TITLE NO. 64-12

How Safe Are Our Large Reinforced Concrete Beams?

By G. N. J. KANI

To answer this question, four series of test beams, with depths of 6, 12, 24, and 48 in., were tested at the University of Toronto and the results compared. Considerable influence of the absolute depth became apparent to such an extent that the safety factor for the largest beams was approximately 40 percent lower than the otherwise similar small beams. This trend indicates that, with a further increase in depth, a correspondingly further decrease in the safety factor can be expected.

Keywords: beams (structural); cracking (fracturing); deep beams; depth; diagonal tension; reinforced concrete; research; safety factor; shear strength.

■ TO DATE (1966), the majority of reinforced concrete beams which have been tested to failure range in depth from 10 to 15 in. Essentially, these are the beams on which all our design practices and safety factors are based. The immediate aim of the test program described in this paper was to answer the question: How representative are the test results derived from such relatively small beams for the safety factors of large beams?

In Eq. (17-2) of the ACI Code:[1]

$$v_u = 0.85 \left(1.9\sqrt{f_c'} + 2500\, p\, \frac{Vd}{M}\right) \qquad (1)$$

no parameter has been included which takes into account the influence of the effective depth d of the beam, thereby suggesting that beams of different depths will have the same safety factor if the concrete strength f_c', the percentage of main reinforcement p, and the shear arm ratio, $a/d = M/Vd$, are equal. Yet, as we shall see later, this particular influence is even greater than the term, $2500\, p\, Vd/M$, in Eq. (1). A comparison of this formula and the test results which illustrate the influence of p and a/d is shown in Fig. 1 and is discussed more extensively in Reference 2.

The author's attempt[3] to provide a rational theory of diagonal failure was based essentially on consideration of equilibrium and resulted in some predictions, in particular the following three, which were contrary to general expectations:

1. The influence of concrete strength on the so-called shear strength of rectangular reinforced concrete beams without web reinforcement is negligible and can be omitted in strength analysis.

2. Provided that the reinforcing bars obtain effective anchorage, beams without bond have a higher load-carrying capacity than beams with good bond.

3. All other factors being equal, the safety factor decreases as the depth of the beam increases.

Statement 1 has been substantiated in Reference 2. As for Statement 2, concerning the negative influence of bond, two independent test reports can be quoted:

(a) Leonhardt and Walther[4] state: "Now, the beams with smooth bars produced considerably higher load-carrying capacities than the corresponding beams with deformed bars."*

(b) Lorentsen[5] reports ". . . beam 7 (grouted) carried less than beam 21 (ungrouted) . . ."

The substantiation of Statement 3 is the chief purpose of this paper.

Many investigators assume, contrary to the prediction of the rational theory advanced in Reference 3, that the influence of the effective depth d on the relative strength of reinforced concrete beams exists only for depths smaller than 15 in. In the chapter on influence of the beam size of their report,[6] Rüsch, Haugli, and Mayer state: "It seems that for beams tested under uniformly distributed load, a change of depth beyond a critical value does not have any influence on the load-carrying capacity. This critical beam depth is 15 to 20 cm (6 to 8 in.). This concept of a critical beam depth has also been confirmed by the tests of Forsell.[7] According to his investigation, the critical effective depth for beams tested under point loading lies within the range of 30 to 40 cm (12 to 16 in.)." However, in the University of Toronto tests, no such critical depth could be found.

Due to this contradiction with the prediction of the rational theory,[3] it was decided to provide experimental evidence to substantiate one or the other of the above-mentioned assertions. To accomplish this, four series of test beams of different depths were designed. Each series comprised beams of one depth only (6, 12, 24, and 48 in.).

*See also the author's discussion of these tests in Reference 3, pp. 464-465 under "Influence of Bond" and the discussion of Reference 3 in the December 1964 issue of the ACI JOURNAL, pp. 1626-28.

†For notation see p. 141.

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All series had the same nominal percentage of main reinforcement (p = 2.80 percent) and the same grade of concrete (f_c' = 3800 psi). Within each series, the a/d ratio was varied over the entire range of the "valley of diagonal failure," i.e., from a/d = 1.0 to the transition point T, at which full flexural failure was attained.

THEORETICAL CONSIDERATIONS†

The expectation that beams, different only in depth, should have different relative strengths was derived from the principal formula of the rational theory of diagonal failure developed in Reference 3:

$$M_u = M_o \cdot \frac{\Delta x}{s} \cdot \frac{a}{d} \qquad (2)$$

where M_u denotes ultimate moment, $\Delta x/s$ is the

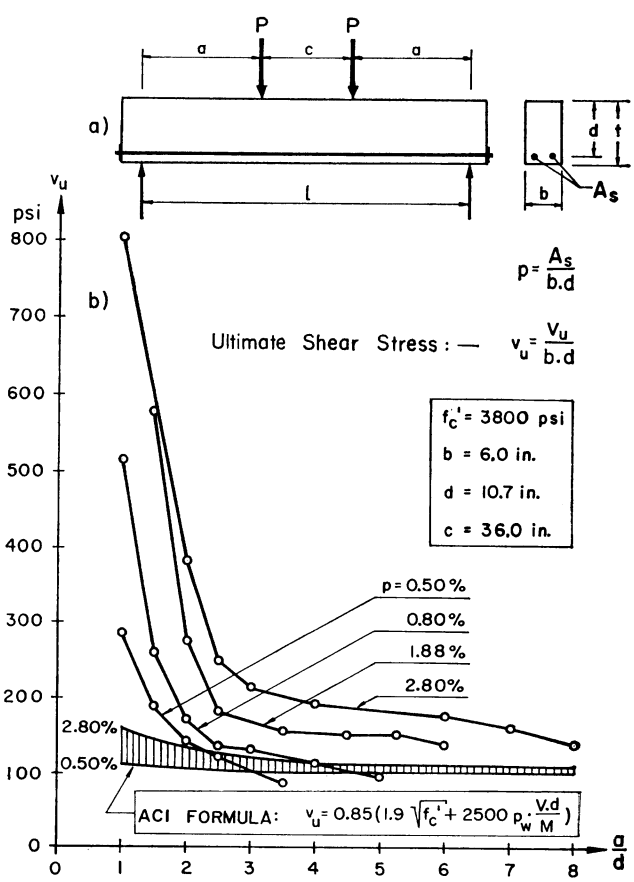

Fig. 1—Shear stress at failure versus a/d

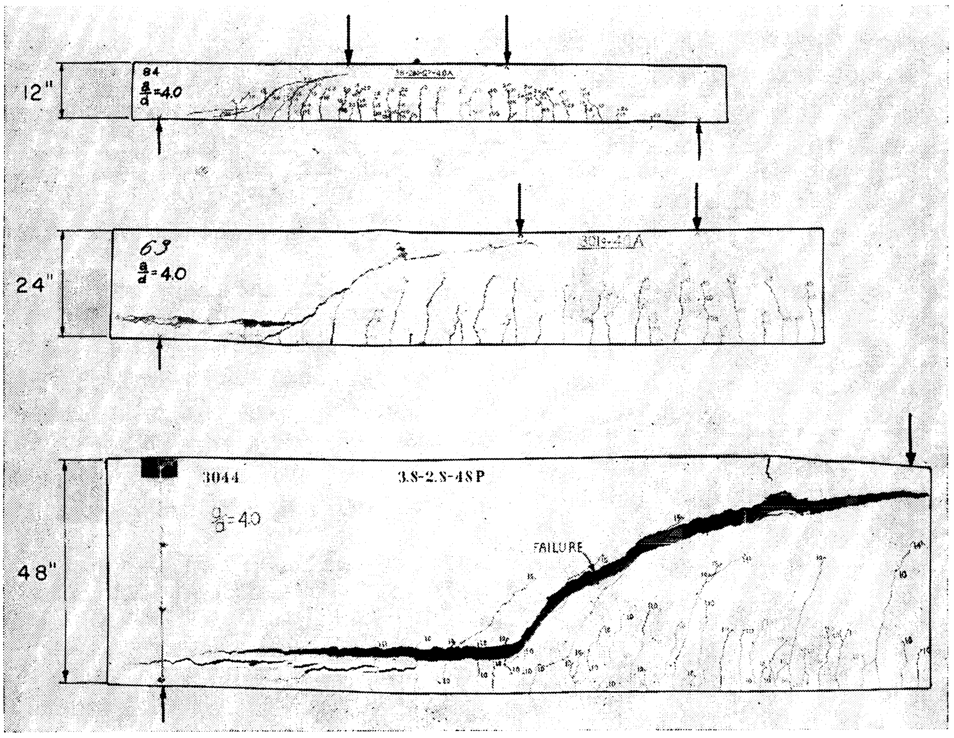

Fig. 2—Differences in crack patterns among beams of different depths

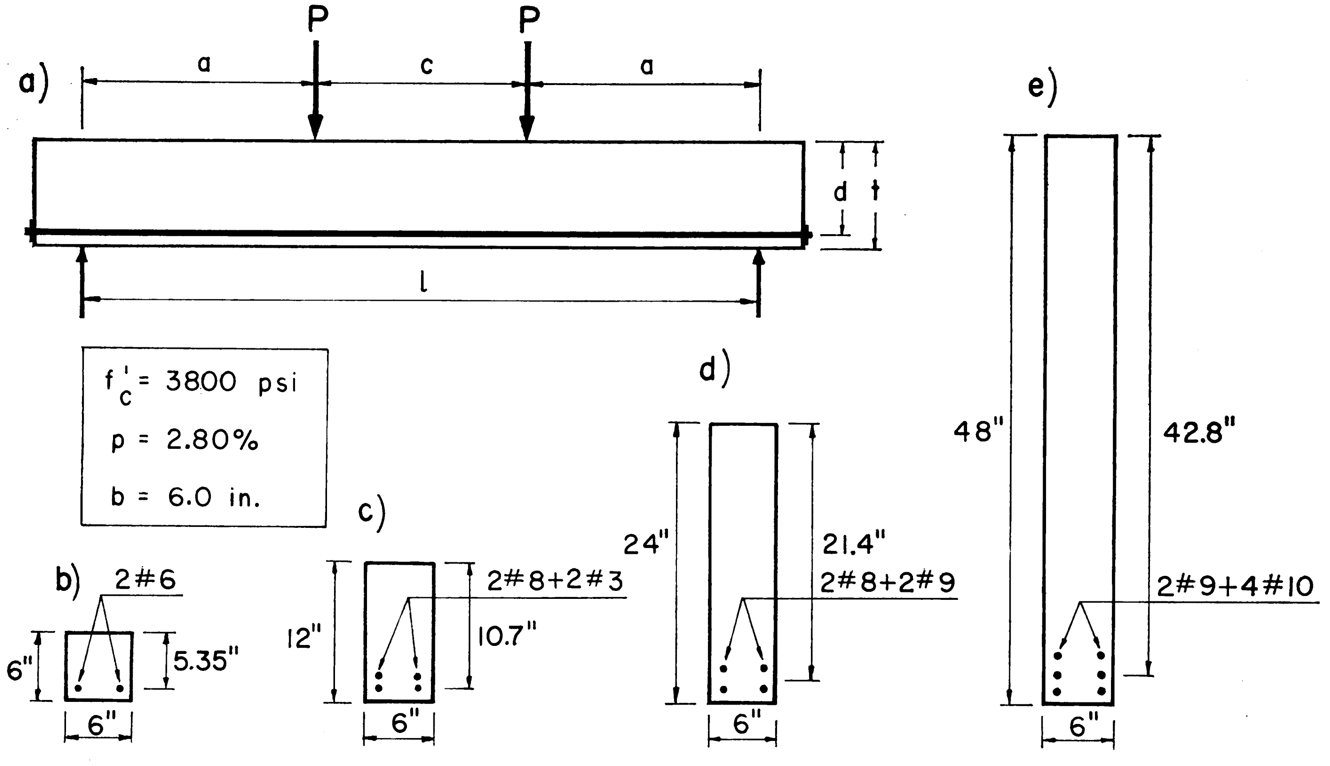

Fig. 3—Typical loading arrangement and cross sections of the four beam series

crack factor, i.e., the ratio of the spacing of cracks to the average length of concrete teeth, and:

$$M_o = f_t' \frac{jbd^2}{6}$$

where f_t' is the modulus of rupture of the concrete. By dividing Eq. (2) by the flexural failure moment:

$$M_{fl} = Tjd = pbdf_yjd$$

we obtain the following expression for the relative beam strength:

$$r_u = \frac{M_u}{M_{fl}} = \frac{f_t'}{6pf_y} \cdot \frac{\Delta x}{s} \cdot \frac{a}{d} \qquad (3)$$

For a given grade of concrete and steel and a given percentage of main reinforcement, the factor $f_t'/(6pf_y)$ is a constant. Therefore, the relative beam strength M_u/M_{fl} depends, except for a/d, only on the crack factor $\Delta x/s$.

However, the spacing of cracks Δx is almost constant, virtually independent of the beam depth, whereas the crack length s is greatly influenced by a change in depth. In all the tests, the crack spacing was approximately 4 in. and did not vary appreciably with the beam depth. On the other hand, the length s of the fully developed cracks was found to be proportional to the depth d. Thus, the first impression derived from Eq. (3) is that the relative beam strength M_u/M_{fl} should decrease rather quickly with increasing beam depth.

Fortunately, the concrete teeth of large beams are not merely "longer" but are geometrically quite different from those of smaller beams. Fig. 2 illustrates the typical crack configurations of 12, 24, and 48 in. beams. The development of shorter secondary cracks between fully developed primary cracks has been extensively discussed by Broms.[8]

The initial meaning of Δx as introduced in Reference 3 was the crack spacing and s, the effective

