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Tests on Concrete Slab-Column Connections with Stud-Shear Reinforcement Subjected to Shear-Moment Transfer


by Adel A. Elgabry and Amin Ghali

Test results of five full-scale reinforced concrete flat-plate connections with interior columns subjected to shear-moment transfer are reported. One specimen had no shear reinforcement, and the remaining four contained various arrangements of stud-shear reinforcement (vertical rods mechanically anchored at their top and bottom). The results confirmed the effectiveness of this type of shear reinforcement in increasing the shear strength and ductility of the connection. Code provisions suggested earlier for the design of shear studs are verified. Requirements for the dimensions of the anchor heads are suggested. The design of shear-stud reinforcement is demonstrated by a numerical example.

Keywords: columns (supports); connections; deformation; ductility; **flat concrete plates; moments;** punching shear; reinforced concrete; **shear strength.**

Gravity, wind, or earthquake forces produce shear and bending moment in flat-plate floors, which are transferred between the slab and the columns. Such bending moment is often referred to as the "unbalanced" moment, e.g., clause 11.12 of ACI 318-83.[1] Flat-plate buildings usually have structural systems to provide adequate stiffness to resist lateral loads. Despite the existence of such systems some shear-moment transfer must take place. The shear force and the unbalanced moment produce shear, bending moment, and torsional moment in the slab. A brittle failure in the column vicinity can occur due to the high shear stresses that are produced by such a transfer. Thus, shear strength and ductility of the slab-column connection must be considered in design.[2] The use of shear reinforcement in the form of stirrups, bent-up bars, or structural shear heads can increase the shear strength and ductility of the connection. However, installation of such reinforcement in relatively thin flat plates is difficult, and providing adequate anchorage is a problem.

A relatively new type of shear reinforcement, in the form of vertical rods (studs) mechanically anchored at their top and bottom ends, has been investigated extensively at the University of Calgary, Canada. The bottom anchors for the studs may be preferably in the form of steel strips that serve an additional purpose of holding the studs in their appropriate position in the forms. The top anchors for the studs can be in the form of plates of area at least 10 times the area of the stem. A semiautomatic welding procedure[3] can be used to weld the studs to the anchor heads. Cold-formed anchor heads and other welding procedures may also be used. Details of this type of shear reinforcement and its effectiveness to resist uniform shear stresses in the column vicinity are presented in References 3, 4, 5, and 6.

The stud-shear reinforcement for slabs is adopted by the Canadian Code (CAN3-A23.3-M84[7]) and by the West German Construction Supervising Authority (Approval Certificate No. Z-4.6-70 dated July 29, 1980).

The present paper reports new tests conducted on a series of five full-scale specimens of reinforced concrete interior flat-plate-column connections subjected to shear V and unbalanced bending moment M. One specimen had no shear reinforcement while the remaining four contained various arrangements of stud-shear reinforcement. The objectives were to study the effectiveness of the stud-shear reinforcement in resisting the shear stresses in the column vicinity and to verify the validity of the design code provisions suggested by Dilger and Ghali[4] when the connection is subjected to shear-moment transfer. A numerical example for the design of the stud-shear reinforcement is presented.

RESEARCH SIGNIFICANCE

This research is related to the design of stud-shear reinforcement for slabs subjected to shear-moment transfer. The shear strength and ductility of interior slab-column connections reinforced with shear studs are investigated. Suggested code provisions for the design

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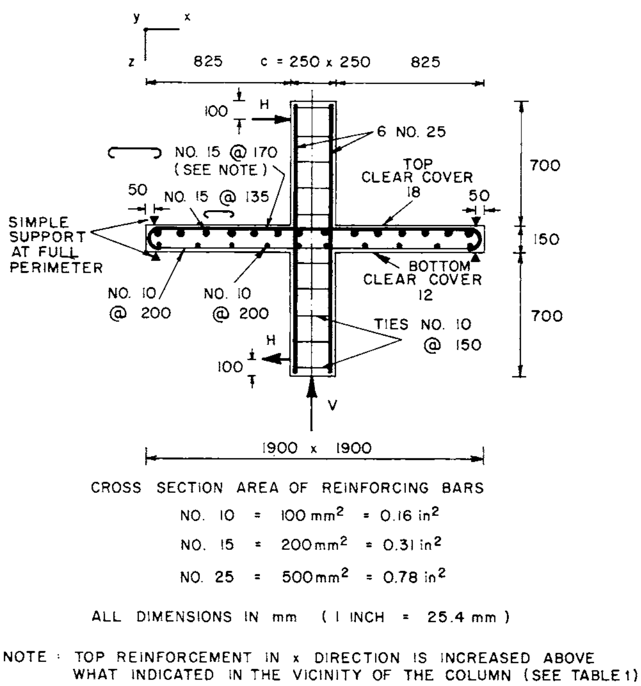

Fig. 1 — Test specimen

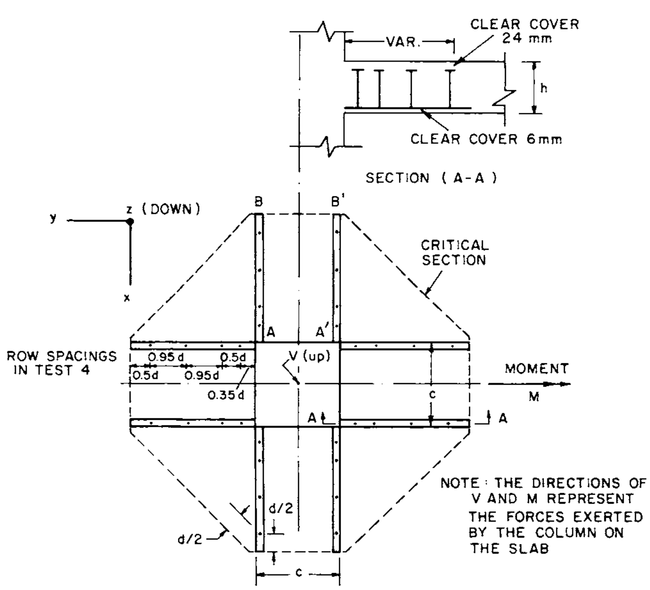

Fig. 2 — Typical stud-shear reinforcement arrangement

of this type of shear reinforcement are discussed, and a numerical design example is presented.

