Shear Flow Zone in Torsion of Reinforced Concrete

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Abstract: Rausch's classical formula overestimates the torsional strength of reinforced concrete members to an unacceptable degree. The error is traced to the incorrect determination of the centerline of the circulating shear flow. The position of the centerline of shear flow is directly related to the thickness of the shear flow zone (t_d). The determination of t_d in torsion is analogous to the determination of the depth of the compression zone in bending. This paper presents a simple theoretical method to calculate t_d based on the softened truss model theory. The method utilizes the equilibrium and compatibility conditions, as well as a softened stress-strain relationship for concrete struts. Since t_d is calculated by a rigorous procedure, an accurate torsional strength can be predicted. The prediction of the torsional strengths of 61 beams found in the literature compares extremely well with the test values. In addition, a very simple formula for t_d is also proposed for the practical design of members subjected to torsion.

Introduction

The basic formula for calculating the torsional strength of reinforced concrete members was developed by Rausch (1929) using the space truss concept. Unfortunately, Rausch's equation may be unconservative by more than 30% for under-reinforced beams (Hsu 1968a, 1968b). The error is traced to the incorrect determination of the centerline of the circulating shear flow, resulting in the overestimation of the lever arm area A_0. The correct determination of the centerline of shear flow depends on a logical way to find the thickness of the shear flow zone, t_d.

Since the late 1960s, the truss model theory for shear and torsion has undergone four major developments. First, the introduction of the variable-angle truss model and the discovery of the bending phenomenon in the diagonal concrete struts were made by Lampert and Thurlimann (1968, 1969). Second, compatibility equation was derived by Collins (1973) to determine the angle of the diagonal concrete struts. Third, the softening phenomenon in the concrete struts was discovered by Robinson and Demorieux (1972), and this behavior was quantified by Vecchio and Collins (1981), using a softening coefficient. Fourth, combining the equilibrium, compatibility and softened stress-strain relationships, a softened truss model theory was developed (Hsu 1988), which was able to analyze the shear and torsional behavior of reinforced concrete members throughout the post-cracking loading history.

Using the softened truss model theory, the thickness of the shear flow zone t_d can expeditiously be calculated for the torsional strength of reinforced concrete members. This method is presented in this study. In addition, a simple expression for t_d is proposed for practical design.

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Analogy Between Torsion and Bending

A prismatic member of arbitrary cross section subjected to torsion is shown in Fig. 1(*a*). The circulating shear stresses are restricted to an outer ring area with a thickness of t_d. Within this thickness a shear flow q acts along a certain centerline, s. Taking equilibrium about the center of twist O, the external torque T is resisted by the internal moment:

$$T = q\oint a\,ds \qquad (1)$$

where a = the lever arm of shear flow q measured from the center of twist, O. The product ads is represented graphically by twice the shaded triangular area shown. Therefore, $\oint ads$ is twice the area within the centerline of shear flow, and will be denoted as $2A_0$. A_0 will be called the lever arm area and is proportional to the square of the level arm a. Substituting $2A_0$ into Eq. 1 gives

$$T = q(2A_0) \qquad (2)$$

Eq. 2 is Bredt's thin-tube theory (1896), but it should also be applicable to a thick tube, if the position of the centerline of shear flow can be determined.

In a reinforced concrete member, after cracking, as shown in Fig. 2(*a*), an element isolated from the tube defined by the shear flow zone with a thickness t_d, Fig. 2(*c*), can be represented by a truss model in Fig. 2(*d*). The element has a vertical length as well as a horizontal length of unity. The diagonal lines representing the cracks are inclined at an angle α. Taking equilibrium of forces on the horizontal face and assuming yielding of steel gives:

$$q = \frac{A_t f_{ty}}{s} \cot\alpha \qquad (3)$$

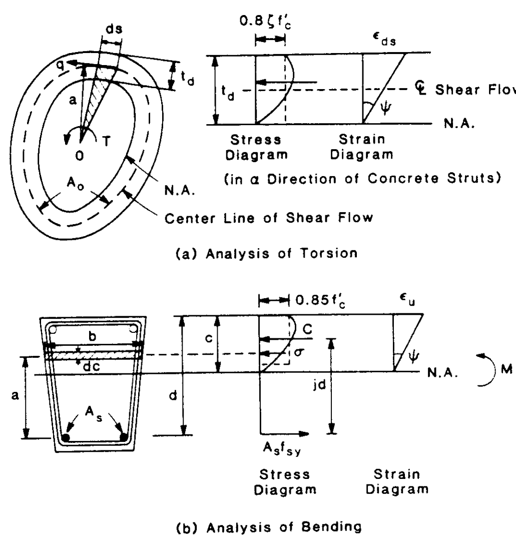
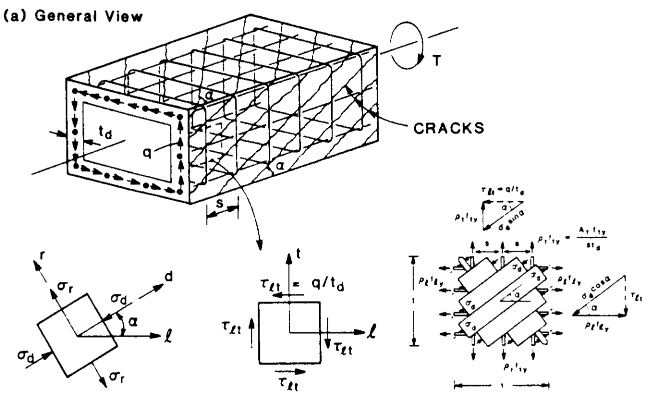


FIG. 1. **Analogy Between Torsion and Bending**



(b) Shear Flow Zone Element in d-r Coordinate (c) Shear Flow Zone Element in ℓ-t Coordinate (d) Truss Model of Element in Shear Flow Zone

FIG. 2. Reinforced Concrete Members Subjected to Torsion

Inserting q from Eq. 3 into Eq. 2:

$$T_n = \frac{A_t f_{ty} \cot \alpha}{s} (2A_0) \quad (4)$$

Eq. 4 is the fundamental equation for torsion in the variable-angle truss model. It reduces to the well-known Rausch's equation (1929) when α is taken as 45°. When equilibrium of forces is taken on the vertical face of the truss model element in Fig. 2(d), q and T_n can be expressed in terms of the longitudinal steel, i.e., $q = (A_l f_{ly}/p_0) \tan \alpha$ and $T_n = [(A_l f_{ly}/p_0) \tan \alpha](2A_0)$.

The analysis of torsion shown is analogous to the analysis of bending in a prismatic members shown in Fig. 1(b). Taking the moment about the centroid of the tension steel, the external moment, M, is resisted by the internal moment

$$M = \int(\sigma b) a dc = C(jd) \quad (5)$$

where C = the resultant of stresses σ in the compression zone and jd = the lever arm of the resultant. Comparing Eq. 5 to Eq. 2, the term of twice the lever arm area $2A_0$ in Eq. 2 is equivalent to the resultant lever arm jd in Eq. 5, and the shear flow q is similar to the resultant of compressive stresses C.

