine was used to fit curves to the points on the interaction diagram.

4. For a given end eccentricity ratio, the moment-curvature diagrams from step 3 were used to compute the deflected shape of the column at each axial load level. This calculation involved fitting a deflected shape to the column which matched the moments and curvatures at the ends and midheight of the column. Several iterations were required at each load level. The maximum moment in the column and the corresponding axial load were one point on a load-maximum moment curve for a column with the properties chosen in step 2 and the given end eccentricity ratio. Failure was assumed to occur when the load-maximum moment curve intersected the interaction diagram of step 3 (material failure) or when dM/dP approached infinity (stability failure). The resulting failure strength R was divided by the factored resistances ϕR_n from step 1 for the end eccentricity ratio under consideration, giving one value of $R/\phi R_n$ for each of the four ϕ_s values studied.

5. Step 4 was repeated for each of the six end eccentricity ratios studied. This gave one value of $R/\phi R_n$ for each combination of e/h and ϕ_s-values.

6. Steps 2–5 were repeated 1,000 times for the column under consideration, resulting in a population of 1,000 values of $R/\phi R_n$ for each combination of e/h and ϕ_s values studied.

7. A log-normal probability distribution was fitted using the five and one percentile values to each of the populations of $R/\phi R_n$ in step 6. These log-normal distributions were found to agree with the corresponding generated populations of $R/\phi R_n$ in the lower half of the probability curves and were used in the reliability analysis in place of the actual ones.

The properties of probability distributions obtained for the columns studies are listed in Table 2. Note the resistance statistics with e/h equal to 0.5, 1.0, and 1.5 were also developed and used in the reliability analysis, but are not shown in Table 2.

As indicated in Table 1, slenderness ratio (l/h), longitudinal reinforcement ratio (ρ_g), and end eccentricity ratio (e/h) were varied for five columns studied in this paper. All other variables were found to have an insignificant effect on the variability of slender columns and were not included in the reliability analysis. Further details on strength variability of reinforced concrete slender columns will be reported in a separate paper and will not be repeated here.

Description of Loads Used in Reliability Analysis

In analysis of safety, it is necessary to deal with load effects rather than the loads themselves. Thus, it is necessary to describe the probability distributions of effects of different types of loads. The load effects can be obtained by combining the variability of loads with the variability introduced by the structural analysis. The latter component is small and was neglected except in the case of dead load.

The load effect statistics used in the reliability analysis are shown in Table 3. The probability distribution of dead-load effect shown in Table

TABLE 3.—Load Statistics Used in Reliability Analysis

Load type (1)	Mean value of actual to nominal load ratio (2)	Coefficient of variation (3)	Type of probability distribution (4)	Basic time interval (5)	Frequency of occurrence (6)
Dead load	1.050	0.100	Normal	Lifetime[a]	Always present
Sustained live load	0.390[b]	0.45[b]	Gamma	5 yr	Always present
Transient live load	0.400[c]	0.190	Extreme type I	7 hr	Once a month
Snow load	0.880	0.227	Extreme type I	8 wk	Once a year
Wind load	0.875	0.177	Extreme type I	4 hr	Once a month

[a]The lifetime of a structure was assumed to be 50 yr.

[b]These values are functions of the tributary area. The mean value and the coefficient of variation shown are for a tributary area of 1,000 sq ft (93 m^2).

[c]This value is a function of the tributary area. The mean value shown is for a tributary area of 1,000 sq ft (93 m^2).

3 was based on the variability of dead loads in concrete structures resulting from variations in dimensions, densities, and superimposed loads combined with the variability caused by structural analysis. The descriptions of the remaining loads shown in Table 3 were taken from Ref. 10.

Selection of Target Safety Index Used in Reliability Analysis

In computing the stability resistance factor ϕ_s, the target safety index was chosen to furnish a level of safety comparable to that for beams designed according to the current safety provisions of the ACI Building Code. Thus, the 1983 ACI Building Code was calibrated for typical reinforced concrete beams. For calibration studies, the ACI 318-83 load and resistance factors were included in the analysis. The β values were calculated for different ratios of live load to dead load, snow load to dead load, and wind load to dead load. These values were then weighted according to the relative occurrences of different loads to obtain a single value of β for each combination of certain types of loads for each of the beams calibrated. The weighted β was 3.0–3.7 for beams subjected to dead plus snow load, 2.7–3.6 for beams subjected to dead plus live load, and 2.7–3.3 for beams subjected to dead plus live plus wind load. The overall average of β for all the beams studied was 3.1.