TEST DATA

The dimensions and the reinforcement details of a typical specimen are shown in Fig. 1. The specimen represents a full-scale interior column connected to a slab part bound by the lines of contraflexure around the column. In the tests, the slab was simply supported near the edges along the perimeter of a square of side length = 1800 mm (71 in.). The slab was tested in a vertical position in which the shear force V was applied in a horizontal direction along the column axis and the unbalanced moment was introduced by two equal and opposite vertical forces H near the column tips. The shear studs were arranged in rows around the column as shown in Fig. 2. Each row had eight studs, and the spacing between the rows varied as shown in Fig. 3. All specimens were cast with normal density concrete and Specimen No. 1 had no shear reinforcement. Studs in Tests 2 to 5 had a 24-mm (0.94-in.) cover above the top anchor heads and 6 mm (0.24 in.) below the bottom anchor strips. A semiautomatic welding procedure[3] was used in the tests to weld the studs to the top and bottom anchors. A summary of test data is presented in Table 1.

The studs used in the tests were made of stems provided with flux at the two ends for electric welding. The commerically available stems had an overall length = 108 mm (4.25 in.). This gave stud-shear reinforcement units with a total height of 120 mm (4.72 in.). Using these units in a 150-mm (5.9-in.) slab leaves 30 mm (1.18 in.) for the sum of the covers above and below the anchor heads. The 6-mm (0.24-in.) cover used below the bottom anchor strips in the tests may be increased in practical applications to reduce the likelihood of honeycombs or voids underneath the bottom plate (e.g., covers 0.5 in. and 0.75 in. at bottom and top, respectively). However, the covers should be kept to a practical minimum because earlier tests[3] indicated that elimination of the concrete covers improves the effectiveness of the studs.

To establish the required minimum dimensions of the top anchor heads and the bottom strips, these dimensions were differed for the studs on Line AB (Fig. 2) from those on other lines (see Table 2). In Tests 2 and 3 the bottom strip along Line AB had a width = $2D$, instead of $2.5D$ along other lines, with D being the stud diameter. In Tests 4 and 5 the thickness of the top anchors and of the bottom strips was increased to $0.66D$, instead of $0.5D$ on the other lines. The dimensions of the anchorage at the top and bottom of the studs in all tests are listed in Table 2. In all tests, the area of the top anchor plate was approximately 10 times the stud cross-sectional area; the small differences reported in Tables 1 and 2 are insignificant.

At service load level, the shear force V was kept constant and the unbalanced moment was cycled 10 times to simulate service load repetition. The shear force was then increased to the value V_{test} (Table 3) and kept constant while the applied moment was increased from zero until failure. The strains in the flexural reinforcement, the shear studs, and the bottom face of slab were measured by electrical strain gages. Dial gages were used to measure deflections on the top face of the slab. The top and bottom faces of the slab and the direction of application of V and M are defined in Fig. 1.

FAILURE MODES

A slab-column connection reinforced by well-anchored shear reinforcement may fail by punching shear or by flexure. Punching shear failure may be within the shear-reinforced zone or at a critical section outside this zone. The suggested code provisions in the following section are concerned with the punching shear failure.

The loads that cause flexure failure may be calculated by the yield line theory, which is adequately treated in the literature.[2,8,9] It is assumed herein that the slab has adequate flexural reinforcement to prevent this type of failure.

PROPOSED CODE PROVISIONS

Dilger and Ghali[4] suggested code provisions for the design of stud-shear reinforcement for slabs. These rules are presented in the following paragraphs in a clearer form compatible with the ACI Building Code 318-83.[1] The specified compressive strength of concrete f'_c must be in psi when used with the equations following. The corresponding equations, when the SI units are used, are given in Appendix A.

Suggested code clauses

Shear reinforcement consisting of vertical rods (studs) or the equivalent may be used in slabs when axial force or when axial force combined with moment is transferred from the slab to the column. Shear studs shall be mechanically anchored at each end by a plate or head having an area at least 10 times the cross-sectional area of the stem. The shear studs are to be arranged in rows parallel to the perimeter of the column section.

Design of critical slab sections perpendicular to the plane of slab shall be based on

$$v_u \leqslant \phi \, v_n \tag{1}$$

where v_u is the shear stress in the critical section caused by the transfer from the slab to the column of factored axial force or a factored axial force combined with moment. The value of v_u shall be computed in accordance with Sections 11.12.2.1 through 11.12.2.4.

The shear strength shall satisfy Eq. (1) at a critical section perpendicular to the plane of the slab at a distance $d/2$ from the column perimeter and at a critical section located so that its perimeter b_o is minimum but need not approach closer than $d/2$ to the outermost row of shear studs.

The shear strength at a critical section at distance $d/2$ from the outermost row of shear studs shall be computed by

