Siddiqui *et al. Int J Concr Struct Mater (2018) 12:58*
https://doi.org/10.1186/s40069-018-0289-9

International Journal of Concrete Structures and Materials

ORIGINAL ARTICLE

Reliability Assessment of HFRC Slabs Against Projectile Impact

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Abstract

In the present study, a probabilistic procedure is presented for estimating the reliability of hybrid fiber reinforced concrete (HFRC) slabs against the impact of hemispherical nose projectiles considering uncertainties involved in the material, geometric and impact parameters. The influence of hybrid fibers in improving the safety level of reinforced concrete slabs against impact loads has also been studied on a parametric basis. The failure of the HFRC slabs was assumed to occur when the impact velocity of the projectile exceeds the ballistic limit of the slab i.e. perforates the slab. To illustrate the procedure, a probabilistic analysis was carried out on the impact test results of HFRC slabs containing different proportions of hooked-end steel, polypropylene and Kevlar fibers, recently published by the authors. Reliability assessment was performed for a range of applied nominal impact loads by varying the impact velocity of the given projectile. Reliability analysis yields the safety level of all the HFRC slabs against the impact of the above projectile. Effect of fibers, especially steel fibers, and slab thickness on the reliability of HFRC slabs are also investigated on a parametric basis.

Keywords: RC slabs, hybrid fibers, impact, reliability, ballistic limit, concrete, fibers, FRC, HFRC

1 Background

Reinforced concrete (RC) slabs and walls are many times required to resist impact loads generated due to the strike of missiles, projectiles and blast debris (Abbas et al. 1996; Frew et al. 1998; Chen and Li 2002). In the literature, a few methods have been proposed to increase the impact resistance of RC slabs against the strike of projectiles. Use of hybrid fibers in the concrete mixes is one of the effective techniques to improve impact resistance of concrete slabs. In the past, limited research has been conducted on impact response of hybrid fiber reinforced concrete (HFRC) slabs and plates (Almusallam et al. 2013, 2015; Daghash et al. 2016). A slab is assumed to be safe if the energy imparted by the impacting projectile remains below the perforation energy of the slab. In the present study, ballistic limit, which is the minimum impact velocity at which the slab gets perforated, was used for the assessment of perforation limit. If a slab appears to be safe (e.g. the deterministic value of slab's ballistic limit is more than the deterministic value of projectile impact velocity) for a given set of material and geometric properties of target (slab) and projectile, the slab may not be actually safe due to uncertainties involved in the governing parameters (in the above example, impact velocity may be substantially higher than the assumed velocity or ballistic limit could be lower than the estimated value). It is due to this reason that an effort is required to estimate the safety level of the slab in a probabilistic sense. That is, whether a slab is having desired probability of not failing under the impact of a design projectile. The probability of not failing could be termed as reliability. In the present study, a probabilistic procedure is presented for estimating the reliability of HFRC slabs against the impact of hemispherical nose projectiles considering uncertainties involved in the material, geometric and impact parameters. The influence of hybrid fibers in improving the safety level of RC slabs against impact loads will also be studied on a parametric basis.

In the literature, limited researchers are available on the reliability assessment of structures subjected to the impact loads of aircrafts, missiles and projectiles on RC

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Journal information: ISSN 1976-0485 / eISSN 2234-1315

structures (Pandey 1997; Han and Ang 1998; Choudhury et al. 2002; Siddiqui et al. 2003; Penmetsa 2005; Siddiqui et al. 2009, 2014a, b). Pandey (1997) presented a quantitative reliability-based procedure for evaluating the containment integrity as a function of bonded prestressing systems' condition. The procedure uses the results of destructive, lift-off, and flexural tests to modify the probability distribution of prestressing force. He then revised the estimated reliability against cracking of containment components. He also established an acceptable criterion for maintaining adequate reliability during service life of the containment. Han and Ang (1998) followed probabilistic approach and derived load factors for the limit state design of RC containment structures. As the prime purpose of RC containment structures is to provide protection against radioactive release, they considered serviceability limit state against crack failure as the critical limit state. They proposed load factors for designing RC containments and performed the reliability analysis against the above critical (serviceability) limit state. Choudhury et al. (2002) studied the reliability of a buried concrete target against normal impact of a missile. They derived the limit state functions (mathematical functions that assume zero or negative value at failure) using the formulations of penetration depth available in the literature. They also located the design points on the failure surface and carried out sensitivity analysis to study the effect of various random parameters on concrete target reliability. The outcomes of the study are useful in probabilistic design of buried concrete targets. Siddiqui et al. (2003) carried out the reliability assessment of a nuclear containment against the strike of a Boeing jet airplane. They obtained the probabilities of failures and reliabilities at a number of vulnerable locations of the containment using first order reliability method (FORM). FORM is a semi-probabilistic reliability analysis method devised to evaluate the reliability of structural elements or system. They used these values to estimate conditional and annual reliabilities of nuclear containment. Penmetsa (2005) presented an efficient methodology for system reliability analysis employing analytical equations for the penetration depth and buckling strength of the missile. Using this methodology, he determined the probability of destroying buried concrete targets with the help of deep penetration weapons. He has also employed above equations to find out the sensitivity of mission success to different parameters. Siddiqui et al. (2009) presented a methodology for reliability assessment of concrete targets, lying at a certain depth in the soil, against a rigid missile impact. They derived general expression for velocity of missile at any depth in the soil and deduced limit state functions based on penetration depth in the concrete target. The reliability assessment of concrete target was then carried out using first order reliability method (FORM). Sensitivity analysis and a few parametric studies were also included to obtain the results of practical interest.