After cracking of the flexural member, the truss model concept in bending is reduced to the so-called internal couple concept. Assuming the yielding of steel gives

$$C = A_s f_{sy} \quad (6)$$

Substituting C from Eq. 6 into Eq. 5:

$$M_n = A_s f_{sy}(jd) \quad (7)$$

Eq. 7 shows that the bending moment capacity, M_n, is equal to the longitudinal steel force, $A_s f_{sy}$, times the resultant lever arm, jd. Similarly, in Eq.

4 the torsional moment capacity, T_n, is equal to a certain stirrup force per unit length, $(A_t f_{ty}/s) \cot \alpha$, times twice the lever arm area, $2A_0$.

In bending, an increase of the nominal bending strength M_n due to increasing reinforcement results in an increase of the depth of the compression zone c, and a reduction of the resultant lever arm jd. The relationships among M_n, c and jd can be derived from the stress and strain diagrams. Similarly, in torsion, an increase of the nominal torsional strength T_n due to increasing reinforcement results in an increase of the thickness of shear flow zone t_d and a reduction of the lever arm area A_0. The relationships among T_n, t_d and A_0 can also be derived from the stress and strain diagrams. The understanding of these relationships is a purpose of this study. The crucial problem in torsion of reinforced concrete is to find the thickness of the shear flow zone t_d, which is analogous to finding the depth of the compression zone c in bending.

Various Definitions of Lever Arm Area A_0

When Rausch derived Eq. 4 (with α = 45°) in 1929, a reinforced concrete member after cracking was idealized as a space truss. The longitudinal and hoop bars are assumed to take tension and the diagonal concrete struts are in compression. Each diagonal concrete strut is idealized as a straight line lying in the center surface of the hoop bars. Hence, the lever arm area, A_0, is defined by the area within the center surface of the hoop bars. This area is commonly denoted as A_1. It has been adopted since 1958 by the German Code, and others. Using the bending analogy, this definition is equivalent to assuming that the resultant lever arm, jd, is defined as the distance between the centroid of the tension bars and the centerline of the stirrups in the compression zone. In terms of torsional strength this assumption is acceptable near the lower limit of the total steel percentage of about 1%, but becomes increasingly unconservative with an increasing amount of steel (Fig. 3). For a large steel percentage of 2.5–3% near the upper limit of under reinforcement (both the longitudinal steel and stirrups reach yielding), the over prediction of torsional strength by Rausch's equation using A_1 exceeds 30%. This large error is caused by two conditions. First, the thickness of the shear flow zone t_d may be very large, in the order of 1/4 of the outer cross-sectional dimension, due to the softening of concrete (to be explained in the next section). Second, in contrast to the bending strength M_n, which is linearly proportional to the resultant lever arm jd, the torsional strength T_n is proportional to the lever arm area A_0, which, in turn, is proportional to the square of the lever arm, a, [Fig. 1(a)].

In order to reduce the unconservatism of using A_1 in Rausch's equation, Lampert and Thurlimann (1969) have proposed that A_0 be defined as the area within the polygon connecting the centers of the corner longitudinal bars. This area is commonly denoted as A_2 and has been adopted by the CEB-FIP Code ("Model Code" 1978). In terms of the bending analogy, this definition is equivalent to assuming that the resultant lever arm, jd, is defined as the distance between the centroid of the tension bars and the centroid of the longitudinal compression bars. The introduction of A_2 has reduced the unconservatism of Rausch's equation for high steel percentages. However, the assumption of a constant lever arm area (not a function of the thickness of shear flow zone) remains unsatisfactory.

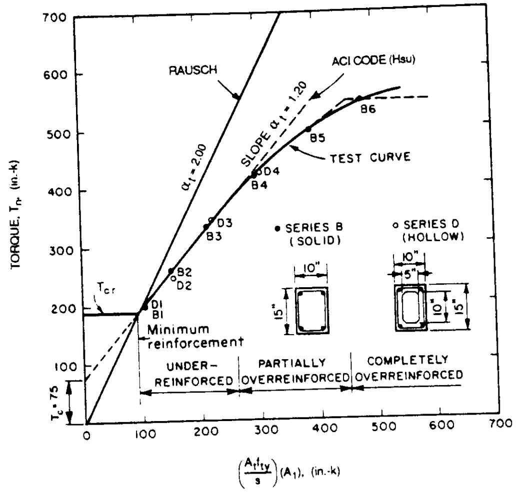


FIG. 3. Comparison of Rausch's Formula and ACI Code Formula with Tests (1 in. = 25.4 mm; 1 in.-kip = 113 N-m)

Another way of modifying Rausch's equation has been suggested by the writer (Hsu 1968a, 1968b) and adopted by the ACI Building Code ("Building Code" 1971).

$$T_n = T_c + \frac{A_t f_{ty}}{s}(\alpha_t A_1) \quad (8)$$

where $\alpha_t = 0.66 + 0.33\, y_1/x_1 \leq 1.5$; x_1 = shorter center-to-center dimension of a rectangular closed stirrup; y_1 = longer center-to-center dimension of a rectangular closed stirrup; T_c = nominal torsional strength contributed by concrete = $0.8x^2y\sqrt{f'_c}$ where x and y = the shorter and longer sides, respectively, of a rectangular section.

Two modifications of Rausch's equation are made in Eq. 8 based on tests. First, a smaller lever arm area $(\alpha_t/2)A_1$ is specified, where α_t varies from 1 to 1.5. Second, a new term T_c is added. This term represents the vertical intercept of a straight line in the T_n versus $(A_t f_{ty}/s)(A_1)$ diagram (Fig. 3). Although the addition of T_c allows the test curve to be closely approximated by a straight line in the under-reinforced region, the complexity that is generated by T_c is certainly undesirable.

The definitions of the lever arm areas, A_1, A_2 or $(\alpha_t/2)A_1$, all have a common weakness. They are not related to the thickness of the shear flow zone or the applied torque. A logical way to define A_0 must start with the determination of the thickness of shear flow zone.

Analysis of Thickness of Shear Flow Zone, t_d

The determination of t_d requires three equations, derived from compatibility, equilibrium, and material law. The derivations are shown.

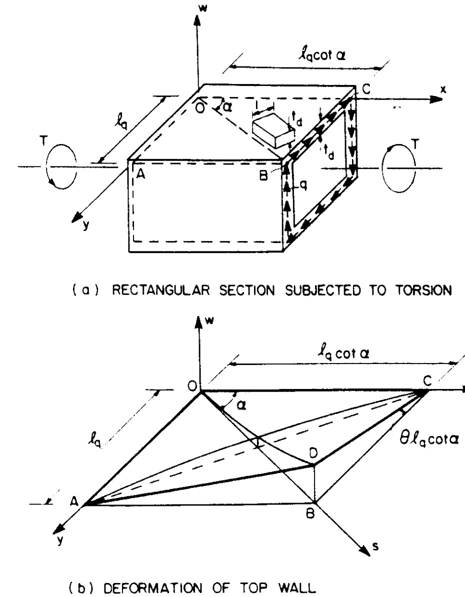


FIG. 4. Bending of Concrete Struts in Torsion

Compatibility Equations

When a hollow reinforced concrete member is subjected to torsion as shown in Fig. 2, each cross section will rotate, producing an angle of twist, θ, in the member and a shear strain γ_{lt} in the shear flow tube. According to Bredt's theory for circulatory torsion (see Chapter 1 in Hsu 1984) θ and γ_{lt} are related by the compatibility condition:

$$\theta = \frac{p_0}{2A_0}\gamma_{lt} \quad (9)$$

where p_0 = the perimeter of the centerline of shear flow.