$$v_n = 2\sqrt{f'_c}\left[1 + \frac{2(4 - \alpha)}{3\beta_c}\right) \tag{2}$$

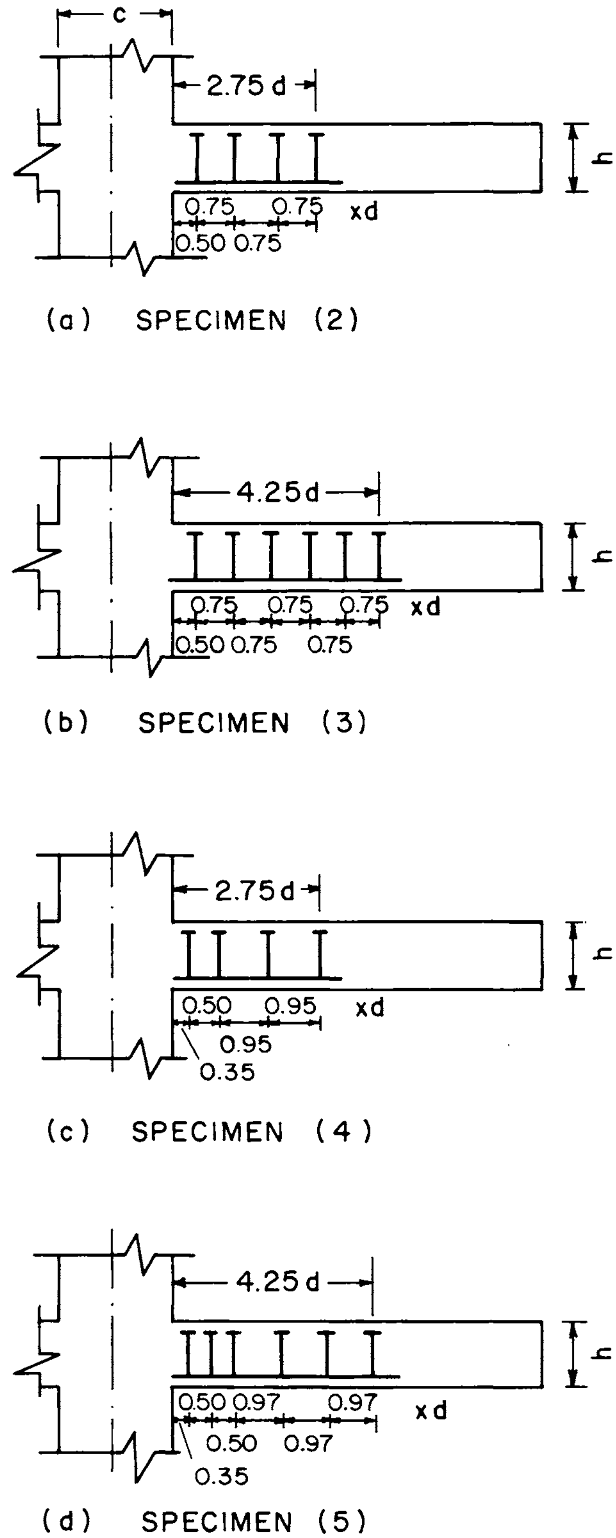

Fig. 3 — Stud row spacings

Table 1—Summary of test data

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8	Col. 9	Col. 10	Col. 11	Col. 12	Col. 13
Test	f'_c, MPa (psi)	f_y Compression reinforced MPa, ksi	f_y Flexural reinforced MPa, ksi	Steel ratio within $c + 3h$	f_{yv} MPa, ksi	Total number of studs	Stud diameter mm, in.	Ratio of area of top anchor head and stem section	Distance from column face to first row of studs	Distance from column face to last row of studs	Spacing between rows of studs	Number of stud rows
1	35 (5075)	375 (54.4)	452 (65.5)	1.1%								
2	33.7 (4887)	377 (54.7)	452 (65.5)	1.1%	460 (66.7)	32	12.7 (0.5)	11	0.5*d*	2.75*d*	0.75*d*	4
3	39 (5655)	377 (54.7)	452 (65.5)	1.23%	460 (66.7)	48	12.7 (0.5)	11	0.5*d*	4.25*d*	0.75*d*	6
4	40.8 (5916)	377 (54.7)	446 (64.7)	1.39%	500 (72.5)	32	9.5 (0.375)	10	0.35*d*	2.75*d*	0.5*d* up to distance of 0.85*d* then spacing is increased to 0.95*d*	4
5	45.6 (6612)	377 (54.7)	446 (64.7)	1.39%	500 (72.5)	48	9.5 (0.375)	10	0.35*d*	4.25*d*	0.5*d* up to distance of 1.35*d* then spacing is increased to 0.97*d*	6

Table 2—Dimensions of anchor heads and bottom strips

Test	Stud diameter D, mm (in.)	Top anchor head dimensions, thickness × width × length, divided by stud diameter D		Bottom anchor strip dimensions, thickness × width × length, divided by stud diameter D		Steel type and yield strength
		Line AB	Other lines	Line AB	Other lines	
2	12.7 (0.5)	0.5 × 3 × 3	0.5 × 3 × 3	0.5 × 2 × 29.7	0.5 × 2.5 × 29.7	Hot-rolled steel flats min. f_y = 276 MPa (40 ksi)
3	12.7 (0.5)	0.5 × 3 × 3	0.5 × 3 × 3	0.5 × 2 × 43.4	0.5 × 2.5 × 43.4	Hot-rolled steel flats min. f_y = 276 MPa (40 ksi)
4	9.5 (0.375)	0.66 × 2.66 × 2.93	0.5 × 2.66 × 2.93	0.66 × 2.66 × 39.6	0.5 × 2.66 × 39.6	Cold-finished steel flats min. f_y = 372 MPa (54 ksi)
5	9.5 (0.375)	0.66 × 2.66 × 2.93	0.5 × 2.66 × 2.93	0.66 × 2.66 × 57.9	0.5 × 2.66 × 57.9	Cold-finished steel flats min. f_y = 372 MPa (54 ksi)

Table 3—Summary of test results

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6	Col. 7	Col. 8
Test	Axial load V_{test}, KN (kip)	M_o, kN-m (kip-ft)	M_{u1}, kN-m (kip-ft)	M_{u2} kN-m (kip-ft)	M_{u3} kN-m (kip-ft)	M_{test} kN-m (kip-ft)	$\frac{M_{test}}{M_{u3}}$
1	150 (33.7)	57 (42.0)	57 (42.0)		57 (42.0)	130 (95.9)	2.28
2	150 (33.7)	55 (40.6)	186 (137.2)	160 (118.0)	160 (118.0)	162 (119.5)	1.01
3	300 (67.4)	16 (11.8)	162 (119.5)	128 (94.4)	128 (94.4)	142 (104.7)	1.11
4	300 (67.4)	19 (14.0)	123 (90.7)	133 (98.1)	123 (90.7)	150 (110.6)	1.22
5	450 (101.2)	0.0	80 (59.0)	100 (73.8)	80 (59.0)	105 (77.4)	1.31

but not less than $2\sqrt{f'_c}$; where α is the distance between the column face and the critical section divided by d, but α must not be smaller than 1. β_c is the larger of 2 and the ratio of the long side to the short side of the column cross section.