Siddiqui et al. (2014b) designed a simple experiment that simulates a double-wall containment and studied the impact response of RC shielded steel plates against the strike of hard projectiles. They also performed the reliability analysis of the tested RC shielded steel plates using Monte Carlo Simulation (MCS) technique for a range of impact velocities and correlated the results of their reliability assessment with different scenarios of the failure of the test specimens. MCS is a simulation method that use random numbers to estimate the failure probability. They further extended their work (Siddiqui et al. 2014a, b) for studying the reliability of a double-wall containment structure against impact load of a hard projectile on its outer reinforced concrete wall. They correlated the safety of the containment, measured in terms of reliability indices and probabilities of failure, with the ballistic limit of its outer RC wall. A number of parametric studies were also carried out for studying the influence of some governing parameters on the reliability of double-wall containment structures.

The above literature review indicates that although a limited research is available on reliability studies of reinforced concrete slabs and steel plates, the studies on reliability of HFRC slabs against impact loads are not widely available. In the present study, a probabilistic procedure is presented for estimating reliability of HFRC slabs against the impact of hemispherical nose projectiles considering uncertainties involved in the material, geometric and impact parameters.

2 Formulation for Reliability Analysis

The reliability assessment of a structure or structural member is primarily concerned with the estimation of its probability of not violating/exceeding the limit state during the working life. In the current study, limit state is said to be violated when ballistic limit of HFRC slab become less than the impacting velocity of the projectile. In other words, a slab is said to have failed if its ballistic limit is less than the impact velocity of the projectile. In the present study, a MCS-based procedure is presented to estimate the reliability index and probability of failure of HFRC slabs. To illustrate the influence of some of the governing parameters, a few parametric studies have also been included.

In order to carry out the reliability analysis of a structure or structural component against a particular mode or type of failure, the failure is represented by a mathematical function called limit state function. This function can assume a positive, zero or negative value. A zero or negative value of the function represents failure of the structure or structural component against that particular mode of failure. A positive value of the function on the other hand represents the structure or structural component is safe. The probability of failure can then be defined as

$$P_f = P[g(\underset{\sim}{x}) \leq 0] \tag{1}$$

where, P_f represents the probability of failure, $g(\underset{\sim}{x})$ is the limit state function and $\underset{\sim}{x}$ is the vector of basic random variables. It is worth mentioning that, in the present study, the failure due to impact is a local failure caused due to high stress concentration and propagation of stress waves in the vicinity of impact location.

2.1 Limit State Function

To obtain the desired limit state function for carrying out the reliability analysis of HFRC slabs, normal strike of hard (non-deformable) hemispherical nose projectile on the slab was assumed. The slab was assumed to have failed when the impact velocity of the given projectile exceeded the ballistic limit of the slab. If V_0 = impact velocity; and V_p = ballistic limit velocity, the limit state function can be written as

$$g(\underset{\sim}{x}) = V_p - V_0 \tag{2}$$

when the impact velocity of the projectile is higher than the ballistic limit of HFRC slab, the slabs are likely to get fully penetrated (i.e. perforated). In the literature, a good number of researchers have proposed formulae for predicting ballistic limit of reinforced concrete targets (Frew et al. 1998; Chen and Li 2002) but these formulae cannot be as is employed for HFRC targets. In the present study, the ballistic limit expression, recently proposed and validated by the authors (Almusallam et al. 2013, 2015) for HFRC targets containing n-types of fibers was employed. The equation is:

$$V_p = V_a \qquad \text{for } V_a \leq 70\text{ m/s} \tag{3}$$

and

$$V_p = V_a\left[1 + \left(\frac{V_a}{500}\right)^2\right] \qquad \text{for } V_a > 70\text{ m/s} \tag{4}$$

$$\begin{aligned} V_a = {} & 1.3\rho_c^{1/6} f_c'^{1/2}\left(\frac{pH_0^2}{\pi M}\right)^{2/3}(r+0.3)^{1/2} \\ & \times\left[1.2 - 0.6\left(\frac{c_r}{H_0}\right)\right]\exp\left[\frac{1}{2}\left\{\frac{1}{2}RI_v\right\}^{1.5}\right] \end{aligned} \tag{5}$$

in the above equations, H_0 = thickness of the RC target (m); M = projectile mass (kg); p = missile aft body cross-section perimeter (m); c_r = spacing in steel rebars (m); r = steel rebars percentage; f_c' = compressive strength of concrete (MPa); and ρ_c = concrete density (kg/m^3).

In the above equation, RI_v represents reinforcing index which is expressed as

$$RI_v = \sum_{i=1}^{n}(\alpha_i p_i) \tag{6}$$

Here p_i represents volume fraction of ith fiber and α_i is a constant for the ith fiber. α_i depends on the fiber properties and defined as

$$\alpha_i = \frac{k_i l_i}{d_i}\frac{E_i}{E_s} \tag{7}$$

where, k_i = bond factor of ith fiber; l_i = ith fiber length; d_i = equivalent diameter (for non-circular sections) or simply diameter (for circular sections) of ith fiber; E_s and E_i = modulus of elasticity of the steel and ith fiber respectively. k_i = 1.0 for hooked and crimped fibers and k_i = 0.8 for plain fibers (Almusallam et al. 2013).

In the absence of rebars, the above equation will simplify into the following:

$$V_p = 1.3\rho_c^{1/6} f_c'^{1/2}\left(\frac{pH_0^2}{\pi M}\right)^{2/3}\exp\left[\frac{1}{2}\left\{\frac{1}{2}RI_v\right\}^{1.5}\right] \tag{8}$$

In the above equations, the value of $f_c^{'}$ is limited to 70 MPa. In other words, the value of $f_c^{'}$ shall not be taken more than 70 MPa.