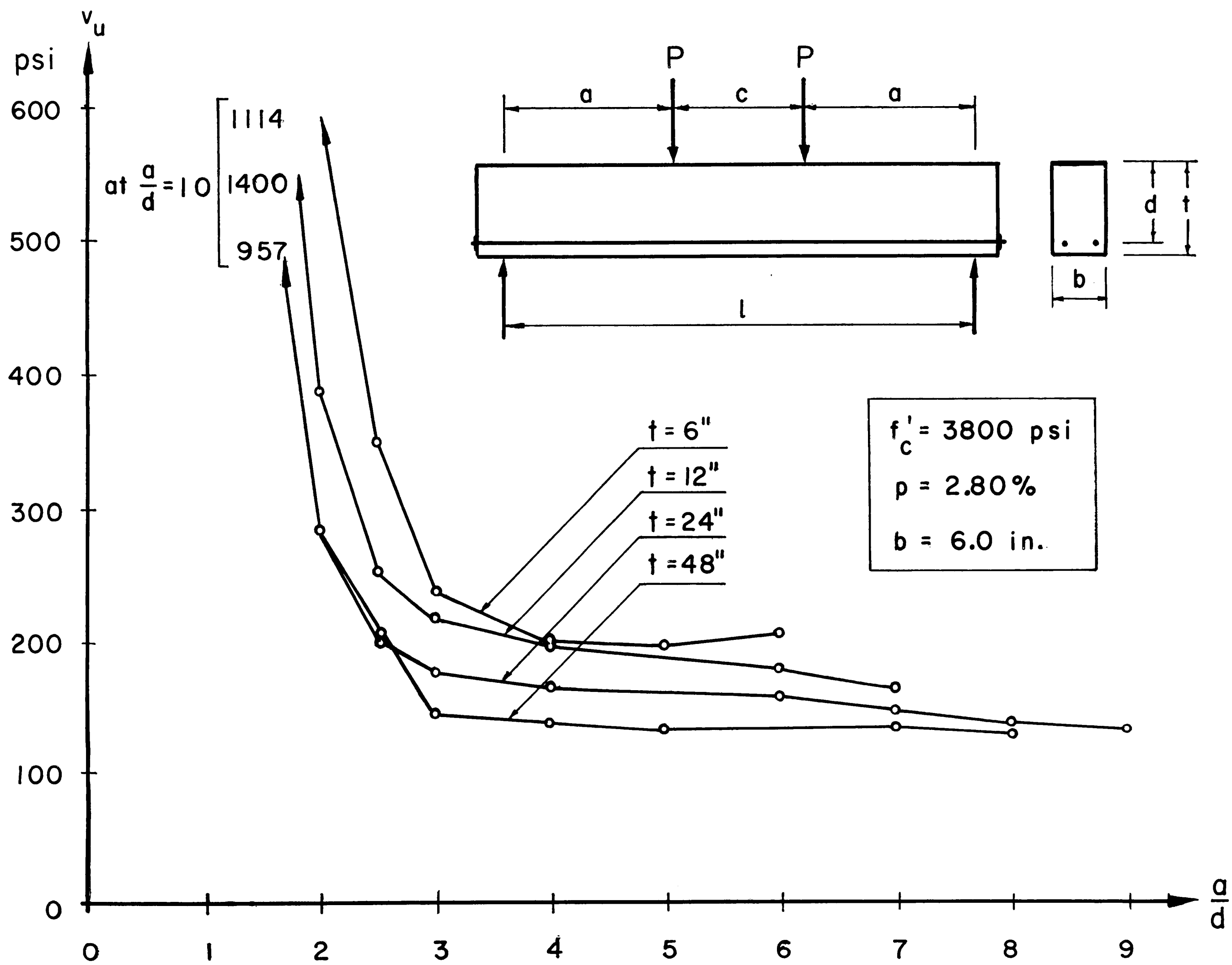

Fig. 4—Shear stress at failure v_u versus a/d for beams of various depths

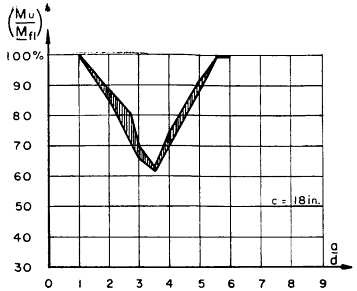

TABLE 1—SERIES 3.8-2.80-6P

Beam No.	Measured dimensions: b, in.	d, in.	a, in.	l, in.	a/d, in. per in.	A_s, sq in.	$p = \frac{A_s}{bd}$, percent	Test values: f_c', ksi	f_y, ksi	$2P_u$, kips	M_u, in.-kips	Type of failure[a]	Calculated values: M_{fl},[b] in.-kips	$\frac{M_u}{M_{fl}}$, percent	$\left(\frac{M_u}{M_{fl}}\right)^*$, percent	$V_u = \frac{P_u}{bd}$, psi	$(a/d)^*$ in. per in.
40	5.97	5.50	29.42	76.84	5.35	0.85	2.59	3.83	56.2	14.39	212	D_{su}	192	110.2	100	219	5.5
41	6.00	5.56	13.38	44.75	2.41	0.87	2.60	3.95	55.2	23.13	155	D_{su}	200	77.5	75.0	347	2.5
43	5.96	5.40	32.00	82.00	5.92	0.88	2.73	4.06	56.8	13.09	209	D_{sl}	196	107.0	100.0	202	6.0
44	5.98	5.40	32.00	82.00	5.92	0.88	2.72	4.06	56.8	13.58	216	F	193	112.0	100.0	214	6.0
45	5.95	5.23	10.70	39.40	2.04	0.88	2.83	3.70	56.8	29.04	155	D_{su}	184	84.2	84.7	467	2.0
46	5.95	5.35	10.70	39.40	2.00	0.88	2.76	3.70	56.8	31.04	166	D_{su}	188	88.4	88.4	488	2.0
47	5.95	5.20	26.70	71.40	5.12	0.88	2.85	3.59	56.8	12.67	170	D_{su}	181	93.7	92.0	205	5.0
48	5.95	5.25	26.70	71.40	5.09	0.88	2.81	3.59	56.8	12.19	163	D_{su}	184	88.6	87.4	195	5.0
52	6.00	5.45	21.40	60.80	3.93	0.88	2.69	3.60	56.8	12.99	139	D_{su}	190	73.2	74.2	199	4.0
53	5.95	5.20	5.34	28.68	1.03	0.88	2.84	3.87	56.8	69.80	187	D_{sl}	184	101.0	100.0	1130	1.0
54	5.95	5.35	5.34	28.68	1.00	0.88	2.76	3.87	56.8	70.90	189	D_{sl}	191	98.0	98.0	1114	1.0
55	5.92	5.30	16.00	50.00	3.02	0.88	2.89	3.64	56.8	14.65	117	D_{sl}	181	64.8	65.0	234	3.0
56	6.03	5.41	18.73	55.46	3.46	0.87	2.67	3.95	58.4	12.59	118	D_{su}	193	61.2	61.3	193	3.5
57	6.03	5.46	29.42	76.84	5.39	0.86	2.60	3.83	54.4	14.19	209	D_{su}	192	109.0	100.0	216	5.5
58	6.00	5.45	18.73	55.46	3.44	0.87	2.68	3.95	60.4	13.00	122	D_{su}	195	62.5	62.5	199	3.5
59	6.08	5.50	14.68	47.36	2.67	0.88	2.63	3.86	56.8	22.55	165	D_{sl}	199	83.6	80.5	337	2.75
60	6.10	5.46	16.02	50.04	2.94	0.88	2.64	3.88	56.8	17.67	141	D_{sl}	197	72.0	67.5	266	3.0

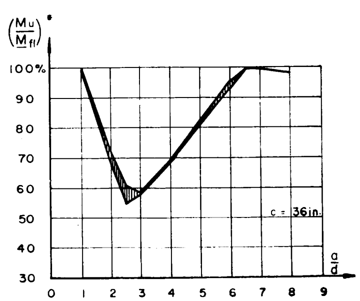

TABLE 2—SERIES 3.8-2.80-12P

Beam No.	Measured dimensions: b, in.	d, in.	a, in.	l, in.	a/d, in. per in.	A_s, sq in.	$p = \frac{A_s}{bd}$, percent	Test values: f_c', ksi	f_y, ksi	$2P_u$, kips	M_u, in.-kips	Type of failure[a]	Calculated values: M_{fl},[b] in.-kips	$\frac{M_u}{M_{fl}}$, percent	$\left(\frac{M_u}{M_{fl}}\right)^*$, percent	$V_u = \frac{P_u}{bd}$, psi	$(a/d)^*$ in. per in.
81	6.04	10.80	64.08	164.16	5.94	1.80	2.76	3.99	49.8	23.00	736	D_{su}	792	93.0	93.3	176	6.0
82	6.09	10.68	85.44	206.88	8.00	1.80	2.77	3.99	49.7	18.10	772	F	783	98.6	98.6	139	8.0
83	6.14	10.68	32.04	100.08	3.00	1.80	2.73	3.98	49.7	29.20	467	D_{sl}	784	59.6	58.5	223	3.0
84	5.95	10.67	42.72	121.44	4.01	1.80	2.83	3.98	49.6	24.90	532	D_{sl}	777	68.5	68.9	196	4.0
88	6.01	10.47	10.68	57.36	1.02	1.77	2.81	4.56	58.2	161.70	862	F	773	112.0	100.0	1400	1.0
91	6.08	10.58	64.08	164.16	6.05	1.74	2.71	3.98	52.8	22.91	735	D_{su}	754	97.5	95.5	178	6.0
92	6.00	10.63	74.76	185.52	7.03	1.74	2.72	3.98	53.5	20.61	771	F	756	102.0	100.0	161	7.0
93	6.10	10.74	69.42	174.84	6.46	1.74	2.66	4.39	54.0	24.19	839	D_{su}	783	107.0	100.0	184	6.5
94	6.03	10.76	21.36	78.72	1.99	1.80	2.78	3.67	51.0	49.70	531	D_{su}	775	68.5	68.0	384	2.0
95	6.04	10.83	26.70	89.40	2.46	1.80	2.75	3.67	49.0	32.70	437	D_{su}	780	56.0	54.7	250	2.5
96	6.03	10.83	42.72	121.44	3.94	1.80	2.76	3.67	48.6	25.30	540	D_{su}	780	69.2	69.6	193	4.0
97	6.00	10.88	32.10	100.20	2.95	1.75	2.68	3.95	53.1	28.10	451	D_{sl}	781	57.8	57.6	215	3.0
98	6.03	10.81	26.75	89.50	2.47	1.75	2.69	3.80	53.1	34.30	459	D_{sl}	770	59.6	57.0	263	2.5
99	6.00	10.70	26.75	89.50	2.50	1.75	2.73	3.80	53.1	34.70	464	D_{sl}	758	61.8	60.2	270	2.5
100	6.03	10.62	21.40	78.80	2.02	1.76	2.74	3.95	53.1	50.30	538	D_{sl}	760	70.9	71.8	382	2.0

[a]Type of failure:
F = flexural failure
D_{sl} = slow diagonal failure
D_{su} = sudden diagonal failure
[b]$M_{fl} = A_s f_y d(1 - 0.4k)$, where $k = A_s f_y / (0.75 f_c' bd)$
*Relative beam strength $(M_u/\underline{M}_{fl})^*$ has been corrected to bring P to $\underline{p}$, a/d to $(a/d)^*$ and f_y to $\underline{f}_y$.

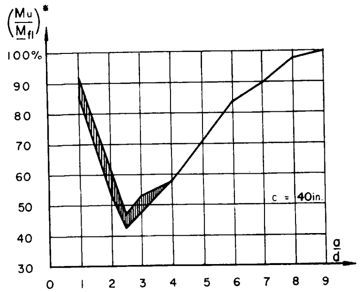

TABLE 3—SERIES 3.8-2.80-24P

	Measured dimensions							Test values					Calculated values				
Beam No.	b, in.	d, in.	a, in.	l, in.	a/d, in. per in.	A_s, sq in.	$p = \frac{A_s}{bd}$, percent	f_c', ksi	f_y, ksi	$2P_u$, kips	M_u, in.-kips	Type of failure[a]	M_{fl},[b] in.-kips	$\frac{M_u}{M_{fl}}$, percent	$\left(\frac{M_u}{M_{fl}}\right)^*$, percent	$V_u = \frac{P_u}{bd}$, psi	$(a/d)^*$ in. per in.
61	6.16	21.32	42.72	125.44	2.00	3.35	2.58	3.88	50.6	73.4	1565	D_{su}	2950	53.1	53.1	280	2.0
63	6.08	21.37	85.44	210.88	4.00	3.60	2.77	3.80	51.0	41.9	1790	D_{su}	3095	57.9	57.9	161	4.0
64	6.15	21.28	170.88	381.76	8.01	3.60	2.75	3.73	51.0	35.5	3030	D_{su}	3100	97.8	97.7	136	8.0
65	5.89	21.75	153.50	147.00	2.46	3.61	2.82	3.91	54.2	50.5	1350	D_{sl}	3170	42.6	42.6	198	2.5
66	6.15	21.31	128.16	296.32	6.02	3.60	2.75	3.83	51.0	40.8	2610	D_{su}	3110	83.9	83.9	155	6.0
67	6.19	20.80	21.36	82.72	1.03	3.54	2.75	4.40	59.0	246.3	2630	D_{sl}	3070	85.7	85.5	957	1.0
68	6.17	21.21	192.24	424.48	9.05	3.53	2.71	3.94	59.0	34.0	3275	F	3050	107.0	100.0	130	9.0
69	6.11	21.35	21.36	82.72	1.00	3.48	2.67	3.97	54.1	263.2	2810	D_{sl}	3054	92.0	92.0	1008	1.0
71	6.10	21.42	64.08	168.16	2.99	3.48	2.66	3.97	54.1	45.9	1470	D_{sl}	3070	47.9	47.9	175	3.0
72	6.00	21.62	42.80	125.60	1.96	3.51	2.70	3.60	55.7	88.5	1890	D_{sl}	3040	62.2	61.7	341	2.0
74	6.00	20.60	64.20	168.40	3.11	3.51	2.84	3.95	53.0	48.4	1554	D_{sl}	2920	53.2	52.6	196	3.0
75	6.00	20.63	64.20	168.40	3.11	3.51	2.83	3.96	53.2	48.5	1556	D_{su}	2930	53.1	52.6	196	3.0
76	6.00	20.38	53.50	147.00	2.62	3.51	2.87	4.46	54.0	51.6	1380	D_{su}	2960	46.7	47.4	211	2.5
79	6.03	21.90	149.80	363.00	6.83	3.59	2.72	3.79	55.3	37.6	2820	D_{su}	3170	89.0	89.9	144	7.0