The distance s_o between the first row of shear studs and the column face shall not be smaller than $d/4$. The upper limits for s_o and for the spacing s between the rows shall be based on the value v_u at a critical section at $d/2$ from the column face

$$s_o \leqslant \frac{d}{2} \qquad \text{when}$$

$$s \leqslant \frac{3}{4} d \qquad 4\sqrt{f'_c} < \frac{v_u}{\phi} \leqslant 6\sqrt{f'_c} \tag{3}$$

$$s_o \leqslant 0.35d \qquad \text{when}$$

$$s \leqslant 0.5d \qquad 6\sqrt{f'_c} < \frac{v_u}{\phi} \leqslant 8\sqrt{f'_c} \tag{4}$$

The upper limit of s_o is intended to eliminate the possibility of shear failure between the column face and the innermost row of studs.

When stud shear reinforcement is provided, shear strength v_n shall not be taken greater than $8\sqrt{f'_c}$. The shear strength at a critical section within the shear-reinforced zone shall be computed by

$$v_n = v_c + v_s \tag{5}$$

where

$$v_c = 2\sqrt{f'_c}\left(1 + \frac{4 - \alpha}{3\beta_c}\right) \tag{6}$$

but not less than $2\sqrt{f'_c}$; and

$$v_s = \frac{A_v f_{yv}}{b_o s} \tag{7}$$

where A_v is the cross-sectional area of the shear studs in one row parallel to the perimeter of the column section; the spacing s is measured perpendicular to the column face.

Unless the spacing s between the rows of shear reinforcement is increased away from the column, no other section needs to be checked within the shear-reinforced zone. When the spacing s is increased away from the column, the increased distance s shall satisfy Eq. (3) or (4), based on the value of v_u at a critical section midway between the rows of studs where s is first changed, and Eq. (1) shall be satisfied at the same critical section.

TEST RESULTS

At failure, the measured values of the shear force v_{test} and the unbalanced moment M_{test} are given in Table 3. Because of M, the shear stress near one face of the column (Face AA′ in Fig. 2) is larger than shear stresses at the other faces. Shear failure occurred near Face AA′ in an inclined plane and was followed by punching of the column through the slab and splitting of the slab at the top flexural reinforcement layer, as shown in Fig. 4. In Test 2, the punching shear failure was accompanied by compression failure on the bottom face of the slab.

The maximum factored shear stress at Side AB of the critical section (Fig. B.1; see Appendix B) due to the combination of V_u and M_u is given by (Clause 11.12.2.4 of ACI 318-83 commentary[1]

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u C_{AB}}{J_c} \tag{8}$$

where

A_c = area of concrete of assumed critical section
J_c = property of assumed critical section analogous to polar moment of inertia
C_{AB} = distance between centroidal axis and Side AB of critical section (Fig. B.1)
V_u = factored shear force transferred between slab and column
M_u = factored unbalanced moment transferred between slab and column
γ_v = fraction of moment between slab and column that is considered transferred by eccentricity of the shear about the centroid of the assumed critical section

Equations that may be used in the calculations of the properties of the critical section — A_c, J_c, and C_{AB} — are given in Appendix B. The value M_o given in Table 3 represents the bending moment that when combined with V_{test} produces, at a critical section at $d/2$ from the column face, a maximum shear stress $v = 0.33 \sqrt{f'_c}$ (MPa) [$4\sqrt{f'_c}$ (psi)]. The value M_o represents the theoretical failure moment when no shear reinforcement is provided. The values M_{u1} and M_{u2} in the same table produce a maximum shear stress $v_u = v_n$ at critical sections outside and within the shear-reinforced zone, respectively. Here v_u is calculated by Eq. (8) and v_n by Eq. (2) or (5). The theoretical failure moment M_{u3} (Col. 6 of Table 3) is the smaller of M_{u1} and M_{u2}. The values of M_{test}/M_{u3} (Col. 8 of Table 3) are greater than one indicating the validity of the design equations to calculate a safe M_{u3}.

Table 4 gives a comparison between the allowable nominal shear stress and the experimental values at ultimate. The value v_{n1} calculated by Eq. (2) and listed in Col. 2 is the allowable stress at a critical section at $d/2$

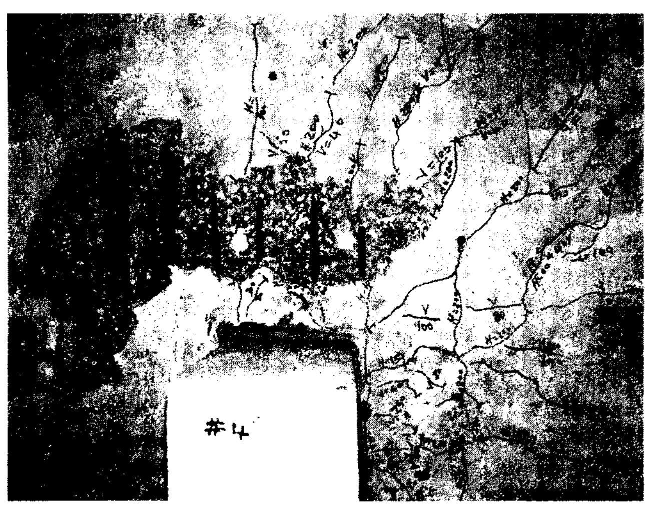

Fig. 4 — Top face of slab after shear failure (Test 4)

Table 4—Comparison between allowable nominal shear stresses and actual stresses at ultimate in terms of $\sqrt{f_c'}$

Col. 1	Col. 2	Col. 3	Col. 4	Col. 5
	Allowable stresses		Actual stresses	
Test	At $d/2$ outside shear-reinforced zone [Eq. (2)], v_{n1}	At section within shear-reinforced zone at $d/2$ from column face [Eq. (5)], v_{n2}	At $d/2$ outside shear-reinforced zone [Eq. (8)]	At $d/2$ from column face [Eq. (8)]
1	0.33 (4.0)		0.56 (6.8)	
2	0.21 (2.5)	0.67 (8.0)	0.19 (2.3)	0.68 (8.2)
3	0.17 (2.0)	0.67 (8.0)	0.16 (1.9)	0.71 (8.6)
4	0.21 (2.5)	0.67 (8.0)	0.23 (2.8)	0.72 (8.7)
5	0.17 (2.0)	0.67 (8.0)	0.18 (2.2)	0.69 (8.3)

All stresses in MPa; psi values in parentheses.