In the above limit state given by Eq. (2), the variables V_0, RI_v, $f_c^{'}$, ρ_c, M, H_0, p, c_r and r may have significant uncertainties due to various reasons. Owing to this reason, in the present study, these variables are treated as random. Arraying the above variables in a vector form yields to

$$\underset{\sim}{x} = \left(V_0, RI_v, f_c^{'}, \rho_c, M, H_0, p, c_r, r\right) \tag{9}$$

Having derived the limit state functions, the next step is the calculation of failure probability and reliability of the HFRC slabs against the normal impact of the hemispherical nose shaped projectile. MCS technique (Nowak and Collins 2012) has been employed for this purpose.

2.2 Algorithm for Reliability Analysis

The algorithm followed in carrying out the reliability analysis was based on MCS technique, which can be summarized as given below:

Step 1. Input:

- Nominal values of all random and deterministic variables;
- Statistical properties of all random variables (e.g. COV and bias factors) and select their probability distributions.

Step 2. Estimate the parameters of probability distributions.

Step 3. Set the number of simulations N to perform MCS (e.g. $N = 500{,}000$).

Step 4. In each simulation cycle generate random values for all the random variables as per their probability distributions.

Step 5. Distributions feed the generated random values into derived limit state function $g(\underline{x})$ given by Eq. (2).

Step 6. Repeat the above steps 4 and 5 for N simulations. Count the number of simulations in which $g(\underline{x}) < 0$; say it is N_f.

Step 7. Calculate probability of failure P_f and reliability index β by

$$P_f = N_f / N \tag{10}$$

and

$$\beta = -\Phi^{-1}(P_f). \tag{11}$$

Step 8. Check the convergence of MCS by estimating the coefficient of variation $\mathrm{COV}(P_f)$ of the estimated probability of failure using the following equation (Nowak and Collins 2012):

$$\mathrm{COV}(P_f) \cong \frac{\sqrt{\frac{(1-P_f)P_f}{N}}}{P_f} \tag{12}$$

The smaller the coefficient of variation, the better is the accuracy of the estimated probability of failure. In real life calculations, that many simulation cycles (N) are considered sufficient for which the coefficient of variation $\mathrm{COV}(P_f)$ of the estimated probability of failure is less than 5%.

3 Data for Reliability Analysis

To illustrate the above procedure, probabilistic analysis was carried out for HFRC slab specimens of 600 × 600 × 90 mm size, recently tested and published by the authors in (Almusallam et al. 2013). The specimens (Fig. 1) were prepared using three types of fibers viz. hooked-end steel, polypropylene and Kevlar fibers (Fig. 2) in different proportions and tested against the normal strike of steel projectiles of hemispherical nose shape (Fig. 3). These fibers in different proportions were added into the fresh concrete during casting of the specimens. The constituents of plain concrete mix employed in the preparation of reinforced concrete slab specimens are listed in Table 1. The mechanical and physical properties of fibers are shown in Table 2. Table 3 provides the volume percentages of fibers added in concrete for producing different mixes.

In order to carry out the reliability analysis of the HFRC slabs/plates, the variables which have substantial uncertainties were identified and their statistical characteristics including their meaningful probability distributions were proposed. The variables which are considered random are shown in Table 4. In this table, the bias factor is calculated by dividing the mean value of the random variable by its nominal value (value which is fixed on a non-statistical basis). When this factor is one, it indicates that the mean value is assumed same as non-statistical nominal value. In general, bias factor is taken greater than one for resistance related variables. This factor is less than 1.0 for load related variables.

To carry out the reliability analysis of HFRC slab specimens, probability distribution of the expected extreme impact load, measured in terms of impact velocity for the given projectile, is also essential. In the current study, the impact velocity was described using Extreme Type I distribution (Table 4). The PDF (probability density function) and the CDF (cumulative distribution function) of this distribution are given by Nowak and Collins (2012):

$$\text{PDF:}\, f(x) = \alpha \exp\left\{-e^{-\alpha(x-u)}\right\} \exp\{-\alpha(x-u)\} \tag{13}$$

$$\text{CDF:}\, F(x) = \exp\left\{-e^{-\alpha(x-u)}\right\} \quad \text{for} -\infty \le x \le \infty \tag{14}$$

where u and α are parameters of the distribution. For known mean and standard deviation, values of the distribution parameters can be approximately estimated using Nowak and Collins (2012) as