After diagonal cracking and the formation of the truss action, the shear strain γ_{lt} in the shear flow tube will cause tensile strains in the longitudinal and transverse reinforcement ϵ_l and ϵ_t in the l-t direction and the principal compressive strains ϵ_d and ϵ_r in the d-r direction. The angle between the l axis and the d axis is denoted as α, Fig. 2(b). The shear strain γ_{lt} can be expressed in terms of ϵ_d, ϵ_r, and α (Hsu 1988) as follows:

$$\gamma_{lt} = 2(\epsilon_d - \epsilon_r)\sin\alpha\cos\alpha \quad (10)$$

In addition to the strain ϵ_d in the d-direction, a diagonal concrete strut will also be subjected to a bending action resulting from the angle of twist θ, (Fig. 4). The plane OABC lying in the shear flow tube through the centerline of the shear flow is isolated in Fig. 4(b). After twisting, this plane becomes a hyperbolic paraboloid surface OADC. The diagonal line OB, which represents a concrete strut with an angle of inclination α, becomes a curve OD. The curvature of the concrete struts, ψ, can be related by geometry to the

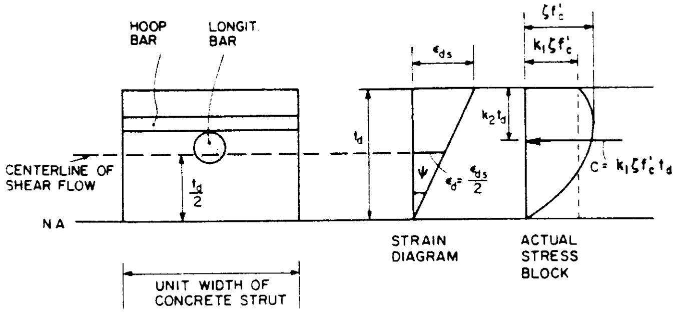


FIG. 5. Strain and Stress Distribution in Concrete Struts

angle of twist, θ, (Lampert and Thurlimann 1968; Hsu 1984) by:

$$\psi = \theta \sin 2\alpha \qquad (11)$$

It should be noted that Eq. 11 is applicable not only to a rectangular section, but also to arbitrary bulky sections. In Fig. 4 a rectangular section is selected to demonstrate the bending curvature in a diagonal concrete strut due to twisting. This is because the imposed curvature is easier visualized in a plane than in a curved surface.

Due to the bending of the diagonal concrete struts the tension area in the inner portion of the cross section will be neglected, Figs. 4 and 5. The compression area will be considered as effective. The depth of the compression zone is denoted t_d, which is identical to the thickness of the shear flow zone. Within the thickness t_d, the strain distribution is assumed to be linear. Therefore, t_d can be related to the maximum strain at the surface ϵ_{ds} and the curvature ψ by

$$t_d = \frac{\epsilon_{ds}}{\psi} \qquad (12)$$

Moreover, because of the linear strain distribution, the maximum strain at the surface ϵ_{ds} should be related to the average strain ϵ_d by

$$\epsilon_{ds} = 2\epsilon_d \qquad (13)$$

The thickness of the shear flow zone t_d in Eq. 12 can be expressed in terms of ϵ_d, ϵ_r, and α by a series of substitution: (1) Substitute γ_{lt} from Eq. 10 into Eq. 9; (2) substitute θ from Eq. 9 into Eq. 11; (3) substitute ψ from Eq. 11 into Eq. 12; and (4) substitute ϵ_{ds} from Eq. 13 into Eq. 12. The resulting expression is:

$$t_d = \frac{A_0}{p_0 \sin^2\alpha \cos^2\alpha}\left(\frac{\epsilon_d}{\epsilon_d - \epsilon_r}\right) \qquad (14)$$

Notice that A_0 and p_0 in Eq. 14 are also functions of t_d:

$$A_0 = A_c - \frac{t_d}{2} p_c + \xi t_d^2 \qquad (15)$$

$$p_0 = p_c - 4\xi t_d \qquad (16)$$

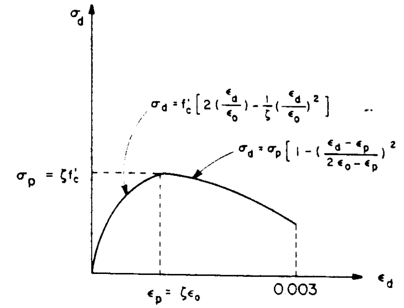


FIG. 6. Stress-Strain Curve for Softened Concrete

where A_c = area bounded by the outer perimeter of concrete cross section; p_c = outer perimeter of concrete cross section; ξ = coefficient equal to 1 for rectangular section and $\pi/4$ for circular section. ξ can be taken as unity for all shapes of cross sections with only negligible loss of accuracy for A_0 and p_0.

A_0 and p_0 have been expressed in Eqs. 15 and 16 by assuming that the centerline of the shear flow lies midway in the thickness of the shear flow zone t_d. This assumption provides two advantages: First, it simplifies the expressions of A_0 and p_0. Second, it is slightly on the conservative side and is desirable to compensate for the slight unconservatism inherent in using the term $A_t f_{ty}/s$ in Eq. 4 to express the stirrup force per unit length. The effect of the stirrup spacing, s, on the shear strength has been carefully explained in Section 4.4.3.1 of Hsu (1984). It should also be pointed out that A_0 and p_0 in Eqs. 15 and 16 are applicable to a thick tube even when ξ is taken as unity.

Material Law

Being subjected to axial stress and bending, the distribution of the compressive stresses in a diagonal concrete strut within the thickness t_d is shown by the solid curve in Fig. 5. This stress-strain relationship is based on a softened stress-strain curve, Fig. 6, proposed by Vecchio and Collins (1981). Their concrete test panels were reinforced in both the longitudinal and transverse directions and were subjected to pure shear at the edges. Their tests clearly show that after diagonal cracking the stress and the strain in the concrete struts, σ_d and ϵ_d, are softened by the tensile strain in the perpendicular direction, ϵ_r. The softened coefficient ζ is a function of the ratio ϵ_r/ϵ_d:

$$\zeta = \sqrt{\frac{\epsilon_d}{\epsilon_d - \epsilon_r}} \qquad (17)$$

The softening coefficient ζ, which is less than unity, is the reciprocal of the coefficient λ given by Vecchio and Collins (1981). In their paper, ϵ_d in the denominator is multiplied by a constant $(1 - \mu)$, where μ is Poisson's ratio for concrete. The omission of μ produces negligible difference (Hsu 1984). Note also that ϵ_d is negative and $\epsilon_r = \epsilon_l + \epsilon_t - \epsilon_d$ is positive.