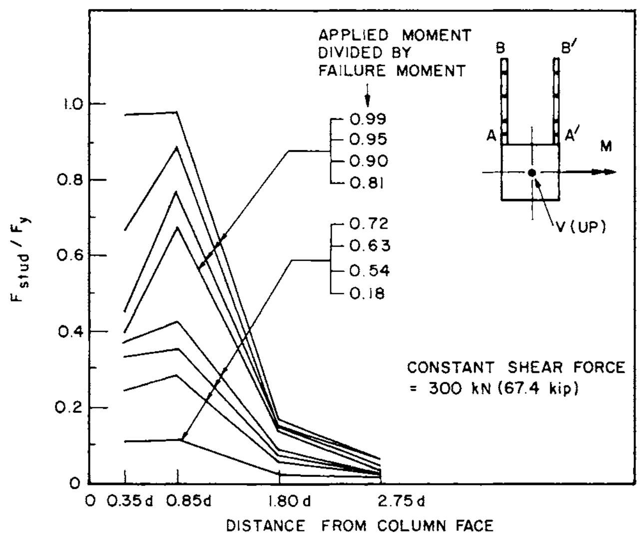

Fig. 5 — Force in studs situated on Line A'B' of Test 4

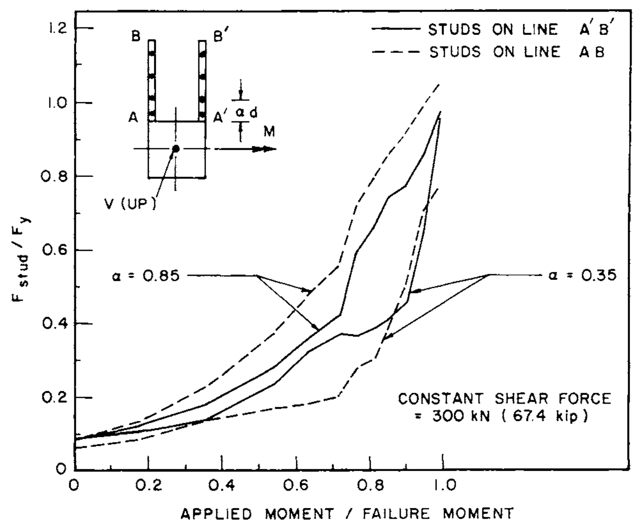

Fig. 6 — Force in studs versus applied moment on Lines AB and A'B' of Test 4

from the column face in Test 1, or from the outermost row of studs in other tests. The value v_{n2} in Col. 3 is the stress allowed at a critical section within the shear-reinforced zone at $d/2$ from column face; v_{n2} is the smaller of 0.67 $\sqrt{f_c'}$ (MPa) [$8\sqrt{f_c'}$ (psi)] and the value given by Eq. (5). The punching shear failure in Tests 2 to 5 occurred within the shear-reinforced zone at a maximum shear stress calculated by Eq. (8) and listed in Col. 5. All values in this column are greater than the limit 0.67 $\sqrt{f_c'}$ (MPa) [$8\sqrt{f_c'}$ (psi)], which verifies that the suggested limit for v_n is safe.

The suggested code clauses require that the zone reinforced for shear extends such that the value v_n calculated by Eq. (2) is not exceeded at $d/2$ outside the outermost row of studs. The fact that failure did not occur at this section in Tests 4 and 5, although the allowable v_n is exceeded, is an indication that the limit set by Eq. (2) for the stress resisted by concrete outside the shear-reinforced zone is safe.

Fig. 5 shows the forces in studs situated on Line A'B' of Specimen 4. At failure, the first two studs near the column face reached or became close to yield. Fig. 6 presents stud force versus applied moment for the first two studs from column face on lines AB and A'B' of Test 4. Similar results were obtained in Test 5. Fig. 5 and 6 indicate that top anchor plates with area equal to 10 times the stud cross-sectional area and thickness equal to 0.5 the stud diameter provide sufficient anchorage through the full range up to yielding of the studs. No significant difference in behavior was observed for the studs on Line AB, which had thicker anchor plates (0.66*D* instead of 0.5*D*).

Specimens 2 and 3 were overreinforced for shear; therefore, the studs in these tests did not reach yield.

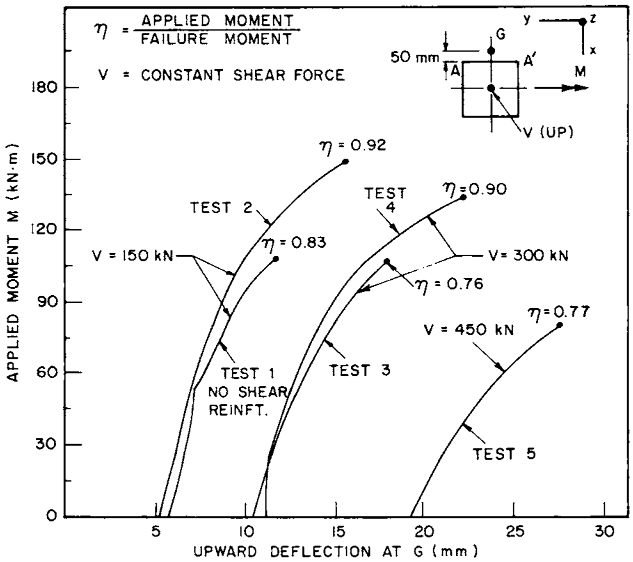

Fig. 7 — Deflection versus moment in Tests 1 to 5

Thus, the adequacy of a reduced width of the bottom strip ($2D$) used in these tests (Table 2) could not be checked. However, a bottom anchor strip with width and thickness equal to $2.5D$ and $0.5D$, respectively, should provide enough anchorage to develop yield in studs.