$$\alpha \approx \frac{1.282}{\sigma_x} \tag{15}$$

$$u \approx \mu_x - 0.45\sigma_x \tag{16}$$

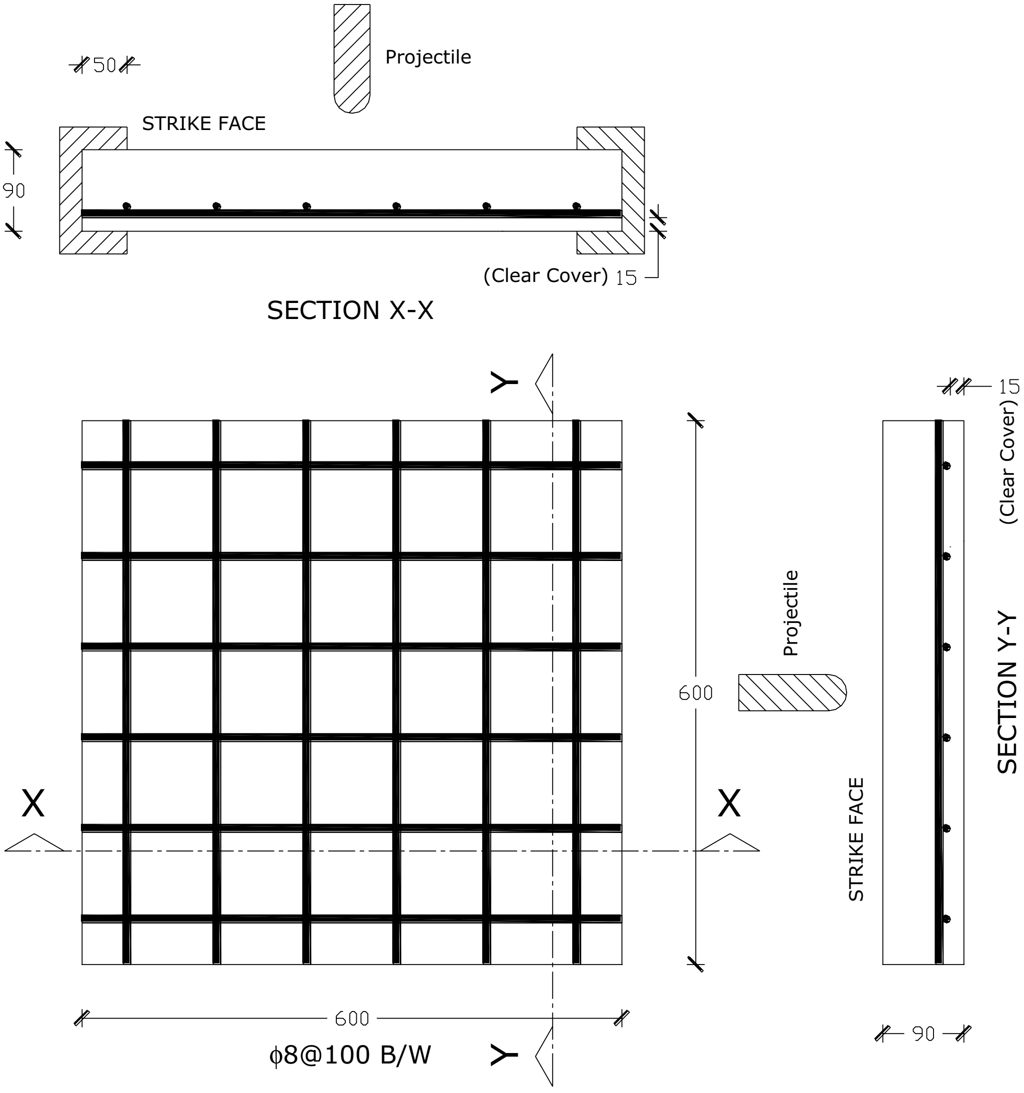


Fig. 1 Details of reinforcement in HFRC slabs (dimensions are in mm).

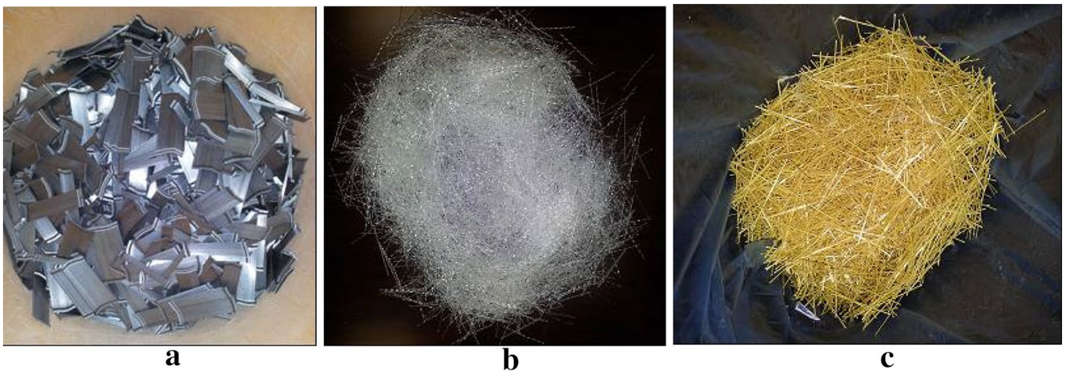


Fig. 2 **a** Hooked-end steel fibers, **b** polypropylene fibers, and **c** Kevlar fibers (Almusallam et al. 2015).

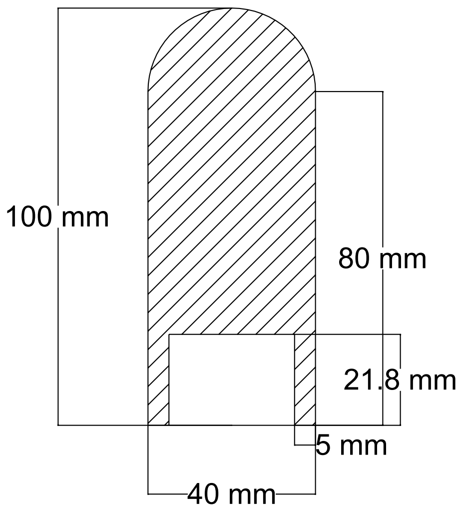


Fig. 3 Details of the spherical nose projectile (Almusallam et al. 2015).

Table 1 The constituents of the plain concrete mix.