A range of β values is plotted in Fig. 1 for the flexural strength of reinforced concrete beams of average construction quality designed to fail in tension with flexural reinforcement ratios ranging from 0.14–0.73 times the balanced steel ratio and subjected to dead, live, and wind loads. Fig. 1 indicates a range of β from approximately 2.5–3.9 with a weighted value of 3.1. Previous studies have shown that the representative values of β for gravity loads are 2.8–3.2 for reinforced concrete beams, 2.75–

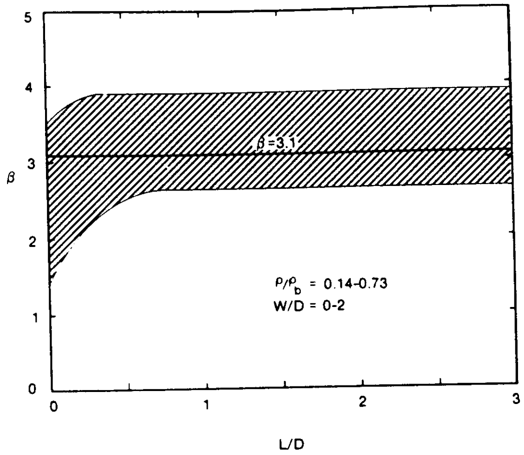

FIG. 1.—Safety Index for Flexural Strength of Reinforced Concrete Beams Designed According to ACI 318-83 Load and Resistance Factors [A_t = 400 ft² (37 m²); ϕ = 0.9]

3.5 for tied columns failing in tension, and 3.0–3.5 for tied columns failing in compression (7). Based on these values and Fig. 1, the target values of β were set at 3.0 and 3.25 for columns exhibiting tension and compression failures, respectively.

Calculation of Stability Resistance Factors

Reinforced concrete columns are subjected to axial load and flexure ranging from pure axial load to pure bending moment. Thus, the ratio of applied bending moment to applied axial load expressed in terms of end eccentricity ratio (e/h) varies from zero for pure axial load to infinity for pure bending moment. In calculations of stability resistance factors, three typical categories of columns were considered. These categories based on analyses of uses and loadings of typical reinforced concrete buildings (8,9) are: (1) Columns supporting more than three floors in addition to the roof with e/h of the order of 0.1; (2) columns supporting 1–3 floors plus the roof with e/h around 0.3; and (3) edge columns supporting just the roof of a building with e/h approximately 0.7. These three column categories are shown in Table 4 along with the weightings assigned to different load combinations. The column cases in Table 4(*c*) were subjected to snow loads in addition to dead loads and wind loads since these columns support just the roof. The columns in the remaining two categories were mainly subjected to live loads plus dead and wind loads.

All of the columns studied with an end eccentricity ratio e/h = 0.1 failed in compression, whereas those with e/h = 0.7 experienced tension failures. For the columns studied with an end eccentricity ratio of 0.3, however, the mode of failure depended on the slenderness ratio l/h and the longitudinal reinforcement ratio ρ_g. Such columns with a high l/h

TABLE 4.—Weighting Factors for Applied Loads on Reinforced Concrete Columns (%)

W/D (1)	L/D or S/D: 0.12 (2)	0.37 (3)	≥0.62 (4)
(*a*) Columns Supporting More than Three Floors Plus Roof[a]			
0.12	13	7	0
(*b*) Columns Supporting 1–3 Floors Plus Roof[b]			
0.12	0	33	17
(*c*) Columns Supporting Roof Only[c]			
0.12	0	0	3
0.37	5	12	0
1.50	0	0	5
2.50	0	0	5