Based on the softened stress-strain relationship in Fig. 6, the peak stress is $\zeta f'_c$ and the average compressive stress, σ_d, can be defined as

$$\sigma_d = k_1 \zeta f'_c \quad (18)$$

where k_1 = the ratio of the average stress to peak stress in the stress block. The k_1-ratio can be obtained by integrating the stress-strain curve in Fig. 6 and has been tabulated in Table 7.4 of Hsu (1984) as a function of the maximum strain ϵ_{ds} and the softening coefficient ζ. For under-reinforced members, the maximum torque occurs when the maximum concrete strain ϵ_{ds} varies from 0.0015 to 0.0030, and ζ varies from 0.35 to 0.50. Within those ranges, the table shows that k_1 varies in a narrow range from 0.85 to 0.77. Taking an average value of $k_1 = 0.80$, and treating f'_c as positive, then σ_d becomes

$$\sigma_d = -0.80 \zeta f'_c \quad (19)$$

Substituting the softening coefficient ζ from Eq. 17 into Eq. 14, t_d can be expressed in terms of ζ and α:

$$t_d = \frac{A_0 \zeta^2}{p_0 \sin^2\alpha \cos^2\alpha} \quad (20)$$

It is interesting to note that t_d in Eq. 20 is no longer a function of the strains ϵ_d or ϵ_r. Physically, this means that t_d is independent of the loading history.

The substitution of the softening coefficient ζ from Eq. 17 into Eq. 14 involves an assumption. Since Eq. 17 is obtained from tests of reinforced concrete panels subjected to pure shear alone, the strains ϵ_d and ϵ_r in this equation represent the uniform in-plane strains of an element without bending. By contrast, Eq. 14 is derived from an element in a concrete strut subjected to in-plane strains as well as bending, so that ϵ_d and ϵ_r represent the average strains in the mid depth of the thickness t_d. Therefore, Eq. 20 is obtained by assuming that the softening of a concrete strut subjected to compression and bending is identical to the softening of a concrete strut subjected to the average compression strain without bending. This assumption has yet to be proven by tests, but it should provide a very good approximation.

The thickness of shear flow zone, t_d, can be solved by Eq. 20 in conjunction with two equilibrium equations.

Equilibrium Equations

From the truss model of a reinforced concrete element shown in Fig. 2(*d*) it can be demonstrated that the stresses in the concrete satisfy Mohr's stress circle (Hsu 1984). Assuming that the steel will yield at failure (for under-reinforced members) and the concrete cannot resist tension in the direction perpendicular to the cracks, i.e., $\sigma_r = 0$, then the superposition of the concrete stresses and steel stresses gives the following three equilibrium equations:

$$\sigma_l = \sigma_d \cos^2\alpha + \rho_l f_{ly} \quad (21)$$

$$\sigma_t = \sigma_d \sin^2\alpha + \rho_t f_{ty} \quad (22)$$

$$\tau_{lt} = \sigma_d \sin\alpha \cos\alpha \quad (23)$$

where σ_l, σ_t = normal stress in the l and t directions, respectively (positive for tension); τ_{lt} = shear stresses in the l-t coordinate (negative, as shown in Fig. 2); ρ_l, ρ_t = reinforcement ratio in the l and t directions, respectively. $\rho_l = A_l/p_0 t_d$, where A_l is the total area of longitudinal steel in the cross section; and $\rho_t = A_t/st_d$, where A_t is the area of one leg of a hoop bar and s is the spacing of the hoop bars; f_{ly}, f_{ty} = yield strength of the longitudinal and transverse steel, respectively.

For the case of pure torsion, $\sigma_l = \sigma_t = 0$. Adding Eqs. 21 and 22 and inserting $\sigma_d = -0.80\zeta f'_c$ result in:

$$\zeta = \frac{\left(\frac{A_l f_{ly}}{p_0}\right) + \left(\frac{A_t f_{ty}}{s}\right)}{0.80 f'_c t_d} \quad (24)$$

Substituting $\sigma_d = -0.80\zeta f'_c$ from Eq. 19 into Eq. 21 and utilizing Eq. 24 give:

$$\cos^2\alpha = \frac{\left(\frac{A_l f_{ly}}{p_0}\right)}{\left(\frac{A_l f_{ly}}{p_0}\right) + \left(\frac{A_t f_{ty}}{s}\right)} \quad (25)$$

Solution Method

The compatibility Eq. 20 and the two equilibrium Eqs. 24 and 25 provide three equations involving three unknown variables, t_d, ζ, and α. The solution of these three simultaneous equations can be obtained by a simple trial-and-error procedure as follows:

1. Assume an initial value of t_d and calculate A_0 and p_0 by Eqs. 15 and 16.
2. Compute ζ and α from Eqs. 24 and 25, respectively.
3. Substituting ζ and α into Eq. 20 gives t_d. If the resulting t_d is close enough to the initial value, then a solution with a set of t_d, ζ, and α values are obtained. If the resulting t_d is not close enough to the initial value, assume another t_d, and repeat the cycle.

Once a solution is obtained, the ultimate shear stress τ_{lt} can be calculated from Eqs. 23 and 19, the torsional strength T_n can be obtained from Eq. 4. An example problem showing the solution procedures is given in Appendix I.

Comparison with Tests

This method of calculating the thickness of shear flow zone and the torsional strength has been applied to analyze the 61 test beams available in literature (McMullen and Warwaruk 1967; Hsu 1968a; Lampert and Thurlimann 1968, 1969; Bradburn 1968; Leonhardt and Schelling 1974; Mitchell and Collins 1974; McMullen and Rangan 1978). The calculated $t_{d,\text{calc}}$ and $T_{n,\text{calc}}$ are recorded in Table 1. The $T_{n,\text{calc}}$ values are also compared to the test values $T_{n,\text{test}}$. The average $T_{n,\text{test}}/T_{n,\text{calc}}$ is 1.010 and the standard deviation is 0.051.

The 61 beams available in literature satisfy the following four criteria (Hsu and Mo 1985a, 1985b): (1) The member should have sufficient reinforce-

TABLE 1. Comparison of Test Strengths to Calculated Strengths Using t_d From Eq. 20