Constituent	Quantity
Cement	520 kg/m^3
Sand (fine)	586 kg/m^3
10 mm size coarse aggregate	850 kg/m^3
5 mm size coarse aggregate	315 kg/m^3
Water	145 kg/m^3
Super-plasticizer (Gli-110)	3.0 L
Retarder (LD10)	1.5 L

where μ_x and σ_x represent mean and standard deviation respectively. The references followed for selecting the coefficient of variation (COV) and the probability distributions of the random variables are given in the last column of Table 4.

To carry out the reliability analysis using MCS, it is essential to know the required number of simulations. Figure 4 shows the variation of COV(P_f) with the number of simulations for HFRC slab specimen prepared using the Mix M3 and impacted by 60% of its estimated nominal ballistic limit. This figure shows that with the increase in the number of simulation cycles (N), COV(P_f) is decreasing. This figure indicates that, for the present study, N greater than 100,000 can give an adequately accurate failure probability (i.e. COV(P_f) less than 5%). In the present study, in general, 500,000 simulations were used to perform the MCS. However, wherever probability of failure was sufficiently small, much higher than 500,000 simulations were used to estimate the failure probability.

4 Discussion of Results

Employing the data presented in Table 4 and using the MCS technique, failure probability (P_f) and reliability indices (β) of HFRC slab specimens were obtained and shown in Table 5. In this table, the nominal impact velocities are taken same as the experiment (Almusallam et al. 2013). During the experiment, HFRC slab specimens of $600 \times 600 \times 90$ mm size, were tested against the normal impact of steel projectiles of hemi-spherical nose shape (Fig. 3).

During the tests, the impact velocity was varied from 0.83 V_{BL} to 1.14 V_{BL}, where V_{BL} is the ballistic limit of the specimens for the given projectile estimated using Eqs. (3) through (5). The last two columns of Table 5 shows the probabilities of failure P_f and reliability indices β of the HFRC slab specimens. The results clearly indicate that as the projectile impact velocity increases, the probability of failure of the HFRC specimens also increases. This is an expected trend. Table 5 shows that for those specimens which have the reliability indices above 1.0 corresponds to a sufficiently low probability of failure. This is because the parameters that affect the ballistic limit of the HFRC specimens for the given projectile, their combined uncertainties do not make the ballistic limit to fall below the impact velocity. For example, a probability of failure of 0.07 for the specimen M5-S1 suggests that if 100 such specimens were subjected to the same impact velocity, only ballistic limit of 7 specimens would fall below the impact velocity.

It is worth mentioning that in a deterministic sense whenever striking velocity is higher than the ballistic

Table 2 Properties of fibers.

Fiber type	Length (mm)	Shape	Section dimensions (mm)	Aspect ratio	Specific gravity	Tensile strength (MPa)	Modulus of elasticity (GPa)	Bond factor, *k*
SF	60	Hooked ends	0.75 ϕ (circular)	80	7.85	1225	200	1
PP	50	Crimped	1 × 0.6 (rectangular)	57.2[a]	0.9	550	4	1
KF	45	Plain	0.50 ϕ (circular)	90	1.45	3220	131	0.8

[a] Calculated using equivalent diameter.

Table 3 Fiber percent in different concrete mixes.

Concrete mix	Percentage of fiber by volume (by weight)			
	Polypropylene (PP)	Steel (SF)	Kevlar (KF)	Total
M0	0.0 (0.00)	0.0 (0.00)	0.0 (0.00)	0.0 (0.00)
M1	0.0 (0.00)	1.2 (3.93)	0.0 (0.00)	1.2 (3.93)
M2	0.2 (0.08)	1.0 (3.27)	0.0 (0.00)	1.2 (3.35)
M3	0.0 (0.00)	1.4 (4.58)	0.0 (0.00)	1.4 (4.58)
M4	0.2 (0.08)	1.2 (3.93)	0.0 (0.00)	1.4 (3.98)
M5	0.0 (0.00)	0.9 (2.94)	0.3 (0.11)	1.2 (3.05)
M6	0.0 (0.00)	1.1 (3.60)	0.3 (0.11)	1.4 (3.71)
M7	0.2 (0.08)	0.9 (2.94)	0.3 (0.11)	1.4 (3.13)
M8	0.2 (0.08)	0.7 (2.29)	0.3 (0.11)	1.2 (2.48)

limit, failure is certain; however, in probabilistic sense it only means that there is a high probability that failure will occur. Table 5 supports this point as for all those specimens for which V_0/V_{BL} ratio is greater than 1 (Specimens: M0S1, M0S2, M1S2, M2S2, M2S3, M3S2, M4S2, M5S3, M6S2, M7S2, M7S3, M8S1, M8S2, and M8S3) probabilities of failure are substantially high (reliability indices are substantially less than 3.0). But when V_0/V_{BL} ratio is only slightly above 1.0, failure may or may not occur. For example, for M0S2 specimen ($V_0/V_{BL} = 1.06$), failure has not occurred, but for specimen M1S2 ($V_0/V_{BL} = 1.04$) failure was seen in the specimen. It is to be noted that in structural engineering, a structure or its component are generally considered safe enough if their reliability indices are 3.0 or above (Siddiqui et al. 2014a, b; Nowak and Collins 2012; Alsayed and Siddiqui 2013). Present probabilistic analysis of the specimens illustrates that for the considered impact velocities, none of the specimens is as reliable as desired because for all the specimens reliability indices are falling below 3.0. In order to arrive at the impact velocity for which slab specimens can be considered as reliable as desired, Fig. 5 was plotted between reliability index and V_0/V_{BL} ratio. The graph was plotted only for three specimens HFRC-M3, HFRC-M5 and HFRC-M8 in order to avoid clumsy presentation. Figure 5 shows that as the impact velocity increases, reliability decreases sharply. Reliability is around 4 when impact velocity is about half of the ballistic limit ($\sim 0.5\ V_{BL}$) and reduces to almost 0, when it is about 1.4 times the ballistic limit ($\sim 1.4\ V_{BL}$) of the slab. The reliability is 3 and above when the ratio V_0/V_{BL} is 0.7 or less. This concludes that for design purposes the ballistic limit of the HFRC slabs should be kept around $V_0/0.7 \sim 1.4\ V_0$ in order to have desired reliability index of the slab at least 3.0. Since all the graphs are overlapping this conclusion is valid for all the mixes of the HFRC slab.