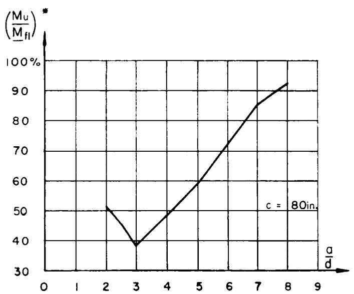

TABLE 4—SERIES 3.8-2.80-48P

	Measured dimensions							Test values					Calculated values				
Beam No.	b, in.	d, in.	a, in.	l, in.	a/d, in. per in.	A_s, sq in.	$p = \frac{A_s}{bd}$, percent	f_c', ksi	f_y, ksi	$2P_u$, kips	M_u, in.-kips	Type of failure[a]	M_{fl},[b] in.-kips	$\frac{M_u}{M_{fl}}$, percent	$\left(\frac{M_u}{M_{fl}}\right)^*$, percent	$V_u = \frac{P_u}{bd}$, psi	$(a/d)^*$ in. per in.
3041	6.00	43.2	86.40	252.80	2.00	7.06	2.73	3.90	54.6	146.6	6310	D_{sl}	12350	51.3	51.3	282	2.0
3042	6.06	43.1	107.75	295.50	2.50	7.06	2.71	3.83	54.4	106.5	5740	D_{su}	12300	46.6	46.6	204	2.5
3043	6.05	43.00	129.00	338.00	3.00	7.06	2.72	3.91	54.5	74.2	4786	D_{su}	12400	38.5	38.5	142	3.0
3044	6.00	43.20	171.80	423.60	3.97	7.06	2.73	4.28	54.5	71.5	6142	D_{su}	12650	48.5	48.5	137	4.0
3045	6.10	43.0	215.00	215.00	5.00	7.08	2.70	4.10	55.2	68.5	7364	D_{su}	12450	59.1	59.1	130	5.0
3046	6.10	43.2	331.40	684.80	7.00	7.12	2.72	3.87	52.2	69.3	10490	D_{su}	12500	85.1	85.1	131	7.0
3047	6.10	43.1	373.80	769.60	8.00	7.06	2.68	3.87	54.5	66.1	11400	D_{su}	12350	92.3	92.3	126	8.0

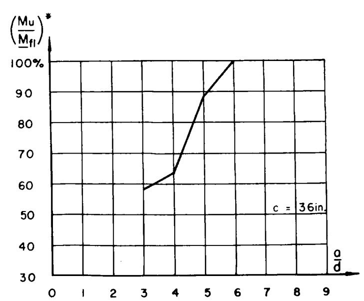

TABLE 5—SERIES W3.8-2.80-12P

	Measured dimensions							Test values					Calculated values				
Beam No.	b, in.	d, in.	a, in.	l, in.	a/d, in. per in.	A_s, sq in.	$p = \frac{A_s}{bd}$, percent	f_c', ksi	f_y, ksi	$2P_u$, kips	M_u, in.-kips	Type of failure[a]	M_{fl},[b] in.-kips	$\frac{M_u}{M_{fl}}$, percent	$\left(\frac{M_u}{M_{fl}}\right)^*$, percent	$V_u = \frac{P_u}{bd}$, psi	$(a/d)^*$ in. per in.
271	24.06	10.58	64.20	164.40	6.06	6.99	2.75	3.91	54.6	97.64	3140	D_{su}	2995	104.6	100.0	193	6.0
272	24.05	10.66	53.50	143.00	5.02	6.99	2.72	3.91	54.6	102.40	2740	D_{su}	3030	90.5	88.3	200	5.0
273	24.10	10.68	42.80	121.60	4.02	6.99	2.72	3.94	54.6	92.66	1980	D_{su}	3050	65.0	63.7	180	4.0
274	24.10	10.64	32.10	100.20	3.02	6.99	2.73	3.94	54.6	112.45	1810	D_{su}	3040	59.5	58.1	219	3.0

[a]Type of failure:
F = flexural failure
D_{sl} = slow diagonal failure
D_{su} = sudden diagonal failure
[b]$M_{fl} = A_s f_y d(1 - 0.4k)$, where $k = A_s f_y/(0.75 f_c' bd)$
*Relative beam strength $(M_u/\underline{M}_{fl})^*$ has been corrected to bring P to $\underline{p}$, a/d to $(a/d)^*$ and f_y to $\underline{f}_y$.

length of a concrete tooth. But, due to the differences in crack patterns among beams of different depths, the crack factor $\Delta x/s$ requires a more general definition than used for the 12-in. beams. However, the basic expectation, that beams of greater depth have a smaller crack factor, is nevertheless true. Ignoring this fact necessitates the acceptance of lower safety factors for large beams, as became clear from the test results.

TEST PROGRAM

For beams without web reinforcement, the reduction in strength caused by premature diagonal failure, increases with $p = A_s/bd$ as was discussed in Reference 2. Therefore, for the determination of the influence of depth on shear failure, a relatively high percentage of reinforcement was chosen, corresponding to a nearly balanced cross section. For the chosen grade of concrete, $f_c' = 3800$ psi, this corresponds approximately to $p = 2.80$ percent, the value selected for all four series. The cross sections of the four test series designed according to the above specifications are shown in Fig. 3b-e. Fig. 3a illustrates the loading arrangement.

Using the same designation for the test series as in Reference 2, the four series were designated:

3.8-2.80-6.0P
3.8-2.80-12.0P
3.8-2.80-24.0P
3.8-2.80-48.0P

The particulars for each beam are given in Tables 1 through 4.

Since anchorage failures were considered to be a special type of failure and outside the scope of this program, precautions were taken to eliminate this factor by welding anchor plates to the ends of bars in the same way as described in Reference 2.

Because the longer 48-in. beams with only a 6-in. width, approach a laterally unstable case, all 48-in. beams had four lateral supports in the form of roller bearings just below the top surface, 5 ft from the center line of the beam.

The reinforcement consisted of ASTM A16 deformed bars with a specified minimum yield strength of 50 ksi. However, differences in the properties among bars of different diameters resulted in variations in the yield strength from 48.6 to 60.4 ksi (see Tables 1 to 5). Because of

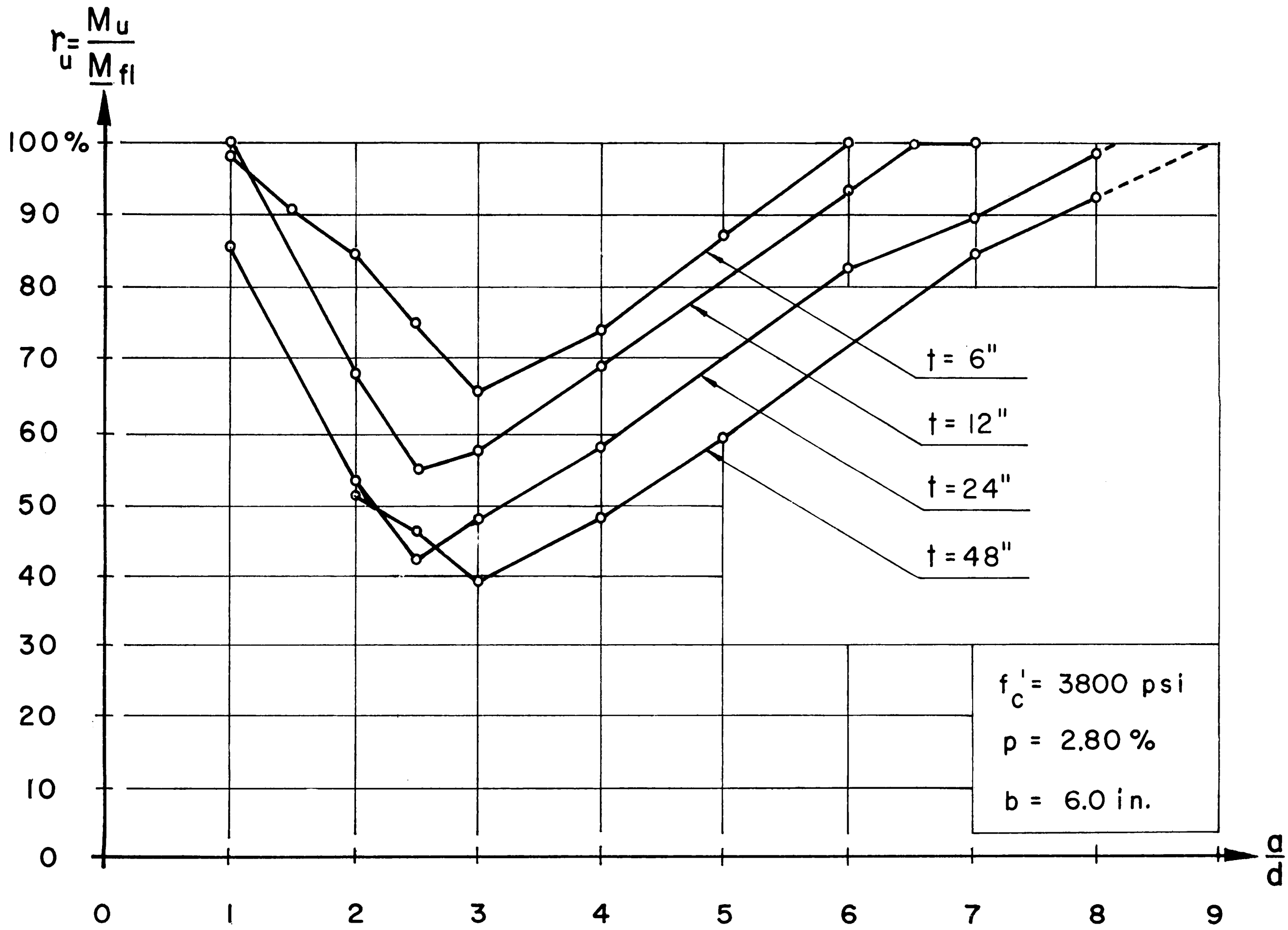

Fig. 5—Relative strength r_u versus a/d for reinforced concrete beams of various depths