It is worth mentioning that Andrä's[5] tests showed that the yield strength of the studs could be developed using cold-formed anchor heads of area 6.25 times the area of the stem. In his tests, the yield stress of the studs was 260 MPa (37.7 ksi) and the concrete strength was 38 to 47 MPa (5510 to 6815 psi). Studs with these anchor heads and yield strength, welded to steel strips known as stud rails or shear combs, are commercially available in Europe.

Fig. 7 shows deflection at Point G versus applied moment. The deflection measurements were stopped when the applied moment reached a fraction η of the ultimate moment; the value η varied in the tests as indicated in the figure. The deflection readings at $M = 0$ is due to the applied shear force V. Comparison of the graph for Test 1 with the other tests indicates that provision of shear reinforcement increases the values of V and/or M at ultimate as well as the deflection. The large deflections exhibited at ultimate in Tests 3 to 5 were accompanied by yielding of the flexural reinforcement running in the x-direction within a column strip of width = $c + 3h$. This yielding enhanced a ductile failure mode.

NUMERICAL EXAMPLE

The design of shear studs is required at an interior column of a flat plate (Fig. 8) with the following data:

Column size c = 10 x 10 in.^{2} (250 x 250 mm^2); slab thickness = 6.75 in. (171 mm); concrete cover = 0.75 in. (19 mm); f_c' = 4350 psi (30.0 MPa); yield strength of studs f_{yv} = 60 ksi (400 MPa); flexural reinforcement diameter = 0.625 in. (16 mm). The factored forces transferred from the column to the slab are: V_u = 65 kip (290 kN) and M_u = 80 ft-kip (108 kN-m).

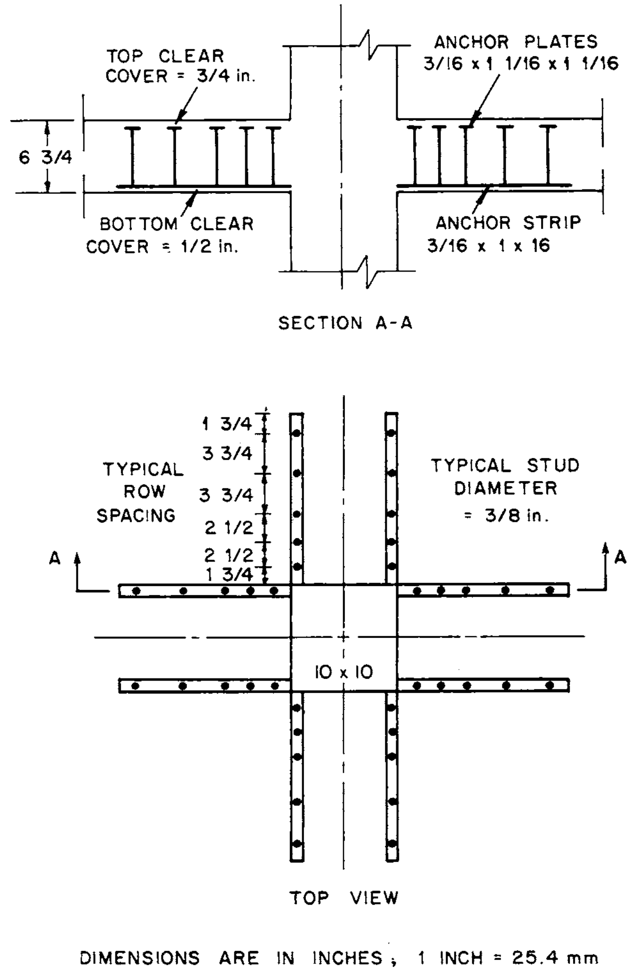

Fig. 8 — Stud arrangement in numerical example

The effective depth of slab

$$d = 6.75 - 0.75 - 0.625 = 5.375 \text{ in. } (136.5 \text{ mm})$$

Properties of a critical section at $d/2$ from column face [Eq. (B.1) to (B.4) and ignoring the last term of Eq. (B.3); see Appendix B]

$$b_o = 4(10 + 5.375) = 61.5 \text{ in. } (1560 \text{ mm})$$

$$A_c = 5.375\,(61.5) = 331 \text{ in.}^2$$

$$J_c = \tfrac{2}{3}\,(5.375)\,(10 + 5.375)^3 = 13{,}000 \text{ in.}^4\ (5400 \times 10^6 \text{mm}^4)$$

$$C_{AB} = \frac{10 + 5.375}{2} = 7.7 \text{ in. } (200 \text{ mm})$$

Maximum shear stress at this section due to the factored forces [Eq. (8)]

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_v M_u C_{AB}}{J_c}$$

$$v_u = \frac{65{,}000}{331} + \frac{0.4\,(80{,}000 \times 12)\,(7.7)}{13{,}000}$$
$$= 424 \text{ psi } (2.92 \text{ MPa})$$

$$\frac{v_u}{\phi} = \frac{424}{0.85} = 498 \text{ psi } (3.44 \text{ MPa})$$
$$= 7.6\sqrt{f_c'}$$

The nominal shear stress that can be resisted without shear reinforcement at the critical section considered [Eq. (2)]

$$v_n = 4\sqrt{f_c'} = 4\sqrt{4350} = 264 \text{ psi } (1.82 \text{ MPa})$$

The quantity v_u/ϕ is greater than v_n indicating that shear reinforcement is required; the same quantity is also smaller than the upper limit $v_n = 8\sqrt{f_c'}$, which means that the slab depth d is adequate. Also, the value of v_u/ϕ sets the following limits on stud spacings [Eq. (4)]

$$s_o \leqslant 0.35d = 1.9 \text{ in.}; \; s \leqslant \frac{d}{2} = 2.7 \text{ in.}$$

At a critical section at $d/2$ from column face, the shear stress resisted by concrete in presence of shear reinforcement [Eq. (6)]

$$v_c = 3\sqrt{f_c'} = 198 \text{ psi } (1.37 \text{ MPa})$$

Use of Eq. (1), (5), and (7) gives

$$v_s \geqslant \frac{v_u}{\phi} - v_c = 498 - 198 = 300 \text{ psi } (2.07 \text{ MPa})$$

$$\frac{A_v}{s} = \frac{v_s\, b_o}{f_{yv}}$$

thus

$$\frac{A_v}{s} \geqslant \frac{300\,(61.5)}{60{,}000} = 0.308 \text{ in. } (7.82 \text{ mm})$$