[a]Average $e/h \le 0.1$.
[b]Average $e/h \simeq 0.3$.
[c]Average $e/h \simeq 0.7$.
Note: All weights in the table add to 100.

ratio and/or a low ρ_g ratio failed in tension since the second-order bending moments were high enough to increase the eccentricity at the failure section (midheight) beyond the balanced eccentricity. The remaining columns with an end eccentricity of 0.3 had either a low l/h or a high ρ_g ratio and failed in compression, because low values of second-order moments were obtained. Thus, the tension failures consisted of all columns with e/h of 0.7 plus some columns with e/h of 0.3 (column 4005, 4006, 4017 in Table 1). The compression failures, on the other hand, consisted of all columns with e/h = 0.1 and the remaining columns with e/h = 0.3 (column 4004, 4029 in Table 1).

Three different tributary areas A_t were examined to establish their effect on safety index β. In Fig. 2, the safety index is plotted against the

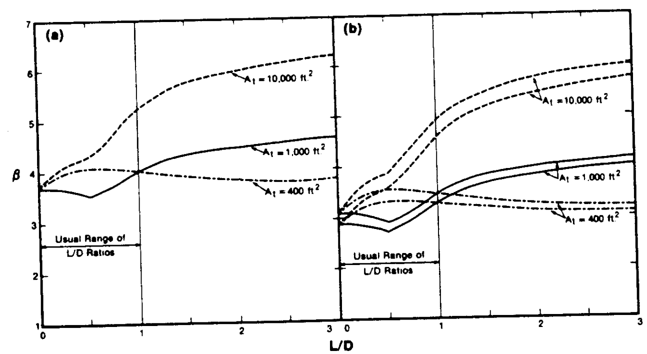

FIG. 2.—Effect of Tributary Area A_t on Safety Index β for Column 4005 [W/D = 0; l/h = 20; ρ_g = 0.022]: (*a*) e/h = 0.1; (*b*) e/h = 0.3–1.5

live to dead load ratios (L/D) for tributary areas of 400 ft^2, 1,000 ft^2, and 10,000 ft^2 (37 m^2, 93 m^2, and 929 m^2) acting on a typical column. The cross-section resistance factors and load factors specified in the ACI Standard 318-83 (5) and the stability resistance factor ϕ_s of 0.8 were included in computation of β-values plotted in Fig. 2. Note that the value of ϕ_s used for Fig. 2 is average ϕ_s currently specified in ACI Code and is close to the one suggested in the later part of this paper.

Reinforced concrete columns are usually subjected to $L/D \leq 1.0$ as indicated in Table 4. In this range of L/D ratios, the lowest β values were obtained for A_t of 1,000 ft^2 (93 m^2) as shown by Fig. 2. For $L/D >$ 1.0, A_t of 400 ft^2 (37 m^2) is shown to be critical in Fig. 2. However, such a low tributary area is usually carried by columns supporting only the roof. These columns are subjected to tension failure and are not expected to impose severe safety problems due to the ductile nature of the tension failure. Thus, 1,000 ft^2 (93 m^2) was taken to be the most critical value for A_t and all the columns studied were assumed to support a tributary area of 1,000 ft^2 (93 m^2).

It was anticipated that the stability resistance factors ϕ_s would fall between 0.7 and 1.0. Thus, safety indices were computed for ϕ_s of 0.7, 0.8, 0.9, and 1.0, and β-ϕ_s curves were developed for tension and compression failures, which then established the desired ϕ_s for the target β. Note that the target values of β were set at 3.0 and 3.25 for tension and compression failures, respectively.

Based on resistance and load statistics (Tables 2 and 3) and a given value of ϕ_s, the safety indices were calculated from the risk analysis computer program, described in Ref. 10, for different combinations of L/D, S/D, and W/D ratios for the columns shown in Table 1. All columns were studied for e/h of 0.1, 0.3, and 0.7. The β-values for each of the columns were then weighted according to the weighting factors of the three column categories shown in Table 4. The computations for the weighted β were carried out for each of the ϕ_s-values studied. The resulting range of weighted β's are plotted against ϕ_s in Figs. 3 and 4 for columns subjected to tension and compression failures, respectively.