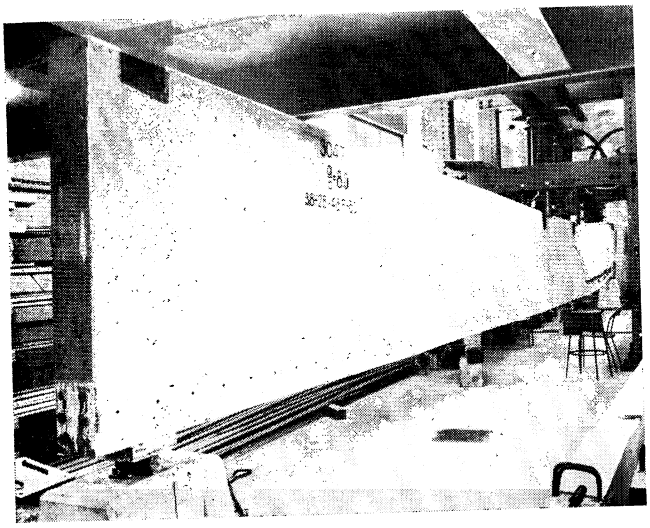

Fig. 6—Longest 48 in. beam ($a/d = 8.0$)

this variation, and for reasons explained in Reference 2, pp. 681, the comparative flexural strength $\underline{M}_{fl}$, based on $\underline{f}_y = 50$ ksi, was used in the analysis of all four beam series. The grade of concrete was maintained at 3800 psi, with a maximum aggregate size of ¾ in. and no admixtures, for all of the test series.

TEST RESULTS

The conventional presentation of the shear stress at failure v_u versus a/d for the four test series is shown in Fig. 4, illustrating the fact that beams, equal in all parameters except depth, exhibit a decreasing shear strength v_u with increasing depth d. Moreover, the strong variation of v_u with respect to a/d exists in large beams, as was known to be the case for small beams.

As extensively discussed in References 2 and 3, the variation of the test results is much smaller if we choose the ultimate moment M_u or the nondimensional relative strength, $r_u = M_u/\underline{M}_{fl}$, instead of v_u as the indicator of beam strength. With r_u known, the ultimate moment of a beam failing in shear appears simply as:

$$M_u = r_u \underline{M}_{fl} \quad (4)$$

Since $\underline{M}_{fl}$ is not only easy to determine, but in general is calculated anyway, the analysis of shear safety is reduced to the determination of the factor r_u.

Using the relative beam strength r_u as the indicator of failure, the test results are presented in Fig. 5 as derived from Tables 1 to 4.

From Fig. 5, several conclusions may be deduced:

1. Since no conventional code formula considers the influence of the absolute value of the beam depth, the safety factor of large beams can be considerably smaller than for small beams. Fig. 5 indicates that the 48-in. beams, when compared to the corresponding 6-in. beams, have a reduction in the safety factor of up to 40 percent. Therefore, if the safety factor obtained from testing 6-in. beams is designated by n_6, then we can expect a reduced factor of safety for the 48-in. beams which may be as low as:

$$n_{48} = 0.60\, n_6$$

This means that design based on data derived from 6 in. test beams, which renders an apparently conservative safety factor of 2.5, would actually result in an equivalent 48-in. beam with a dangerously low safety factor of 1.5.

2. No similarity of behavior exists for beams of different depths, even if they are otherwise equal, i.e., having the same f_c', a/d, p, and b. The 6-in. beams, with $a/d = 6.0$, fail in flexure, attaining 100 percent of the load-carrying capacity of the cross section, whereas an otherwise equal 48-in. beam fails in shear, attaining only some 75 percent of the flexural load-carrying capacity of the cross section. The transition point T, which also varies with the beam depth, moved from $a/d = 6.0$

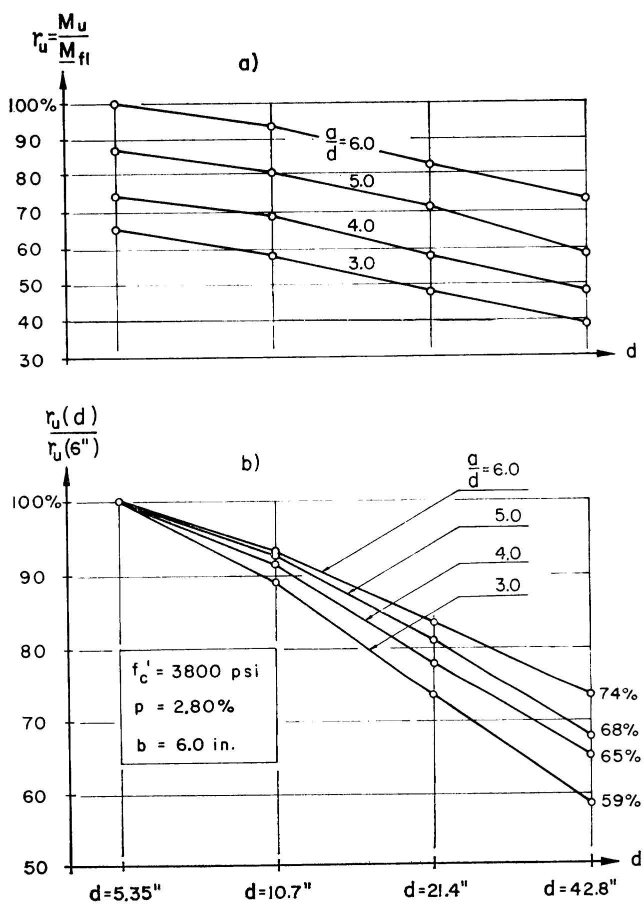

Fig. 7—Variation of relative beam strength (r_u versus depth)

TABLE 6—DECREASE IN RELATIVE BEAM STRENGTH DUE TO INCREASING DEPTH

Beam depth t, in.	6	12	24	48
Relative beam strength r_u	0.650	0.576	0.479	0.385
Loss due to doubling of depth, percent		11.4	18.6	22.2

for the 6-in. beams to $a/d = 9.0$ for the 48-in. beams. The main reason for the loss of similarity in behavior can be found in the obvious differences in the crack pattern (Fig. 2), which dictates the shape of concrete teeth.

The largest 48-in. beam that the laboratory space allowed (see Fig. 6) had $a/d = 8.0$ and a length of 66 ft. The relative beam strength at failure was 92.3 percent, thus not quite reaching the transition point T. However, the curve in Fig. 5 produces by extrapolation a transition point at about $a/d = 9.0$.

3. Beyond the transition points, i.e., where full flexural strength of the cross section is attained, no significant reduction was observed in the relative beam strength due to increased beam depth. Thus, only within the valley of diagonal failure must a reduction of strength due to greater beam depth be considered.

4. There is no indication that the reduction in the safety factor approaches a limiting value with increase in beam depth. Within the test range d = 5.35 to 42.8 in. an almost constant amount of reduction was observed whenever the beam depth was doubled. The amount of reduction is dependent on the a/d ratio, with the greatest strength loss occurring at $a/d = 3.0$. The 48-in. beam produced a relative beam strength r_u, which was 40.7 percent lower than the corresponding value of the 6-in. beam.

5. A critical beam depth as reported by Rüsch, Haugli, and Mayer[6] could not be observed in the tests. Taking the lowest test values for the beams with $a/d = 3.0$, the values for the relative beam strength are given in Table 6.

Not only is there an absence of a critical beam depth, but the percentage loss in strength increases each time the depth is doubled. At that rate, zero strength is reached in approximately 3.8 such steps. Therefore, it appears that a beam with $a/d = 3.0$ and a depth of 20 ft would fail under its own weight. For higher a/d ratios, this critical maximum depth is much lower. Fig. 7(a) demonstrates the relationship of the relative beam strength versus beam depth, using a logarithmic scale along the depth axis for convenience. Each line in the diagram was obtained by joining the test values for beams of different depths, but having the same a/d ratios.

In Fig. 7(b) the test values of the 6-in. beams, which had d = 5.35 in., are taken as a standard. The results of all other beams indicate the proportion of the relative strength which they have with respect to a 6-in. beam of corresponding a/d. The greatest loss of 41 percent was obtained for $a/d = 3.0$.