Figure 6 shows the effect of fibers on reliability of HFRC slabs subjected to same impact velocity taken equal to the ballistic limit of the control slab (118 m/s). Figure clearly illustrates that when there was no fiber (Mix M0), reliability of the slab was substantially low. With the addition of fibers, slab reliability improved substantially. The maximum increase is for mix M3 which contains highest amount of steel fibers (1.4%). The second highest reliability is obtained for mix M1 that contains second highest percentage of steel fibers. The reliability of mix M4 and M6 are also substantially high but slightly less than M1 and M3 that contain only

Table 4 Random variables and statistical data.

Random variable	Nominal	Bias factor	Coefficient of variation (COV)	Distribution	References
HFRC slab					
Concrete density, ρ_c (kg/m^3)	2500	1.05	0.10	Lognormal	Choudhury et al. (2002)
Concrete strength, f_c' (MPa)	Variable	1.10	0.10	Lognormal	Siddiqui et al. (2014b)
Thickness of slab, H (mm)	90	1.00	0.05	Normal	Choudhury et al. (2002)
Reinforcement ratio, r (%)	0.708	1.10	0.10	Normal	Siddiqui et al. (2014b)
Steel rebar spacing, c_r (mm)	100	0.90	0.05	Lognormal	Assumed
Reinforcing index, RI_v	Estimated[a]	1.10	0.12	Lognormal	Assumed
Projectile					
Mass of the projectile, M (kg)	0.8	1.10	0.05	Lognormal	Penmetsa (2005)
Diameter of the projectile, d (mm)	40	1.05	0.05	Normal	Penmetsa (2005)
Impact velocity, V_0 (m/s)	Variable	0.90	0.10	Extreme type I	Choudhury et al. (2002)

[a] Nominal value was estimated using Eq. (6).

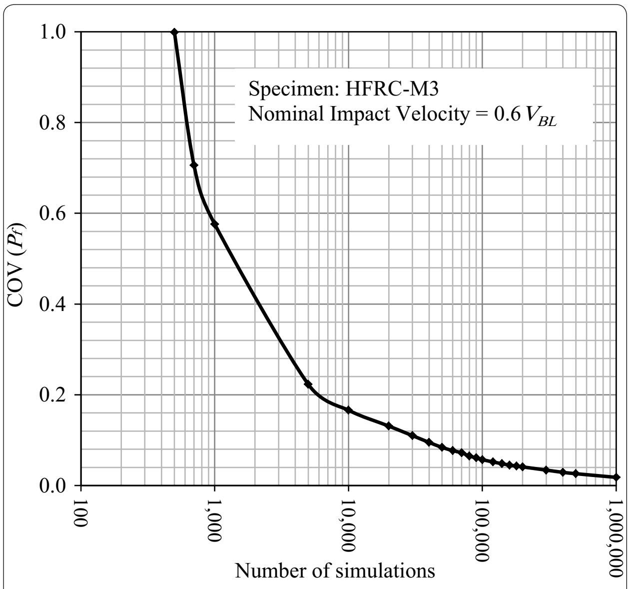


Fig. 4 Effect of simulation cycles on the convergence of P_f (V_{BL} = estimated nominal ballistic limit).

the steel fibers. This suggests that as far as increasing the ballistic limit is concerned the influence of steel fiber is maximum. However, for improving the other properties, as discussed in our earlier paper (Almusallam et al. 2013, 2015), plastic and Kevlar fibers do have their significance.

4.1 Effect of Impact Velocity

Figure 7 shows the effect of projectile impact velocity on the reliability of HFRC slabs. As expected, with the increase in the impact velocity, reliability is decreasing. This is so because as the impact velocity will increase, the safety margin ($V_0 - V_{BL}$) will reduce which will consequently reduce the reliability. The figure also shows that for HFRC-M3 specimen, the reliability is more than the desired reliability (reliability index = 3.0) for impact velocity up to 105 m/s. A relatively higher tolerance for impact velocity (for desired reliability index) is due to the presence of high percentage of steel fibers in HFRC-M3 slabs. Figure 7 also illustrates that HFRC-M5 and HFRC-M8 slabs have adequate reliability (reliability index = 3.0) only up to impact velocity of 90 m/s. The impact velocity tolerance is small in this case due to lower percentage of steel fibers in these slabs.

4.2 Effect of Steel Fiber Proportion

Figure 8 shows how proportion of steel fiber influences the reliability of HFRC slab. This figure clearly illustrates that in order to achieve the desired reliability of 3.0, for a HFRC slab subjected to impact velocity of 118 m/s, the proportion of steel fiber should be around 1.8%. This is so because at this percentage of steel fiber, $(\beta - \beta_D)^2 \approx 0$. In other words, reliability index of HFRC slab will reach to its desired value if the steel fiber percentage increases to 1.8%. Here β and β_D are the actual and desired reliability index values.