Figs. 3 and 4 show that a column-stability resistance factor of 0.8 will provide a target β of at least 3.0 for tension failures and at least 3.25 for compression failures. Exception to this observation are the columns plotted in Fig. 4(b), which were subjected to wind loads and experienced compression failure. For these columns, a value around 0.7–0.75 seems to be more appropriate.

Figs. 3 and 4 were plotted for columns subjected to usual loads and end eccentricity ratios as indicated in Table 4. As a result, the values of ϕ_s obtained from these figures represent slender columns that are common in concrete buildings. Therefore, a study was conducted to test the application of these ϕ_s-values to columns subjected to an unusual range of loads and/or eccentricity of loads. Figs. 5 and 6(a) show the range of β obtained with L/D ratio ranging from 0–3.0 and $W/D = 0$ for all the columns listed in Table 1. In both figures, e/h ratio ranged from 0.1–1.5, so that the figures included tension and compression failures. A value of ϕ_s equal to 0.7 and 0.8 was included in computing β's for Figs. 5 and 6(a), respectively. It should be pointed out that some of the columns included in these figures are impractical for reinforced concrete build-

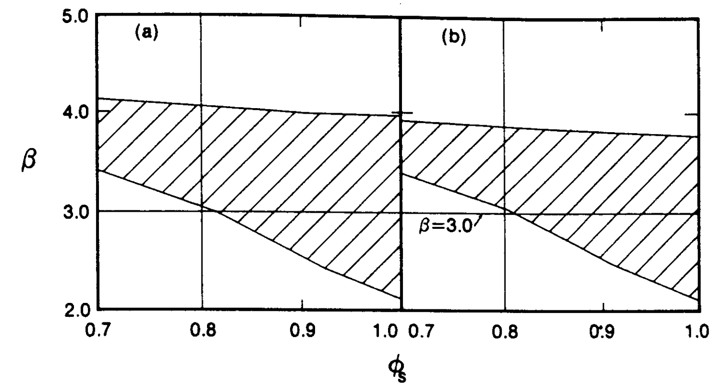

FIG. 3.—Effect of Stability Resistance Factor ϕ_s on Safety Index β for Usual Columns Subjected to Tension Failure [A_t = 1,000 ft^2 (93 m^2)]: (*a*) W/D = 0; (*b*) W/D = 0.12–2.5

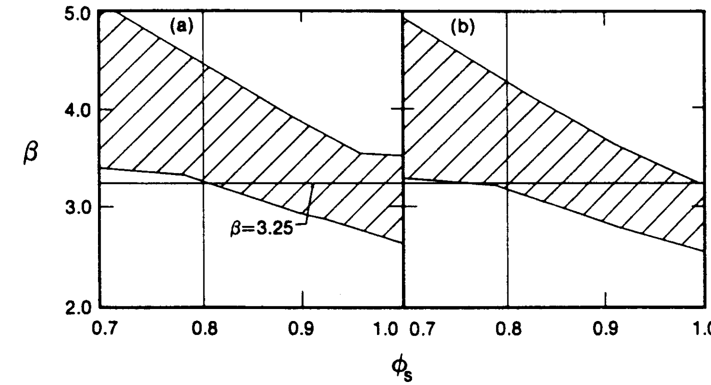

FIG. 4.—Effect of Stability Resistance Factor ϕ_s on Safety Index β for Usual Columns Subjected to Compression Failure [A_t = 1,000 ft^2 (93 m^2)]: (*a*) W/D = 0; (*b*) W/D = 0.12

ings; they were considered only to study the full range of L/D and e/h ratios.

Fig. 5 shows that for columns in which wind effects are not included, ϕ_s = 0.7 will furnish a β value at least equal to 3.0 over almost the entire range of L/D ratios. This is reasonable since the figure includes tension as well as compression failures. Values of β as low as 2.6 were obtained for ϕ_s = 0.8 as indicated in Fig. 6(a). Thus, a value between 0.7–0.75 seems to be a reasonable estimate of ϕ_s for the columns plotted in Figs. 5 and 6(a).