Influence of beam width

For the previously described four series of beams of varying depths (Fig. 3), the width b of the beams was maintained at 6 in. Thus, the ratio of a/d was different for each series. The question

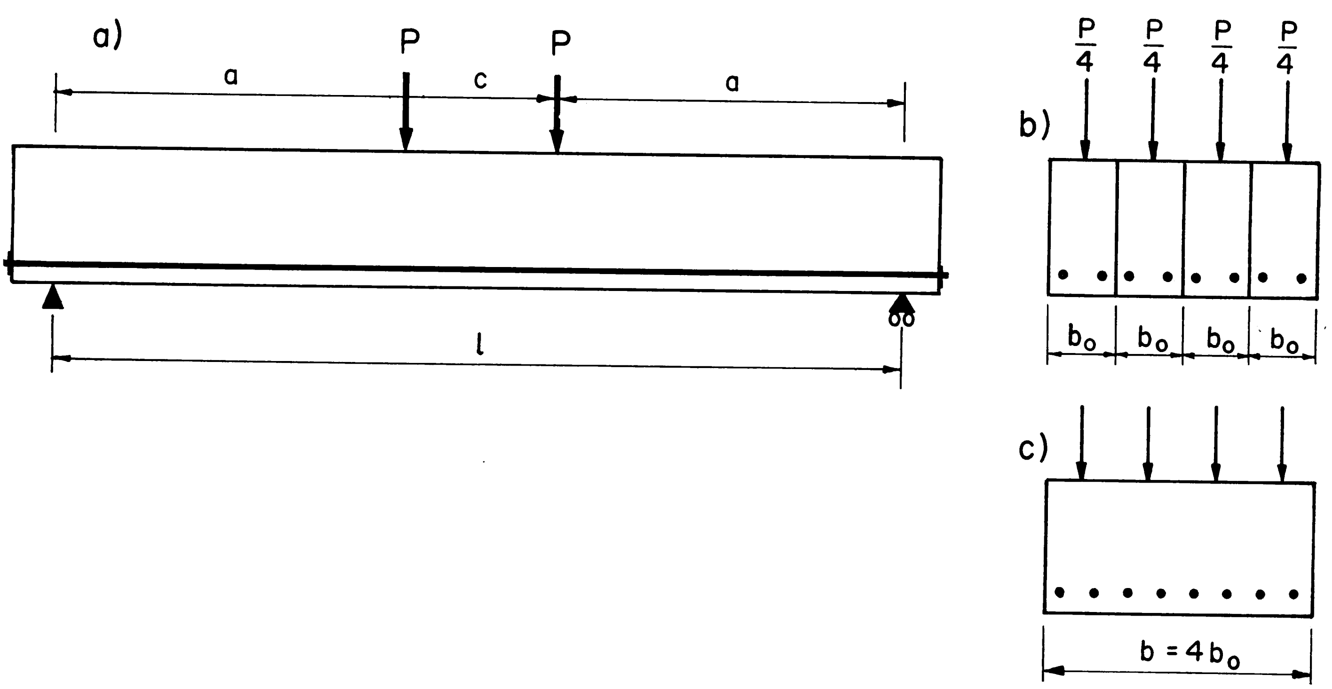

Fig. 8—Influence of beam width

of the influence of width must therefore be considered.

Let us assume that we have four equal beams of 6 in. width and we have decided to test them together (see Fig. 8a and 8b). Since all four beams undergo the same deformations, little interference from beam to beam can be expected. Therefore, the resulting performance of the beams should be the same as if each beam was tested individually. If the four beams were glued or cast together (Fig. 8c) there would be little reason to expect the one-piece beam to behave differently from the group of four narrower beams (Fig 8b), since the depth, over-all width, percentage of reinforcement, etc., would be the same in both test arrangements. On the other hand, the four single beams, having a free surface every 6 in., might contribute less lateral restraint in the inner part of the compressive zone than the one-piece beam, thus producing a small difference in results.

To verify the above considerations experimentally, a series of four beams ($a/d = 3$, 4, 5, and 6) was cast and tested (Fig. 9). These beams had the same characteristics as the previously described series (Series 3.8 - 2.80 - 12P) with one difference, that of $b = 24$ in. (Fig. 10a) instead of 6 in., thus being four times as wide as the standard beam cross section (Fig. 10b).

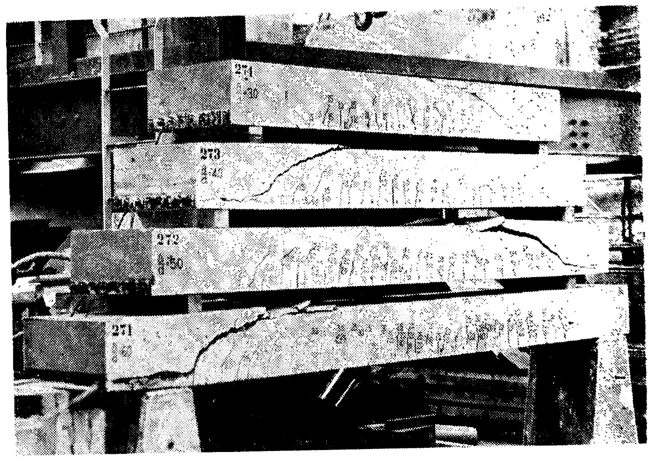

Fig. 9—Test series of beams with a width of $b = 24$ in.

The load arrangement (Fig. 10c) used for testing was the same as for the 3.8 - 2.80 - 12P series. The results are presented in Fig. 10d, which, for comparison, also show the relative beam strength r_u of Series 3.8 - 2.80 - 12P, $b = 6$ in. The details of the test data are presented in Table 5.

The wide beam ($b = 24$ in.) tests produced results which were both above and below the results of the corresponding values for the four times narrower beams ($b = 6$ in.). The difference, however, never exceeded ±10 percent, a scatter which

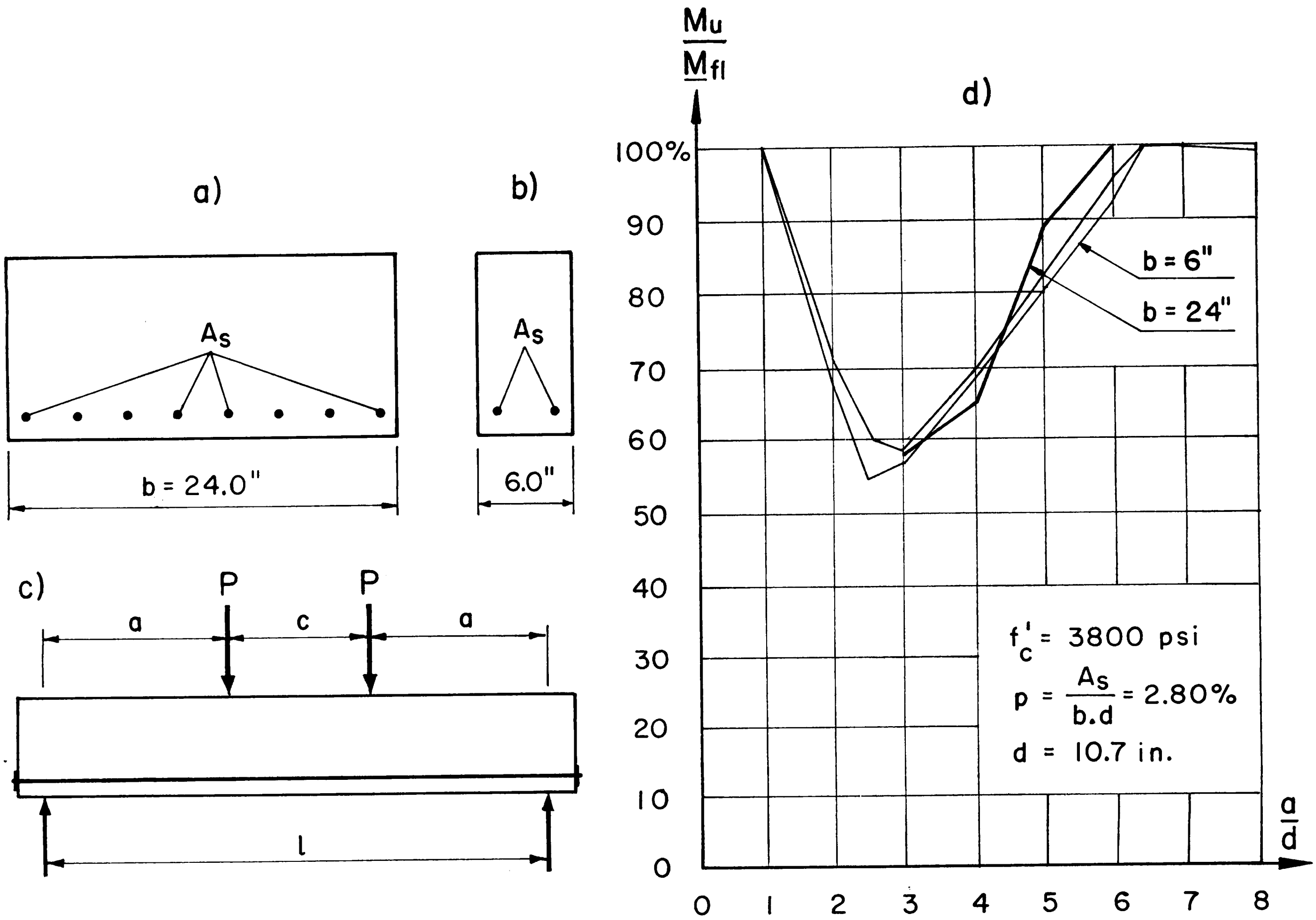

Fig. 10—Comparison of tested beams of different width

is to be expected even in laboratory controlled specimens. Thus, as expected, no significant influence of the beam width b on the relative beam strength r_u could be detected, in spite of the increased width. It appears, therefore, that the omission of the width b from any formulas expressing the relative beam strength of rectangular reinforced concrete beams is justified.

DISCUSSION OF TEST RESULTS

The tests confirmed the expectation that the influence of the absolute depth of beams is significant. By increasing the beam depth from 6 to 48 in., a decrease in relative beam strength r_u in the order of up to 40 percent has been observed (see Fig. 7b). This is more than the maximum influence, over the entire range of all practically important beams, of both, p and $M/Vd = a/d$ of the ACI Code Eq. (17-2) [see Eq. (1)].