4.3 Effect of Slab Thickness

Figure 9 shows that as the HFRC slab thickness increases, the reliability of the slab also increases. This is because as the slab thickness increases, the ballistic limit of the slab increases which as a result increases the reliability of the HFRC slab. This figure illustrates that $(\beta - \beta_D)^2 \approx 0$ when slab thickness is 100 mm. It means, in order to achieve the reliability index of 3.0 for HFRC-M5 and HFRC-M8 slabs, keeping all the variables same, the thickness of HFRC slabs should be at least 100 mm. However, for HFRC-M3 slabs, this thickness requirement reduces to about 95 mm due to the presence of higher percentage of steel fibers in HFRC-M3 slab.

5 Conclusions

In the present paper, a MCS based procedure for carrying out reliability analysis of HFRC slabs is presented. Following conclusions can be derived from the reliability analysis of the tested slabs and related parametric studies:

- As the impact velocity of projectile increases, reliability of HFRC slabs decreases sharply. Reliability is around 4 when impact velocity is about half of the ballistic limit and decreases to nearly 0, when it is around 1.4 times the ballistic limit of the slab. The reliability is 3 and above when the ratio V_0/V_{BL} is 0.7 or less.
- The ballistic limit of the HFRC slabs should be kept around 1.4 V_0 in order to achieve desired reliability index of the slab (i.e. 3.0).
- The addition of fibers improves the slab reliability substantially. The maximum increase was observed for that HFRC slab which contains highest amount of steel fibers (i.e. 1.4%). The second highest reliability

Table 5 Failure probability and reliability indices of each specimen.

Slab specimen	V_0	V_{BL}	V_0/V_{BL}	Failed/not failed	P_f	β
HFRC-M0S1	135.20	118.5	1.14	F	0.232	0.733
HFRC-M0S2	125.10		1.06	N	0.153	1.023
HFRC-M0S3	108.10		0.91	N	0.051	1.635
HFRC-M1S1	135.10	153.6	0.88	N	0.032	1.848
HFRC-M1S2	160.20		1.04	F	0.111	1.220
HFRC-M1S3	147.25		0.96	N	0.064	1.522
HFRC-M2S1	135.10	142.9	0.95	N	0.062	1.540
HFRC-M2S2	160.20		1.12	F	0.184	0.902
HFRC-M2S3	147.25		1.03	N	0.109	1.233
HFRC-M3S1	135.10	162.6	0.83	N	0.018	2.087
HFRC-M3S2	178.50		1.10	F	0.153	1.025
HFRC-M3S3	147.25		0.91	N	0.040	1.748
HFRC-M4S1	135.10	151.8	0.89	N	0.035	1.809
HFRC-M4S2	168.20		1.11	F	0.168	0.962
HFRC-M4S3	147.25		0.97	N	0.069	1.486
HFRC-M5S1	135.15	139.7	0.97	N	0.070	1.475
HFRC-M5S2	125.00		0.89	N	0.036	1.801
HFRC-M5S3	147.10		1.05	F	0.121	1.169
HFRC-M6S1	135.00	147.1	0.92	N	0.044	1.703
HFRC-M6S2	160.20		1.09	F	0.149	1.041
HFRC-M6S3	147.25		1.00	N	0.083	1.384
HFRC-M7S1	135.20	141.4	0.96	N	0.0653×10^{-1}	1.512
HFRC-M7S2	158.10		1.12	F	0.181×10^{-1}	0.911
HFRC-M7S3	147.30		1.04	N	0.114×10^{-1}	1.206
HFRC-M8S1	135.34	134.3	1.01	N	0.983×10^{-1}	1.291
HFRC-M8S2	153.67		1.14	F	0.207×10^{-1}	0.815
HFRC-M8S3	147.30		1.10	N	0.170×10^{-1}	0.954

F failed, *N* not failed.

is obtained for the slab that contains second highest percentage of steel fibers (i.e. 1.2%).

- Those HFRC slabs which have a relatively higher percentage of steel fibers have higher tolerance for projectile impact than HFRC slabs with lower percentage of steel fibers.
- The reliability index of present HFRC slabs will achieve desired value of reliability index (i.e. 3.0) if the steel fiber percentage is increased to 1.8%.
- As the slab thickness increases, the ballistic limit of the slab increases which consequently increases the overall reliability of the HFRC slab.

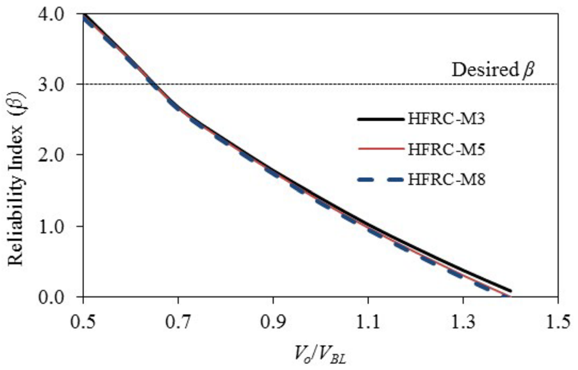
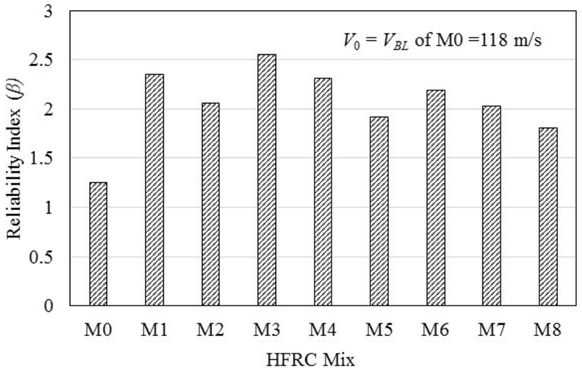


Fig. 5 Variation of reliability index of HFRC specimens with V_0/V_{BL} ratio.