Specimen (1)	$t_{d,calc}$ (in.) (2)	$T_{n,calc}$ (in.-kip) (3)	$T_{n,test}$ (in.-kip) (4)	$T_{n,test}/T_{n,calc}$ (5)
		(a) Hsu (1968a)		
B1	1.452	200.2	197.0	0.984
B3	2.341*	366.4*	332.0	0.906
B4	2.493*	428.9*	419.0	0.977
B7	1.979*	252.3	236.0	0.935
B8	2.206*	275.3*	288.0	1.046
B9	1.887*	265.0*	264.0	0.996
B10	2.253*	276.2*	304.0	1.101
D1	1.510	203.5	198.0	0.973
D3	2.345*	371.7*	346.0	0.931
D4	2.494*	429.4*	424.0	0.987
M1	1.678	252.7	269.0	1.064
M2	2.101	340.1	359.0	1.055
M3	2.770	394.2	388.0	0.984
J1	2.275	181.3	190.0	1.048
G2	1.636	339.1	357.0	1.053
G3	2.272	444.7	439.0	0.987
G4	2.581*	543.4*	574.0	1.056
G6	1.669	337.0	346.0	1.027
G7	2.095	466.3	466.0	0.999
N1	1.058	75.1	80.5	1.072
N1a	1.038	75.4	79.6	1.055
N2	1.473	119.1	128.0	1.075
N2a	1.558	119.2	117.0	0.981
N3	1.426	102.9	108.0	1.049
K1	1.115	131.4	136.0	1.035
K2	1.565	207.9	210.0	1.010
K3	1.831*	237.2*	252.0	1.062
I2	1.425	320.6	319.0	0.995
I3	1.808	429.3	404.0	0.941
I4	2.147	529.3	514.0	0.971
I5	2.424*	611.7*	626.0	1.023
		(b) Leonhardt and Schelling (1974)		
VQ1	1.723	183.8	187.0	1.018
VQ3	1.691	180.8	177.0	0.983
VQ4	2.470*	255.2*	271.0	1.062
VQ9	1.427	179.3	194.0	1.082
VH1	1.875	182.8	189.0	1.034
VM1	1.169	114.9	123.0	1.071
VM2	1.606	326.7	347.0	1.062
VM3	2.113	838.3	893.0	1.065
VM4	3.014	2,591.7	2,472.0	0.954
VS2	1.692	184.0	173.0	0.940
VS3	2.202	253.9	253.0	0.996
VS9	1.958*	203.7*	191.0	0.938
VB3	1.866	426.5	411.0	0.964
VB4	1.590	444.7	430.0	0.967
VU1	1.883*	205.6*	212.0	1.034
VU2	2.174*	256.4*	269.0	1.049
VU4	1.933*	201.5*	230.0	1.141
		(c) McMullen and Rangan (1978)		
A2	1.410	186.5	200.0	1.072
A3	1.765	252.4	246.0	0.975
A4	2.242	327.9	305.0	0.930
B2*	1.260	169.3	184.0	1.087
B3*	1.661	231.0	224.0	0.970
B4*	2.067	295.6	281.0	0.950

TABLE 1. (Continued)

(1)	(2)	(3)	(4)	(5)
		(d) Lampert and Thurlimann (1968)		
T0	3.632*	1,861.2*	1,830.0	0.983
T1	3.225	1,147.4	1,145.0	0.998
T2	3.225	1,147.4	1,162.0	1.013
T4	3.225	1,147.4	1,145.0	0.998
		(e) Mitchell and Collins (1974)		
PT4	2.980	589.3	620.0	1.052
		(f) Bradburn (1968)		
R1	1.323*	101.2*	93.6	0.925
		(g) McMullen and Warwaruk (1967)		
2-1	1.665*	189.5*	181.0	0.955

*Since actual steel stresses in these specimens are slightly less than yield stresses, calculated values are used in computing these values. 1 in. = 25.4 mm; 1 in.-kip = 113 N-m.

ment so that the beam will not fail brittlely at cracking, i.e. $T_n > T_{cr}$. T_{cr} for a hollow beam can be taken as $2A_c t(2.5\sqrt{f'_c})$, where t is the actual wall thickness. For solid beams, t can be taken as A_c/p_c; (2) the member should be under-reinforced so that both the longitudinal bars and the stirrups will yield at failure. To achieve this purpose, α should be greater than $12° + 33°[\tau_n/f'_c(0.27 - 45\epsilon_{ly})]$ but less than $78° - 33°[\tau_n/f'_c(0.27 - 45\epsilon_{ty})]$; (3) stirrup spacing should not be excessive to cause significant drop of torsional strength, i.e., s should be less than $p_1/8$ or 12 in. (30 cm); (4) concrete cover should not be too thick to cause spalling before the maximum torque is reached. In other words, the distance $\bar{c}$ measured from the concrete surface to the inner face of the transverse hoop bars should be less than $0.75t_d$.

The theory presented here has been rigorously derived. The only major inaccuracy introduced is the approximation of $k_1 = 0.80$. This approximation should be quite good after the maximum fiber strain ϵ_{ds} reaches well into the descending branch of the softened stress-strain curve, i.e. $\epsilon_{ds} > 1.5\zeta\epsilon_0$ where ϵ_0 is taken as 0.002. Therefore, the theory is very suitable for finding the torsional strength. At the low load stages when $\epsilon_{ds} < \zeta\epsilon_0$, however, $k_1 = 0.80$ would not be sufficiently accurate, and a more general method of solution (Hsu 1988) should be used.

Thickness of Shear Flow Zone for Design

The thickness of the shear flow zone given in Eq. 20 is suitable for the analysis of torsional strength. It is, however, not convenient for the design of torsional members. In design, the thickness of shear flow zone t_d should be expressed in terms of the torsional strength, T_n. This approach will now be introduced.

The stress in the diagonal concrete struts, σ_d, can be related to the thickness t_d and the shear flow q using the equilibrium Eq. 23:

$$\sigma_d = \frac{q}{t_d \sin\alpha \cos\alpha} \quad (26)$$

At failure, σ_d in Eq. 26 reaches the maximum $\sigma_{d,\max}$, while the torsional moment reaches the nominal capacity T_n. Substituting $q = T_n/2A_0$ at failure into Eq. 26 gives:

$$t_d = \frac{T_n}{2A_0\sigma_{d,\max}\sin\alpha\cos\alpha} \quad \text{(27)}$$

Assuming t_d to be thin, the last term ξt_d^2 in Eq. 15 is neglected, and A_0 can be expressed by the thin-tube approximation:

$$A_0 = A_c - \frac{t_d}{2}p_c \quad \text{(28)}$$

Substituting A_0 from Eq. 28 into Eq. 27 and multiplying all the terms by $2p_c/A_c^2$ results in:

$$\left(\frac{p_c}{A_c}t_d\right)^2 - 2\left(\frac{p_c}{A_c}t_d\right) + \frac{T_n p_c}{A_c^2}\frac{1}{\sigma_{d,\max}\sin\alpha\cos\alpha} = 0 \quad \text{(29)}$$

Define:

- $t_{d0} = A_c/p_c$
- $\tau_n = T_n p_c/A_c^2$
- $\tau_{n,\max} = \sigma_{d,\max}\sin\alpha\cos\alpha$

Eq. 29 becomes

$$\left(\frac{t_d}{t_{d0}}\right)^2 - 2\left(\frac{t_d}{t_{d0}}\right) + \frac{\tau_n}{\tau_{n,\max}} = 0 \quad \text{(30)}$$

When t_d/t_{d0} is plotted against $\tau_n/\tau_{n,\max}$ in Fig. 7, Eq. 30 represents a parabolic curve. Solving t_d from Eq. 30 gives:

$$t_d = t_{d0}\left[1 - \sqrt{1 - \frac{\tau_n}{\tau_{n,\max}}}\right] \quad \text{(31)}$$

This approach of determining the thickness of the shear flow zone, first proposed by Collins and Mitchell (1980) and later adopted by the Canadian Code ("Design" 1984), gives:

$$t_d = \frac{A_1}{p_1}\left[1 - \sqrt{1 - \frac{T_n p_1}{0.7\phi_c f'_c A_1^2}\left(\tan\alpha + \frac{1}{\tan\alpha}\right)}\right] \quad \text{(32)}$$

In Eq. 32 A_c and p_c are replaced by A_1 and p_1, respectively, since the concrete cover is considered ineffective. $\sigma_{d,\max}$ is assumed to be $0.7\phi_c f'_c$, in which the material reduction factor ϕ_c can be taken as 0.6.