When the columns of Figs. 5 and 6(a) were subjected to wind loads, β-values significantly lower than those shown in these figures were obtained, as indicated by a plot for ϕ_s = 0.8 shown in Fig. 6(b). However,

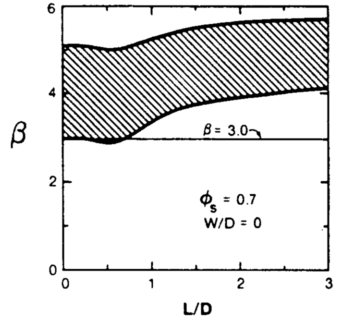

FIG. 5.—Range of Safety Index β Obtained with ϕ_s = 0.7, W/D = 0, L/D = 0–3.0, and e/h = 0.1–1.5 for All Columns Shown in Table 1 [A_t = 1,000 ft^2 (93 m^2)]

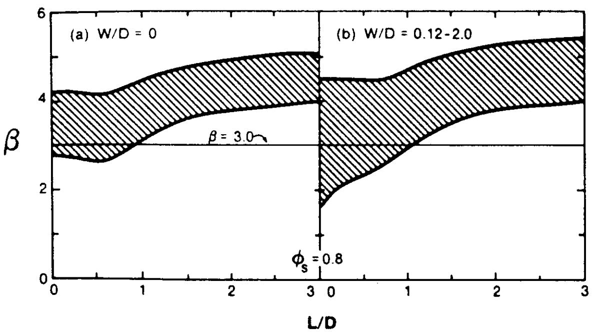

FIG. 6.—Range of Safety Index β obtained with ϕ_s = 0.8, L/D = 0–3.0, and e/h = 0.1–1.5 for All Columns Shown in Table 1 [A_t = 1,000 ft^2 (93 m^2)]: (*a*) W/D = 0; (*b*) W/D = 0.12–2.0

these load cases are not expected to impose severe safety problems because under wind loads a number of cross sections, rather than an isolated one, must fail to form the failure mechanism of the structure (7).

Based on the discussions related to Figs. 3–6, a constant value of ϕ_s between 0.7 and 0.75 is suggested for use in the computation of moment magnification of slender columns. This value of ϕ_s may seem to be low in comparison to the values specified in the ACI Standard 318-83, which vary from 0.7–0.9, depending on the end eccentricity ratio (e/h). Many engineers, however, use a value of 0.7 regardless of the eccentricity ratio.

Conclusions

The discussions and data presented in this paper show that a stability resistance factor of 0.7–0.75 can be used for computing moment magnification of slender columns subjected to single-curvature bending and short-time loads. This value of ϕ_s is applicable to column designs based on (1) The cross-section resistance factors suggested in Refs. 5 and 8; (2) the ACI 318-83 load factors; and (3) the 1983 ACI Building Code design expressions.

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Appendix I.—References

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Appendix II.—Notation

The following symbols are used in this paper:

A_t = tributary area;
C_m = equivalent uniform bending moment diagram factor;
D = specified dead load or its effect;
e = eccentricity of axial load acting at column end;
h = overall thickness of column cross section measured perpendicular to the neutral axis;
L = specified live load or its effect;
l = unsupported length of column measured between column ends;
M_c = magnified factored bending moment;
M_2 = larger factored bending moment applied to the ends of a column;
P_c = critical buckling strength of column;
P_u = factored axial load acting on column;
R = actual resistance of column;

$\bar{R}$ = mean value of R;
R_n = ACI nominal resistance of column;
S = specified snow load or its effect;
U = actual lifetime maximum load effect;
U_n = specified load or its effect;
V_R = coefficient of variation of R;
W = specified wind load or its effect;
Y = R/U;
$\bar{Y}$ = mean value of Y;
α = load factor;
β = safety index as defined in the text;
ρ = ratio of nonprestressed tension reinforcement;
ρ_b = reinforcement ratio producing balanced strain conditions;
ρ_g = longitudinal reinforcement ratio in column cross section;
ϕ = resistance factor (strength reduction factor);
ϕ_c = resistance factor applied to the strength of column cross section; and
ϕ_s = stability resistance factor applied to critical buckling strength of column.