Choose eight studs of diameter ⅜ in. (9.5 mm) per row and the spacing between rows $s = 2\frac{1}{2}$ in. (64 mm)

$$\frac{A_v}{s} = \frac{8\,(0.11)}{2.5} = 0.352 \text{ in.}$$

This value is greater than 0.308 indicating that the choice of studs and their spacing is adequate. It is necessary to find the size of the shear-reinforced zone such that Eq. (1) is satisfied at a critical section at $d/2$ from the outermost row of studs. Try six equally spaced rows of studs. The distance from the column face to a critical section at $d/2$ from the outermost row is

$$\alpha d = s_o + 5s + \frac{d}{2}$$
$$= 1.75 + 5(2.5) + \frac{5.375}{2} = 16.9 \text{ in.}$$
$$\alpha = \frac{16.9}{5.375} = 3.15$$

Properties of this critical section [Eq. (B.5) to (B.12)]

$$\ell_1 = 10 + 0.414\,(5.375) = 12.2 \text{ in.}$$
$$\ell_2 = 10 + 2(3.15)\,(5.375) = 43.9 \text{ in.}$$
$$b_o = 4.(12.2) + 2\sqrt{2}(43.9 - 12.2) = 138.4 \text{ in.}$$
$$A_c = 5.375\,(138.4) = 744 \text{ in.}^2\ (480 \times 10^3 \text{ mm}^2)$$

$$J_c = 5.375\left\{\frac{(12.2)^3}{6} + \frac{12.2\,(43.9)^2}{2} + \frac{\sqrt{2}}{8}\right.$$
$$\left.(43.9 - 12.2)\left[(43.9 + 12.2)^2 + \frac{(43.9 - 12.2)^2}{3}\right]\right\}$$
$$= 170{,}000 \text{ in.}^4\ (71 \times 10^9 \text{ mm}^4)$$

$$C_{AB} = \frac{43.9}{2} = 22.0 \text{ in. } (560 \text{ mm})$$

The maximum shear stress in the section [Eq. (8)]

$$v_u = \frac{65{,}000}{744} + \frac{0.4\,(80{,}000 \times 12)(22.0)}{170{,}000} = 137 \text{ psi}$$

$$\frac{v_u}{\phi} = \frac{137}{0.85} = 161 \text{ psi } (1.11 \text{ MPa})$$

Allowable shear stress at the section considered [Eq. (2)]

$$v_n = 2\sqrt{4350}\left[1 + \frac{2(4 - 3.15)}{3(2.0)}\right] = 169 \text{ psi } (1.17 \text{ MPa})$$

The quantity v_u/ϕ does not exceed v_n, which indicates a satisfactory design. Since s is kept constant, no other critical section needs to be checked and the design may be terminated here. However, as alternate design, reduce the number of rows to five (instead of six) without changing the position of the outermost row, but increase s for the outer rows as shown in Fig. 8. This requires that Eq. (1) and (3) [or (4)] be satisfied at a critical section midway between the rows of studs where s is first increased; that is, between the third and fourth rows in Fig. 8. The distance between this section and the column face is

$$\alpha d = 1.75 + 2 \times 2.5 + \frac{3.75}{2} = 8.6 \text{ in.}$$

or $\alpha = 1.60$. At this section, $v_u/\phi = 283$ psi (1.95 MPa) $= 4.3\sqrt{f_c'}$ and $b_o = 91$ in.

The shear strength at this section is calculated by Eq. (5) through (7)

$$v_c = 2\sqrt{4350}\left(1 + \frac{4 - 1.6}{3 \times 2.0}\right) = 185 \text{ psi}$$

$$v_s = \frac{8(0.11)(60{,}000)}{91(3.75)} = 155 \text{ psi}$$

$$v_n = 185 + 155 = 340 \text{ psi (2.34 MPa)}$$

The quantity v_u/ϕ does not exceed v_n, which means that Eq. (1) is satisfied and the increased stud spacing s = 3.75 in. = $0.7d$ satisfies Eq. (3). The more economical alternate design may therefore be adopted, as detailed in Fig. 8.

CONCLUSIONS

The test results presented herein prove the effectiveness of well-anchored stud-shear reinforcement in increasing the shear strength and ductility of slab-column connections subjected to shear force and unbalanced moment. The tests also verify the validity of the design code provisions suggested by Dilger and Ghali[4] when shear-moment transfer takes place between the slab and the columns. Minimum requirements for the dimensions of top anchor heads and bottom anchor strips are suggested.

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NOTATION

- A_c = area of concrete of assumed critical section
- A_v = cross-sectional area of one row of shear studs distributed over a perimeter b_o of a critical section
- b_o = perimeter of critical section
- C_{AB} = distance between centroidal axis and Part AB of critical section perimeter
- D = stud diameter
- d = effective depth of slab
- f'_c = specified compressive strength of concrete
- f_y = specified yield strength of steel
- f_{yv} = specified yield strength of shear studs
- F_{stud}, F_y = the force measured in a stud and the force that produces yield, respectively
- J_c = property of assumed critical section analogous to polar moment of inertia
- M_u = factored unbalanced moment transferred between slab and column
- s = spacing between stud rows
- s_o = spacing between first row of studs and column face
- V_u = factored shear force
- v_c = nominal shear strength provided by concrete in presence of shear studs
- v_n = nominal shear strength at a critical section
- v_s = nominal shear strength provided by studs
- v_u = maximum shear stress due to factored forces
- α = distance between column face and a critical section divided by d. But when the distance between the column face and the critical section is smaller than d, the value of α in Eq. (2) and (6) must be equal to 1.
- β_c = ratio of long to short side of column cross section. But when this ratio is smaller than 2, the value of β_c in Eq. (2) and (6) must be equal to 2.
- γ_v = fraction of moment between slab and column that is considered transferred by eccentricity of the shear about the centroid of the assumed critical section
- ϕ = strength reduction factor