Eqs. 32 and 31 clearly show that the thickness ratio, t_d/t_{d0}, is primarily a function of the shear stress ratio, τ_n/f'_c. The thickness ratio t_d/t_{d0} is also a function of the crack angle α, but is not sensitive when α varies in the vicinity of 45°.

Eq. 31, $\tau_n < \tau_{n,\max}$ represents the case of under-reinforcement, while $\tau_n > \tau_{n,\max}$ means over-reinforcement. The case of over-reinforcement cannot be expressed by Eq. 31, because it gives a complex number ($\sqrt{-1}$). Fig.

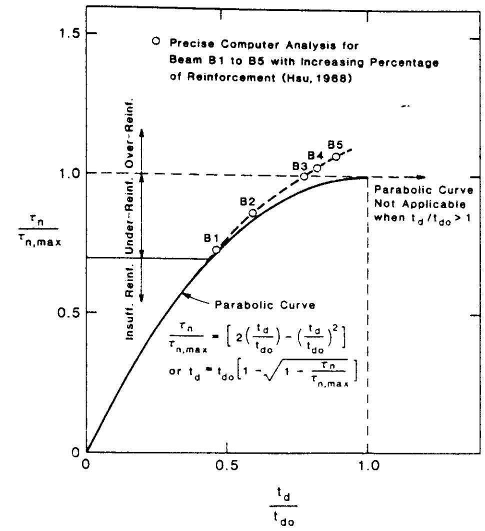


FIG. 7. Graphical Presentation of Eqs. 30 or 31

7 shows that Eq. 31 is applicable when τ_n is less than about 0.9 $\tau_{n,\max}$. However, when τ_n exceeds 0.9 $\tau_{n,\max}$, t_d is increasing unreasonably fast. This problem reflects the difficulty in using the thin-tube approximation for A_0 (Eq. 28) to find t_d. When t_d exceeds about 0.7 t_{d0}, the tube becomes so thick that the term ξt_d^2 cannot be neglected.

To avoid this weakness, the writer has adopted a different approach. Using the softened truss model theory, a computer program was written to analyze the torsional behavior of reinforced concrete members throughout the loading history (Hsu and Mo 1985a). This computer program was used to analyze the 61 eligible torsional members (satisfying the four criteria previously cited) available in literature. The thicknesses of the shear flow zones in the test beams are calculated from the computer program and a linear regression analysis of the thickness ratios t_d/t_{d0} is made as a function of τ_n/f'_c. This analysis provides the following expression (Hsu and Mo 1985b):

$$t_d = \frac{A_c}{p_c}\left(0.082 + 3.405\frac{\tau_n}{f'_c}\right)\frac{1}{\sin 2\alpha} \quad \text{(33)}$$

Eq. 33 is plotted in Fig. 8 for the cases of $\alpha = 45°$ and $\alpha = \tan^{-1}(5/3)$ or $\tan^{-1}(3/5)$, which are the limits adopted by CEB-FIP Code ("Model Code" 1978). The 61 test points are also included and the correlation is shown to be excellent. The t_d/t_{d0} values calculated from Eq. 33 for the writer series B are recorded in Table 2. When compared to the t_d/t_{d0} values obtained from the computer program, the correlation is again excellent. The 10 beams in Series B were chosen because they have total reinforcement ratios varying from a low of 1.07% to a high of 5.28%, and a volume ratio of longitudinal steel to stirrups varying from 0.205 to 4.97. The wide range of application of Eq. 33 is evident. It is not only applicable to under-reinforced members, but also to over-reinforced members.

Although Eq. 33 is found to be excellent, it is considered somewhat un-

TABLE 2. Comparison of Thickness Ratios t_d/t_{d0}

			Computer Program (Hsu 1985a)					Eq. 33	Eq. 36
Beam (1)	$\bar{\rho}_l$ (%) (2)	$\bar{\rho}_t$ (%) (3)	T_n (in.-kip) (4)	α (°) (5)	ζ (6)	t_d (in.) (7)	t_d/t_{d0} (8)	t_d/t_{d0} (9)	t_d/t_{d0} (10)
B1	0.534	0.537	202	46.5	0.372	1.41	0.470	0.464	0.449
B2	0.827	0.823	287	44.8	0.340	1.78	0.593	0.606	0.615
B3	1.77	1.17	370	44.6	0.510	2.32	0.770	0.770	0.808
B4	1.60	1.61	431	44.6	0.531	2.46	0.820	0.818	0.865
B5	2.11	2.13	446	44.5	0.560	2.65	0.883	0.884	0.942
B6	2.67	2.61	466	44.5	0.579	2.77	0.923	0.926	0.991
B7	0.534	1.17	253	53.8	0.433	1.96	0.653	0.619	0.597
B8	0.534	2.61	278	56.7	0.440	2.12	0.707	0.681	0.637
B9	1.17	0.537	269	36.3	0.416	1.82	0.607	0.596	0.572
B10	2.67	0.537	280	33.2	0.444	2.16	0.720	0.691	0.648

Note: $\bar{\rho}_l$ and $\bar{\rho}_t$ are the reinforced ratios of longitudinal steel and transverse hoop steel, respectively, based on total cross-sectional area A_c. Cross section 10 in. × 15 in. $f_y \approx 47{,}000$ psi; $f'_c \approx 4{,}000$ psi; 1 in. = 25.4 mm; 1 psi = 6.89 kPa.

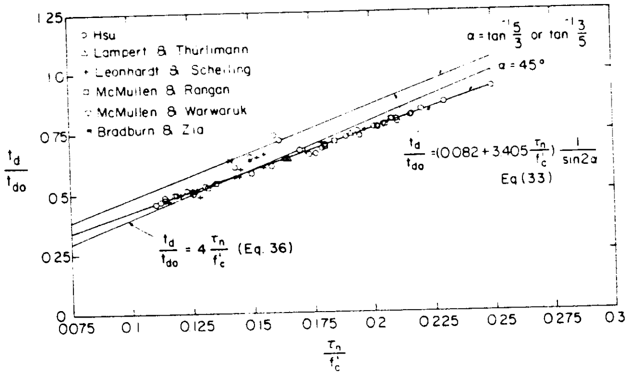


FIG. 8. Thickness Ratio t_d/t_{d0} as Straight Line Functions of Shear Stress Ratio, τ_n/f'_c

wieldy for practical design. In the next section a simplified expression for t_d is proposed. The simplicity is obtained with a small sacrifice in accuracy.