Without the application of any appropriate correction for the absolute depth of the beam, the safety factors derived from tests on small beams produce dangerously low values.

Fig. 4 and 5 indicate that no simple "size factor" can be added to Eq. (1) to consider this influence. A size factor would not only depend on the depth d, but also on the region to which the beam belongs. If we consider the limited range of beams represented by Fig. 5, the largest beams being 4 ft deep, at least four regions of unrelated size factors appear:

1. The region to the left of the minimum point, i.e., between $a/d = 1.0$ and about 2.5.
2. The region between $a/d = 2.5$ and approximately 6.0, i.e., to the transition point of the small beams.
3. The region between approximately $a/d = 6.0$ and 9.0, where the influence of size gradually disappears.
4. The region beyond $a/d = 9.0$, where no size correction for the 4-ft beams is necessary.

The limit $a/d = 9.0$ is, of course, only valid for the 4 ft deep beams. For any other size of beam a different number would apply, adding to the complexity of such a size factor. However, if, instead of v_u, we choose as indicator of failure the relative beam strength r_u, a rather simple expression for the size factor can be developed. This, then, is valid for all regions to the right of the minimum point of r_u (see Fig. 5).

As previously suggested[2,3] and expressed by Eq. (4), the failure moment M_u can be computed from:

$$M_u = r_u M_{fl} \tag{4}$$

where the reduction factor r_u is defined by Eq. (3).

The average or effective crack length s can be expressed as:

$$s = \beta\, s_{max}$$

where s_{max} denotes the theoretical length of the longest cracks in the region of maximum moments, equaling the distance from the reinforcement to the neutral axis:

$$s_{max} = d - kd = d(1 - k)$$

If the average compressive stress in the compressive zone at failure is expressed by $\alpha f_c'$, (at failure α is about 0.75) and the equilibrium condition $T = C$ is used, we obtain:

$$pf_y = \alpha f_c' k \tag{5}$$

(3) then becomes:

$$r_u = \frac{f_t'}{6\alpha f_c'} \cdot \frac{\Delta x}{\beta\, dk(1-k)} \cdot \frac{a}{d} \tag{6}$$

If over-reinforced cross sections are excluded, then k is always less than one-half. Therefore $k(1-k)$ can be replaced with good approximation by $\frac{1}{2}\sqrt{k/2}$, as can be seen from Fig. 11. For $k = 0$ and $k = \frac{1}{2}$, both functions provide the same value. Between these limits, the differences are negligible. If the modulus of rupture f_t' is replaced by $10\sqrt{f_c' \text{ (psi)}}$, we obtain:*

$$r_u = \frac{10}{3\sqrt{\alpha}} \cdot \frac{\Delta x}{\beta d} \cdot \frac{a}{d} \sqrt{\frac{2\text{ (psi)}}{pf_y}} \tag{7}$$

Since the term f_c' has dropped out, this theoretical approach produces the same result as our tests,[2] i.e., that r_u does not depend on the concrete strength f_c'. The quantities α, β, and Δx, have clearly defined physical meanings. Designating their combination, as appearing in Eq. (7) by:

$$R = \frac{\Delta x}{\beta d} \cdot \sqrt{\frac{200}{9\alpha}} \tag{8}$$

---

*It is convenient and mathematically more exact if the dimension (psi) is present in the equation; otherwise, odd dimensions are obtained. For example, if the last factor in Eq. (7) did not contain the unit (psi), then the expression, with $p = 0.004$ and $f_y = 50{,}000$ psi, would appear to have the value $1/10\sqrt{\text{(psi)}}$. In reality, this quantity is nondimensional. The mathematically correct form has also the advantage that the formulas are independent of the dimensions used. For instance, if f_c' is given in cgs units as $f_c' = 257$ kg/cm^2 = 3600 psi, then the equation:

$$f_t' = 10\sqrt{f_c'\text{ (psi)}}$$

would still produce the right answer:

$$f_t' = 10\sqrt{257\text{ kg/cm}^2\text{(psi)}}$$
$$= 10\sqrt{257 \times 14\text{(psi) (psi)}}$$
$$= 600\text{ psi}$$

and by substituting this resistance factor R in Eq. (7), we obtain:

$$r_u = \frac{R}{\sqrt{pf_y}} \cdot \frac{a}{d} \qquad (7a)$$

Because the crack pattern of large beams is not geometrically similar to that of small beams, we cannot expect β to be independent of d, but otherwise, according to Eq. (8), R must be a constant since neither α nor Δx depend on d, p or f_c'.

For any transition point, $r_u = 100$ percent $= 1.0$ and we obtain from Eq. (7a):

$$R = \frac{\sqrt{pf_y}}{(a/d)_{TR}}$$

Since $(a/d)_{TR}$ is experimentally easy to determine, this is the simplest way to determine R. For the 12 in. beam series 3.8 - 2.80 - 12P, we have: $d = 10.7$ in., $p = 2.80$ percent and $(a/d)_{TR} = 6.5$ (see Fig. 5). Thus, for $f_y = \underline{f}_y = 50{,}000$ psi, $R = \sqrt{33.2\,(\text{psi})}$. The formula for all beams with $d = 10.7$ in. is therefore:

$$r_u = \sqrt{\frac{33.2\ (\text{psi})}{p\underline{f}_y}} \cdot \frac{a}{d} \qquad (10)$$

The validity of Eq. (10) can be checked against the results of the eleven 12 in. beam series described in Reference 2. By determining the transition points for varying amounts of main reinforcement, with $r_u = 1.0$ for the transition point $(a/d)_{TR}$, Eq. (10) is used to obtain the values shown in Table 7. The agreement between test and calculated values is more than satisfactory.

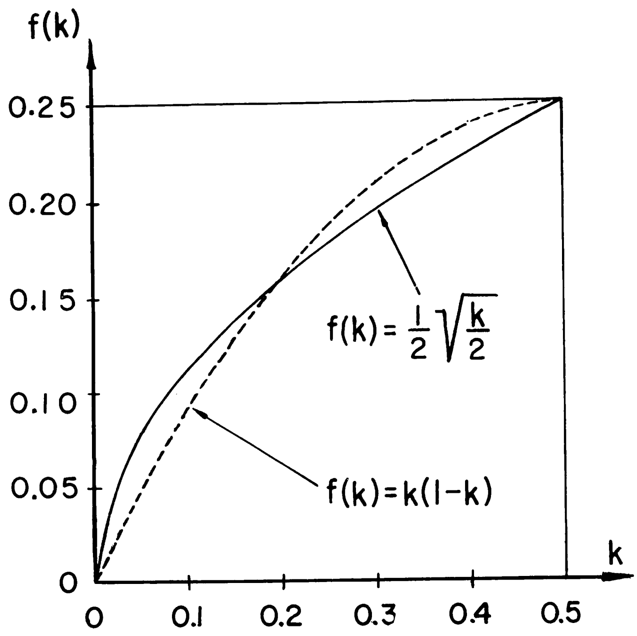

Fig. 11—Comparison of functions $k(1 - k)$ and $\frac{1}{2}\sqrt{k/2}$

TABLE 7—COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OBTAINED FOR THE TRANSITION POINTS

Test series		3.8-2.80-12P	3.8-1.88-12P	3.8-0.80-12P	3.8-0.50-12P
Reinforcement p, percent		2.80	1.88	0.80	0.50
$\left(\frac{a}{d}\right)_{TR}$	Eq. (10)	6.5	5.3	3.5	2.7
	Test values	6.5	5.4	3.0	2.5

To develop a more general formula for any depth d, two simple means are possible: to add to Eq. (10) an additive term or include a factor which will account for the influence of depth. From Fig. 5, it appears that all four lines have about the same slope of approximately 1:9. This would suggest a formula of the first type, i.e.:

$$r_u = \gamma(d) + \sqrt{\frac{c}{100p}} \cdot \frac{a}{d} \qquad (10a)$$

where, for the slope 1:9, the constant c is 0.035. The disadvantage of this solution is that for reliable determination of the transition point T rather accurate values of the function $\gamma(d)$ must be known. Since at present only four r_u lines are known, and these only for $p = 2.80$ and only for rectangular cross sections, more experimental data are necessary before a reliable function of $\gamma(d)$ can be suggested.