CONVERSION FACTORS

1 mm	=	0.0394 in.
1 m	=	3.281 ft
1 kN	=	0.2248 kip
1 kN-m	=	0.7376 ft-kip
1 MPa	=	145 psi
$\sqrt{f'_c}$ (MPa)	=	12 $\sqrt{f'_c}$ (psi)

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APPENDIX A — EQUATIONS IN SI UNITS

The following equations are to be used in lieu of Eq. (1) to (6) when the SI units are used and the specified compressive strength of concrete f'_c is expressed in MPa

$$v_u \leqslant \phi\, v_n \tag{A1}$$

$$v_n = 0.17\sqrt{f'_c}\left(1 + \frac{2(4 - 2)}{3\beta_c}\right) \tag{A2}$$

but not less than $0.17\sqrt{f'_c}$

$$\left.\begin{aligned} s_o &\leqslant \frac{d}{2} \\ s &\leqslant \frac{3}{4}d \end{aligned}\right\} \text{ when } 0.33\sqrt{f'_c} < \frac{v_u}{\phi} \leqslant 0.5\sqrt{f'_c} \tag{A3}$$

$$\left.\begin{aligned} s_o &\leqslant 0.35d \\ s &\leqslant 0.5d \end{aligned}\right\} \text{ when } 0.5\sqrt{f'_c} < \frac{v_u}{\phi} \leqslant 0.67\sqrt{f'_c} \tag{A4}$$

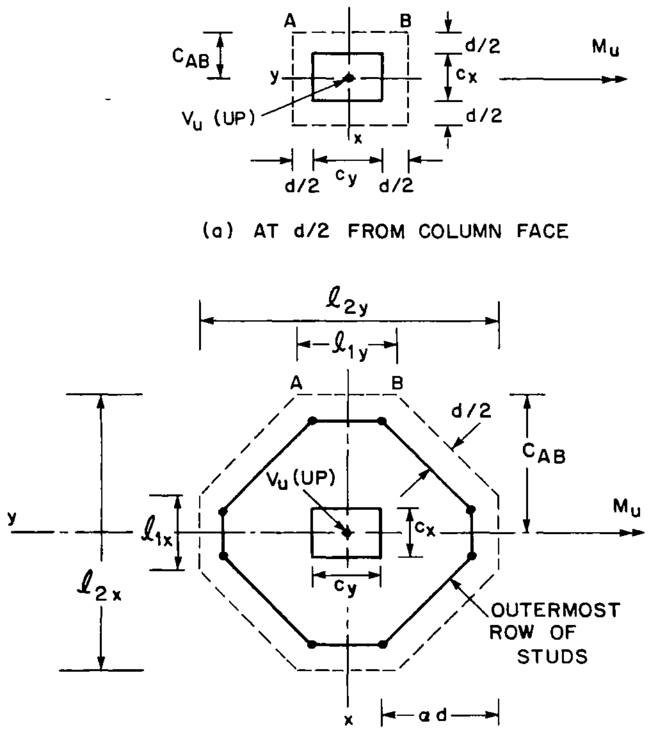
When stud-shear reinforcement is provided, shear strength

$$v_n = v_c + v_s \tag{A5}$$

but not greater than $0.67\sqrt{f'_c}$

$$v_c = 0.17\sqrt{f'_c}\left(1 + \frac{4 - \alpha}{3\beta_c}\right) \tag{A6}$$

but not less than $0.17\sqrt{f'_c}$



(a) AT d/2 FROM COLUMN FACE

(b) AT d/2 FROM THE OUTERMOST ROW OF SHEAR STUDS

Fig. B1 — Critical sections for shear in slab in the vicinity of a rectangular column

APPENDIX B — PROPERTIES OF SECTIONS FOR USE IN CALCULATION OF MAXIMUM SHEAR STRESS

Fig. B.1(a) shows the top view of a critical section at $d/2$ from the face of a rectangular column c_x by c_y. Due to forces V_u and M_u transferred from the column to the slab, the maximum factored shear stress on Side AB of the critical section may be calculated by Eq. (8), which requires the following section properties

$$b_o = 2(c_x + c_y) + 4d \tag{B1}$$

$$A_c = d\, b_o \tag{B2}$$

$$J_c = d\left[\frac{(c_x + d)^3}{6} + \frac{(c_y + d)\,(c_x + d)^2}{2}\right] + \frac{(c_x + d)d^3}{6} \tag{B3}$$

$$C_{AB} = \frac{c_x + d}{2} \tag{B4}$$

Fig. B .1(b) shows a critical section at a distance αd from the faces of a rectangular column. The properties of this section are

$$b_o = 2\,(\ell_{1x} + \ell_{1y}) + 2\sqrt{(\ell_{2x} - \ell_{1x})^2 + (\ell_{2y} - \ell_{1y})^2} \tag{B5}$$

$$A_c = d\, b_o \tag{B6}$$

$$J_c = d\left\{\frac{\ell_{1x}^3}{6} + \frac{\ell_{1y}\,\ell_{2x}^2}{2} + \frac{1}{8}\sqrt{(\ell_{2x} - \ell_{1x})^2 + (\ell_{2y} - \ell_{1y})^2}\left[\left(\ell_{2x} + \ell_{1x}\right)^2 + \frac{1}{3}\left(\ell_{2x} - \ell_{1x}\right)^2\right]\right\} + \frac{\ell_{2y}d^3}{6} \tag{B7}$$

$$C_{AB} = \frac{\ell_{2x}}{2} \tag{B8}$$

where

$$\ell_{1x} = c_x + 0.414d \tag{B9}$$

$$\ell_{1y} = c_y + 0.414d \tag{B10}$$

$$\ell_{2x} = \ell_{1x} + 2\,\alpha\, d \tag{B11}$$

$$\ell_{2y} = \ell_{1y} + 2\,\alpha\, d \tag{B12}$$

The last term is each of Eq. (B3) and (B7) is small and may be ignored; thus, the symbol J_c will simply represent the second moment of area of the critical section about the centroidal axis y.