PROPOSED t_d FOR DESIGN

A simple expression for the thickness of shear flow zone, t_d, can be obtained directly from Eq. 2, noting that $q = \tau_{lt} t_d$ and $T = T_n$:

$$t_d = \frac{T_n}{2A_0\tau_{lt}} \quad (34)$$

Assuming that $A_0 = m_1 A_c$ and $\tau_{lt} = m_2 f'_c$ where m_1 and m_2 are non-dimensional coefficients, substituting them into Eq. 34 gives

$$t_d = C\frac{T_n}{A_c f'_c} \quad (35)$$

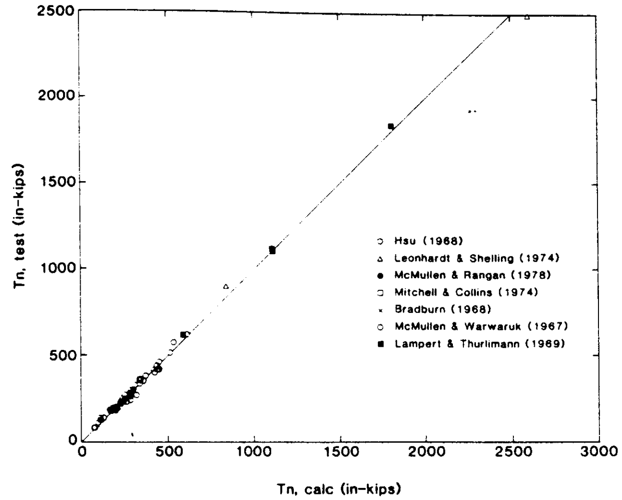


FIG. 9. Comparison of Test Strengths with Calculated Strengths Using Proposed t_d (Eq. 36)

where $C = 1/2m_1m_2$. For under-reinforced members, m_1 varies from 0.55 to 0.85, while m_2 varies from 0.13 to 0.22. These values are obtained from the Appendix of Hsu and Mo's report (1983). The low values of m_2 are due to the softening of concrete. For an increasing amount of reinforcement, m_2 increases while m_1 decreases. Therefore, the product m_1m_2 can be taken approximately as a constant, 0.125, making C a constant of 4. Then

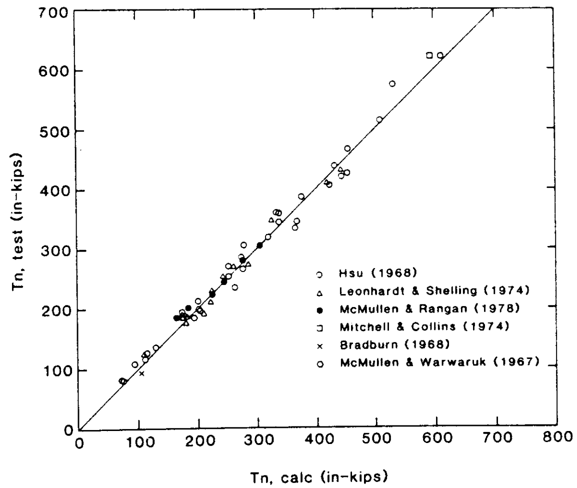


FIG. 10. Comparison of Test Strengths with Calculated Strengths Using Proposed t_d (Eq. 36)—Expended Scale for Lower Portion of Fig. 9

$$t_d = \frac{4T_n}{A_c f'_c} \qquad (36)$$

Eq. 36 is plotted in Fig. 8. Comparison with Eq. 33 shows the difference to be small. Actually, Eq. 36 can be considered a simplification of Eq. 33 by neglecting the small first term with constant 0.082 and increasing the constant in the second term from 3.405 to 4. The small effect of α is also neglected by taking $\sin 2\alpha = 1$, which is the exact value when $\alpha = 45°$. The t_d/t_{d0} ratios calculated from Eq. 36 for the writer's B series are also recorded in Table 2. A comparison with the computer values also shows the correlation to be reasonable.

Inserting Eq. 36 into the thin-tube expression of A_0 in Eq. 28 gives:

$$A_0 = A_c - \frac{2T_n p_c}{A_c f'_c} \qquad (37)$$

A_0 in Eq. 37 is used in conjunction with Eq. 4 to calculate the torsional strength, T_n, for the 61 beams available in literature. The calculated values are compared to the test values in Figs. 9 and 10. The average $T_{n,\text{test}}/T_{n,\text{calc}}$ value is 1.013 and the standard deviation is 0.055.

An example problem is given in Appendix II showing the procedures for designing reinforced concrete members subjected to torsion.

Acknowledgment

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Appendix I. Analysis Example

Beam G7 (Hsu 1968a) has a rectangular cross section of 10 in. by 20 in. (25 cm by 50 cm). It is reinforced with 6 No. 5 longitudinal bars (4 at corners and 2 at midheight of longer side) and No. 4 closed stirrups with uniform spacing of 5.75 in. (14.6 cm). The material properties are: f_{ly} = 46.3 ksi (319 MPa), f_{ty} = 46.8 ksi (322 MPa), and f'_c = 4.49 ksi (30.9 MPa).

Solution:

$A_c = 10(20) = 200$ sq in. (1,290 cm²) $\qquad p_c = 2(10 + 20) = 60$ in. (152.4 cm)

$A_l = 6(0.31) = 1.86$ sq in. (12.0 cm²) $\qquad A_t = 0.20$ sq in. (1.29 cm²)

Assume $t_d = 2$ in. (5.08 cm)

Substitute into Eq. 15.

$$A_0 = A_c - p_c\left(\frac{t_d}{2}\right) + t_d^2 = 200 - 60\left(\frac{2}{2}\right) + (2)^2 = 144 \text{ sq in. } (929 \text{ cm}^2)$$

and into Eq. 16.

$$p_0 = p_c - 4t_d = 60 - 4(2) = 52 \text{ in. } (132 \text{ cm})$$

$$\frac{A_l f_{ly}}{p_0} = \frac{1.86(46.3)}{52} = 1.656 \text{ kip/in. } (290 \text{ kN/m})$$

$$\frac{A_t f_{ty}}{s} = \frac{0.20(46.8)}{5.75} = 1.628 \text{ kip/in. } (285 \text{ kN/m})$$

then Eq. 24.

$$\zeta = \frac{\left(\frac{A_l f_{ly}}{p_0}\right) + \left(\frac{A_t f_{ty}}{s}\right)}{0.80 f'_c t_d} = \frac{1.656 + 1.628}{0.80(4.49)(2)} = 0.4571$$

and Eq. 25.

$$\cos^2\alpha = \frac{\left(\frac{A_l f_{ly}}{p_0}\right)}{\left(\frac{A_l f_{ly}}{p_0}\right) + \left(\frac{A_t f_{ty}}{s}\right)} = \frac{1.656}{1.656 + 1.628} = 0.5042$$

$$\sin^2\alpha = 0.4958$$

and finally into Eq. 20.

$$t_d = \frac{A_0 \zeta^2}{p_0 \sin^2\alpha \cos^2\alpha} = \frac{144(0.4571)^2}{52(0.5042)(0.4958)}$$

= 2.314 in. (5.88 cm) > 2 in. (5.08 cm)

N.G. repeat cycle assuming t_d = 2.10 in. (5.22 cm)

Now

t_d = 2.094 in. (5.32 cm) ≈ 2.10 in. (5.33 cm) o.k.

and finally Eq. 4.

$$T_{n,\text{calc}} = \frac{A_t f_{ty} \cot\alpha}{s}(2A_0) = \frac{0.20(46.8)(1.013)}{5.75}(2 \times 141.4)$$

= 466.3 in.-kip (52.69 kN-m)

The experimental torsional strength ($T_{n,\text{test}}$) of Beam G7 is found to be 466 in.-kip (52.66 kN-m), which is very close to the calculated value of 466.3 in.-kip (52.69 kN-m). It should be noted that the thickness of the shear flow zone t_d of 2.10 in. (5.33 cm) is more than 1/5 of the beam width (10 in. or 25.4 cm) for a beam with moderate total reinforcement ratio of 1.87%. It is obvious that t_d of a reinforced concrete beam could become very big when the reinforcement ratio is large. The definition of the lever arm area A_0 by a constant A_1 in Rausch's Eq. 4 could therefore produce a large error of the torsional strength on the unconservative side.