The other possibility of including a factor produces a simpler expression, especially if a convenient basic depth, e.g., $d_o = 10$ in., is introduced. From Fig. 5, where the transition points can be seen for four different depths, we obtain for $d = 10$ in., by interpolation: $(a/d)_{TR} = 6.4$. For $d = 10$ in., the same calculation as previously used with $\underline{f}_y = 50$ ksi produces the expression:

$$r_u = \sqrt{\frac{0.068}{100p}} \cdot \frac{a}{d} \qquad (10b)$$

Fig. 7 indicates that the loss of relative strength depends not only on d but, to an extent of ± 7.5 percent, also on a/d. Since this amount hardly justifies a more complicated formula, the following expression is, therefore, recommended for the determination of the reduction factor, which considers the amount of reinforcement p, the absolute depth d, and the shear arm ratio a/d:

$$r_u = \sqrt{\frac{0.215}{100\,p\sqrt{\frac{d}{(\text{in.})}}}} \cdot \frac{a}{d} \qquad (11)$$

Of course, r_u cannot exceed 100 percent since this would indicate a flexural failure rather than a diagonal failure. If a value greater than 1.0

TABLE 8—COMPARISON OF THEORETICAL AND EXPERIMENTAL VALUES OBTAINED FOR THE TRANSITION POINTS FOR BEAMS OF VARIOUS DEPTHS

Effective depth d, in.		10.0	10.7	21.4	42.8
$\left(\frac{a}{d}\right)_{TR}$	Eq. (11)	6.4	6.5	7.8	9.2
	Test Values	6.4	6.5	8.1	8.9

were obtained, unity has to be used instead. In Table 8, the calculated transition points, $(a/d)_{TR}$, [Eq. (11)] are compared with the values of the four test series shown in Fig. 5:

In Fig. 12, the calculated lines of relative beam strength, as obtained by Eq. (11), have been added to the lines of Fig. 5 representing the test values. The space between two corresponding lines has been hatched to indicate the differences. With the exception of one point, where the test value was 3 percent below the calculated value, the formula gives conservative values for the relative strengths. In all cases, the theoretical values are zero to 10 percent lower than the test values. For beams with small a/d ratios, i.e., which in Fig. 12 fall to the left of their respective minimum value for r_u, the line $r_u = d/a$, as suggested in previous papers, seems to be satisfactory as long as the concrete above the supports is not overstressed. Such a case belongs to bearing design and therefore is not a shear failure.

All beams of the four series were tested under point loadings, i.e., the investigation of the behavior under uniformly distributed loading has not been included in this program. The behavior of rectangular reinforced concrete beams under uniformly distributed load, as compared to point loading, has been discussed in Reference 2.

CONCLUSION

On the basis of the rational theory developed in Reference 3, increasing the beam depth must result in considerable reduction of the relative beam strength. The tests described in this paper have confirmed this conclusion.

By choosing the relative strength r_u rather than the shear stress v_u as the indicator of failure we obtain the analytical Eq. (11), which produces results within ±10 percent of the test values.

Eq. (11) includes the three major variables on beam strength: p, a/d, and the absolute beam depth d. With the reduction factor r_u known, the ultimate bending moment M_u can be calculated from Eq. (4).

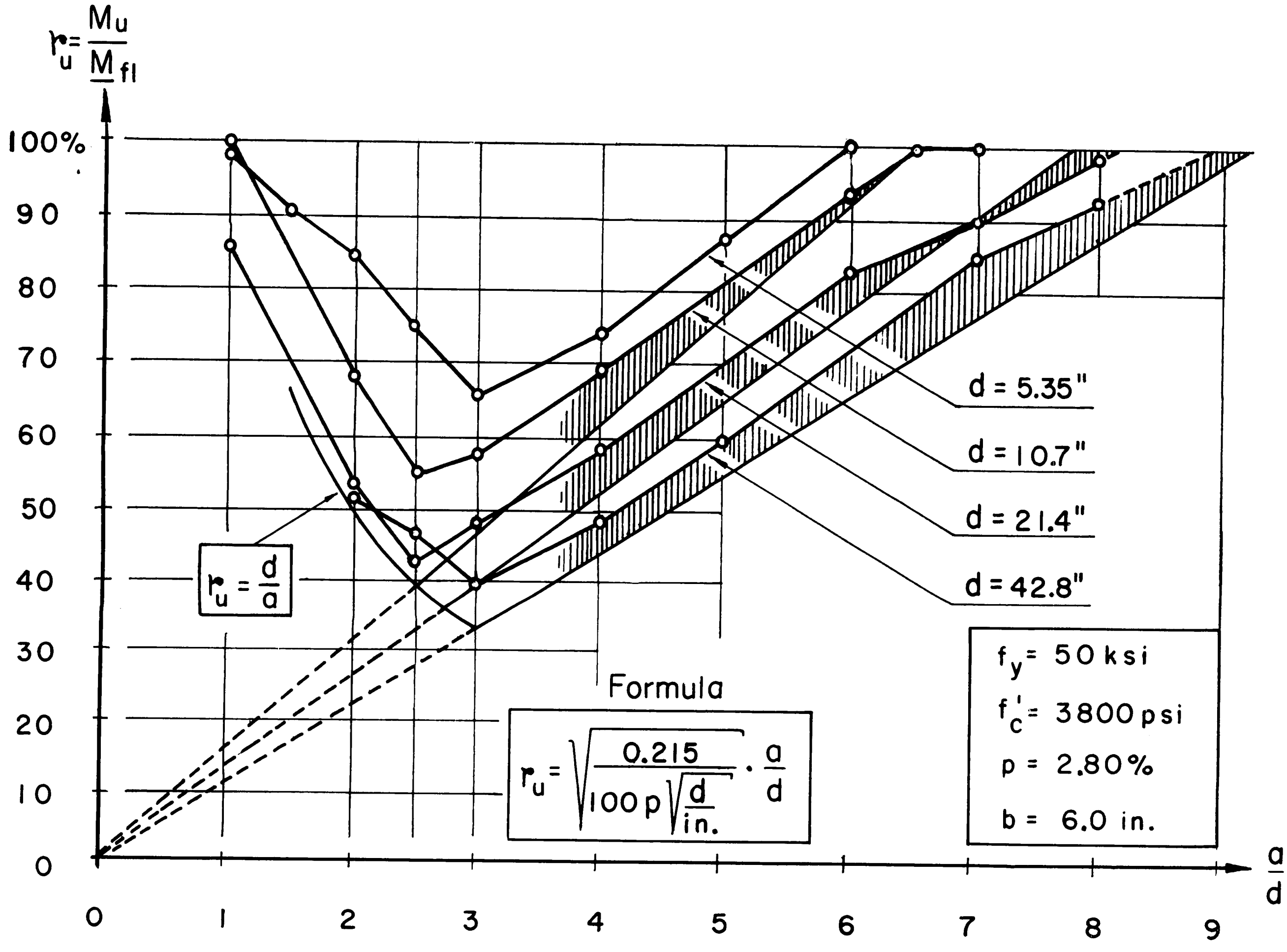

Fig. 12—Comparison of calculated and test values of the relative beam strength r_u

Since r_u gives the percentage reduction in beam strength due to a prematurely developed diagonal crack it is an extremely useful indicator of failure.

ACKNOWLEDGMENTS

The tests described in this paper were carried out in the laboratories of the Department of Civil Engineering at the University of Toronto and sponsored by the National Research Council of Canada. The author would like to thank R. Wittkopp for his invaluable assistance in both carrying out the tests and preparing the manuscript, and also to F. Cico who, as a master of applied science candidate, fabricated and tested the unusually large 48 in. deep beams.

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APPENDIX

NOTATION

b = width of cross section
t = over-all depth of beam
d = depth of cross section to level of main reinforcement
f_c' = concrete strength as determined on 6 x 12-in. cylinders
A_s = cross-sectional area of main reinforcement
k = ratio of depth of compressive zone to effective depth d
p = percentage of main reinforcement in cross section $= A_s/bd$
f_y = yield strength of main reinforcement
a/d = shear arm ratio
$2P_u$ = total applied load at beam failure
M_u = ultimate moment in midspan cross section at failure
M_{fl} = calculated flexural moment capacity of midspan cross section
$\underline{M_{fl}}$ = comparative flexural moment capacity of midspan cross section calculated using $\underline{f_y}$ = 50 ksi (see Reference 2)
r_u = relative beam strength $r_u = M_u/M_{fl}$
v_u = shear stress at failure $v_u = P_u/b\bar{d}$

This paper was received by the Institute Aug. 5, 1966.

Sinopsis—Résumé—Zusammenfassung

¿Qué seguridad Presentan Nuestras Vigas Granles de Concreto Reforzado?

Para responder a esta pregunta se ensayaron en la Universidad de Toronto cuatro series de vigas con peraltes de 6, 12, 24 y 48 pulgadas y se compararon los resultados. Se notó una influencia considerable del peralte efectivo a tal extremo que el factor de seguridad para las vigas más grandes fue aproximadamente 40 por ciento menor que el de las otras vigas similares más pequeñas. Esta tendencia indicó que con un aumento adicional en el peralte se puede esperar una disminución correspondiente adicional en el factor de seguridad.

Quelle est la sécurité réelle des poutres en béton armé de grandes dimensions?

Pour répondre à cette question, quatre séries de poutres d'essai, ayant pour hauteurs 6, 12, 24 et 48 in. (15, 30, 60 et 120 cm) ont été essayées à l'Université de Toronto et les résultats ont été comparés. Il a été trouvé que la valeur de la hauteur absolue avait une grande influence, le coefficient de sécurité réel de la poutre la plus grande étant 40% plus faible que celui des poutres plus petites identiques par ailleurs. Ceci laisse à penser qu'une plus grande augmentation de la hauteur pourrait être susceptible de conduire à une diminution encore plus forte du coefficient de sécurité.

Wie sicher sind unsere grossen Stahlbetonbalken?

Um diese Frage zu beantworten, wurden an der Universität von Toronto vier Versuchsreihen von Stahlbetonbalkengeprüft, deren Höhe 6, 12, 24 bzw. 48 in. betrug. Die Ergebnisse dieser Untersuchungen wurden miteinander verglichen. Ein merklicher Einfluss der absoluten Balkenhöhe wurde deutlich, und zwar zu einem solchen Ausmass, dass der Sicherheitsfaktor für den grössten Balken um annähernd 40 Prozent niedriger als für die sonst gleichartigen kleineren Balken war. Diese Tendenz zeigt, dass bei einer weiteren Vergrösserung der Balkenhöhe auch mit einem weiteren Abfall des Sicherheitsfaktoren zu rechnen ist.