Mix	PF (%)	SF (%)	KF (%)	Total (%)
M0	0	0	0	0
M1	0	1.2	0	1.2
M2	0.2	1	0	1.2
M3	0	1.4	0	1.4
M4	0.2	1.2	0	1.4
M5	0	0.9	0.3	1.2
M6	0	1.1	0.3	1.4
M7	0.2	0.9	0.3	1.4
M8	0.2	0.7	0.3	1.2

Fig. 6 Reliability of all the HFRC slab specimens subject to impact velocity of 118 m/s.

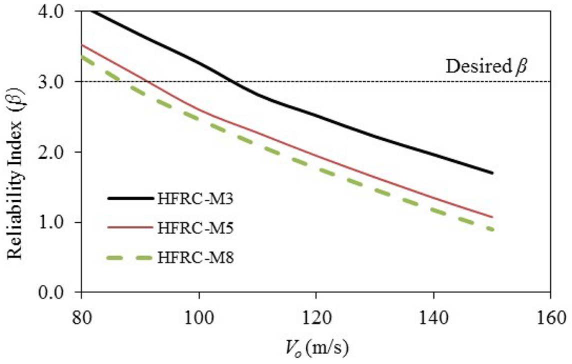


Fig. 7 Variation of reliability index of HFRC specimens with impact velocity.

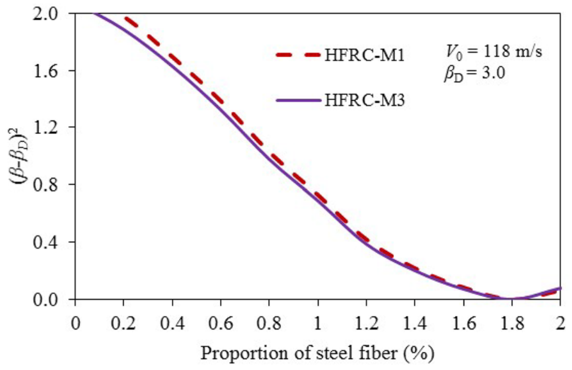


Fig. 8 Effect of steel fiber percentage on reliability of HFRC slab specimens.

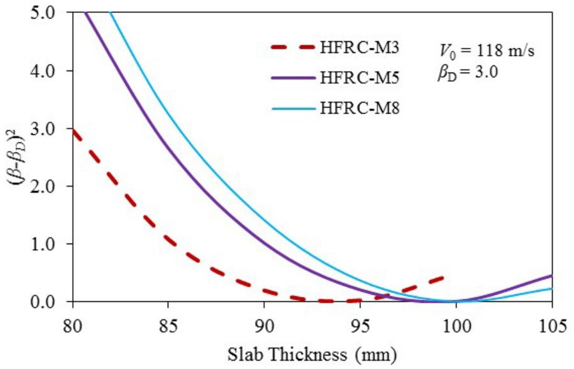


Fig. 9 Effect of slab thickness on reliability of HFRC slab specimens.

- To achieve the reliability index = 3.0 for HFRC-M5 and HFRC-M8 slabs, thickness of slab should be at least 100 mm. However, for HFRC-M3 slabs, this thickness requirement reduces to about 95 mm.

Abbreviations

COV: coefficient of variation; CDF: cumulative distribution function; c_r: spacing of steel rebars; d_i: diameter of *i*th fiber or equivalent diameter for non-circular sections; E_i: modulus of elasticity of *i*th fiber; E_s: modulus of elasticity of steel; f_c': specified compressive strength of concrete; $g(\boldsymbol{x})$: limit state function; H_0: thickness of the RC target; k_i: bond factor of *i*th fiber (= 1.0 for hooked and crimped fibers; = 0.8 for plain fibers); l_i: length of *i*th fiber; *M*: projectile mass; *N*: number of simulations; N_f: number of simulations in which $g(\boldsymbol{x}) < 0$; *P*: missile aft body cross-section perimeter; P_f: probability of failure; p_i: volume fraction of *i*th fiber; *r*: steel rebars percentage; RI_v: reinforcing index; $\boldsymbol{x}$: vector of basic random variables; *u*, *α*: parameters of the probability distribution; V_0: impact velocity; V_p: ballistic limit velocity; α_i: a constant for the *i*th fiber; *β*: reliability index; μ_x: mean; ρ_c: concrete density; σ_x: standard deviation; $\Phi^{-1}(.)$: inverse of standard normal CDF.

Authors' contributions

NAS wrote the computer program for carrying out the reliability analysis and participated in the manuscript preparation; YAA participated in the algorithm development, reviewed the entire manuscript critically and participated in the manuscript preparation; THA carried out the literature review, provided the experimental data for the reliability analysis and participated in the discussion of results; AAA collected the statistical data and carried out the parametric study; HA formulated the limit state function, validated the results and participated in the manuscript preparation. All authors read and approved the final manuscript.

Acknowledgements

This research was supported by Deanship of Scientific Research Chairs at King Saud University, Saudi Arabia for Chair of Research and Studies in Strengthening and Rehabilitation of Structures at Civil Engineering Department.

Competing interests

The authors declare that they have no competing interests.

Publisher's Note

Received: 29 December 2016 Accepted: 26 June 2018

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