Appendix II. Design Example

Design the reinforcement for the hollow box beam with the trapezoidal cross section as shown in Fig. 11. The beam should be able to resist a torsional moment of 7,400 in.-kip (836 kN-m). The net concrete cover is 1.5 in. (3.81 cm) and the material strengths are f'_c = 4,000 psi (27.6 MPa) and f_y = 60,000 psi (413 MPa).

Solution

For the given outer cross-sectional dimensions shown in Fig. 11

$$A_c = \frac{(3 + 4)(3)(12)^2}{2} = 1{,}512 \text{ sq in. } (9{,}755 \text{ cm}^2).$$

$$p_c = (3 + 4 + 2\sqrt{3^2 + 0.5^2})(12) = 157 \text{ in. } (399 \text{ cm}).$$

Check Cracking Torque

$$T_{cr} = A_c t(5\sqrt{f'_c}) = 1{,}512(5)(5\sqrt{4{,}000}) = 2{,}391 \text{ in.-kip } (270 \text{ kN-m})$$

$$T_n = 7{,}400 \text{ in.-kip } (836 \text{ kN-m}) > 2{,}391 \text{ in.-kip } (270 \text{ kN-m}).$$

Reinforcement required.

Calculate t_d, A_0 and p_0

Eq. 36.

$$t_d = \frac{4T_n}{A_c f'_c} = \frac{4(7{,}400)}{1{,}512(4)} = 4.89 \text{ in. } (12.4 \text{ cm}) < 5 \text{ in. } (12.7 \text{ cm})$$

wall thickness o.k.

$$A_0 = A_c - \frac{p_c t_d}{2} = 1{,}512 - \frac{157(4.89)}{2} = 1{,}128 \text{ sq in. } (7{,}277 \text{ cm}^2)$$

$$p_0 = p_c - 4t_d = 157 - 4(4.89) = 137.4 \text{ in. } (347 \text{ cm})$$

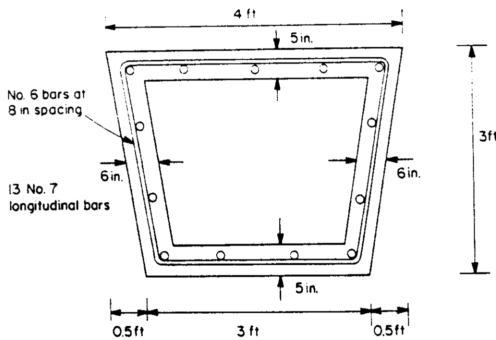

FIG. 11. Design Example

Design of Stirrups

Eq. 4.

$$\frac{A_t}{s} = \frac{T_n \tan\alpha}{2A_0 f_{ty}} = \frac{7{,}400 \tan\alpha}{2(1{,}128)(60)}$$

$$= 0.0547 (\tan\alpha) \text{ sq in./in. } (0.139 (\tan\alpha) \text{ cm}^2/\text{cm})$$

The minimum α is

$$\alpha = 12° + 33°\left[\frac{T_n p_c}{A_c^2 f'_c(0.27 - 45\epsilon_{ly})}\right]$$

$$= 12° + 33°\left[\frac{7{,}400(157)}{(1{,}512)^2(4)(0.27 - 45 \cdot 0.00207)}\right]$$

$$= 35.7° < 45°. \quad \text{Under-reinforced.}$$

Select α = 45° for best crack control.

$$\frac{A_t}{s} = 0.0547(1) = 0.0547 \text{ sq in./in. } (0.139 \text{ cm}^2/\text{cm})$$

Select No. 6 bars $s = \dfrac{0.44}{0.0547} = 8.04$ in. (20.4 cm)

Check stirrup spacing

$$\frac{p_1}{8} = \frac{157 - 4(2)\left(1.5 + \dfrac{0.75}{2}\right)}{8} = 17.75 \text{ in. } (45.1 \text{ cm})$$

s = 8.04 < 12 in. (30 cm) < 17.75 in. (45.1 cm) o.k.

Use No. 6 transverse hoop bars at 8 in. (20.3 cm) spacing.

Design of Longitudinal Steel

$$A_l = \frac{T_n p_0}{2A_0 f_{ly} \tan\alpha} = \frac{7{,}400(137.4)}{2(1{,}128)(60)(1)} = 7.51 \text{ sq in. } (48.4 \text{ cm}^2)$$

Select 13 No. 7 longitudinal bars so that spacing will be less than 12 in. (30 cm).

Actual A_l = 13(0.60) = 7.80 sq in. (50.3 cm²) > 7.51 sq in. (48.4 cm²) o.k.

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Technical Notes

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1. Five copies of an original manuscript are to be submitted to the Journals Department, ASCE, 345 East 47th Street, New York, NY, 10017-2398, along with a request by the author that it be considered as a technical note.
2. Four of the copies will be sent to an appropriate technical division or council for review.
3. If the division or council approves the contribution for publication, it will be returned to Society Headquarters with appropriate comments.
4. The journal staff will prepare the material for use in the earliest possible issue of the journal, after proper coordination with the author.
5. Each technical note is not to exceed 2,500 word-equivalents. As an *approximation*, each full manuscript page of text, tables, or figures is the equivalent of 300 words.
6. The technical notes are grouped in a special section of the journal.
7. A 175-word information retrieval abstract and key words are necessary for technical notes.
8. The final date on which a discussion should reach the Society is given as a footnote with each technical note.
9. Technical notes will be included in *Transactions*.
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The manuscripts for technical notes must meet the following requirements:

1. Titles must have a length not exceeding 70 characters and spaces.
2. The manuscript should be typed double-spaced on one side of 220 mm by 280 mm paper. Five copies of all figures and tables must be included.
3. The author's full name, Society membership grade (if applicable), and a footnote stating present employment must appear on the first page of the note. Authors need not be Society members.
4. All mathematics must be typewritten and special symbols must be properly identified. The letter symbols used must be defined where they first appear, in figures, tables, or text.
5. Standard definitions and symbols must be used. Reference must be made to the lists published by the American National Standards Institute and to the *ASCE Authors' Guide to Journals, Books, and Reference Publications*.
6. Figures must be drawn in black ink on one side of 220 mm by 280 mm paper. Because figures will be reproduced with a width of between 76 mm to 110 mm, the lettering must be large enough to be legible at this width. Photographs must be submitted as glossy prints. Explanations and descriptions must be made within the text for each figure.
7. Tables must be typed on one side of 220 mm by 280 mm paper. An explanation of each table must appear in the text.
8. References cited in text must be typed double-spaced at the end of the technical note in alphabetical order in an Appendix. References.
9. Each author may use the International System of Units (SI), and units acceptable in SI, or other units. When SI units are used, no other units are required. When other units are used, the SI units shall be given in parentheses; in a supplementary or dual-unit table; or an appendix.