Curved-Bar Nodes

A detailing tool for strut-and-tie models

BY GARY J. KLEIN

Since reinforcing bars were first used in concrete, they have been designed with bends for anchorage and transfer of forces in connection regions. Examples include beam-column and wall-slab junctions, where bent bars at the outside of a frame corner resist closing moments.

The forces in these corners can be modeled using the strut-and-tie model (STM) shown in Fig. 1. In this example, the curved region of the bar is modeled as a series of segments with tension forces opposed by a series of fan-shaped compression struts. In this article, I'll explain how this curved region can be modeled as a curved-bar node. In the general case, a curved-bar node is the bend region of a continuous reinforcing bar (or bars) where two tension ties are in equilibrium with a compression strut in an STM. Specific recommendations for design and detailing using STMs with curved-bar nodes in frame corners and dapped-end beams are offered.

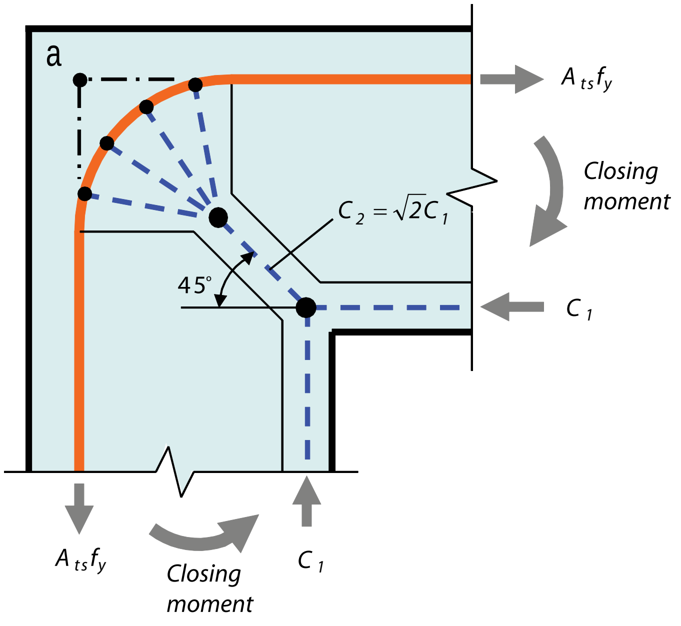

Fig. 1: Strut-and-tie model of forces due to closing moment at a frame corner

Worked example designs for strut-and-tie models using curved-bar nodes are available with the online version of this article at www.concreteinternational.com

STRUT-AND-TIE MODEL

The tie forces at a curved-bar node must be equilibrated by one or more struts. In most cases, the intersection of two ties and a strut at the curved-bar node form a compression-tension-tension (C-T-T) node. Several additional examples of curved-bar nodes in concrete connection regions, or so-called D-regions, are shown in Fig. 2. To simplify analyses of C-T-T nodes such as shown in Fig. 1, 2(a), or 2(b), the curved region can be modeled as a single node at the intersection of the centerlines of the straight ties (Point a in Fig. 1).

As shown by the example C-T-T node in Fig. 3, nodal zones are generally too small to allow development of tie forces through bond alone. If conservative design guidelines for the use of curved-bar nodes in D-regions are developed, curved-bar nodes can provide a cost-effective, simpler alternative to separate mechanical anchorages. Although Appendix A of ACI 318-08[1] does not yet recognize curved-bar nodes, its provisions can be used to develop design recommendations.

COMPRESSIVE STRESS AT CURVED-BAR NODES

In a typical case, a strut bisects the angle formed by the ties extending from the curved-bar node. For 90-degree corners with equal tie forces, the strut angle is 45 degrees, and, using a pressure vessel analogy, the compressive stress acting in the curved region of the bar depends only on the radius of the bend and the tensile force in the bar. It follows that no bond stresses are required within the curved region itself—only a uniform, radial compression stress is required to maintain equilibrium within the region. By placing conservative limits on the compressive stresses acting on the bar, an equilibrium (lower bound) model for the node can be established.

Compressive stress f_{cu} at a curved-bar node is limited by the yield strength of the tie reinforcement. Thus, the maximum value is given by

$$f_{cu} = \frac{A_{ts} f_y}{r_b b} \qquad (1)$$

where A_{ts} is the area of nonprestressed tie reinforcement, f_y is the specified yield strength of the tie reinforcement, r_b is the inside radius of the reinforcing bar bend, and b is the width of the strut transverse to the plane of the STM.

The minimum bend radius in terms of allowable nodal stress, f_{ce}, can be derived by reorganizing Eq. (1) as follows

$$r_b = \frac{A_{ts} f_y}{b f_{ce}} \qquad (2)$$

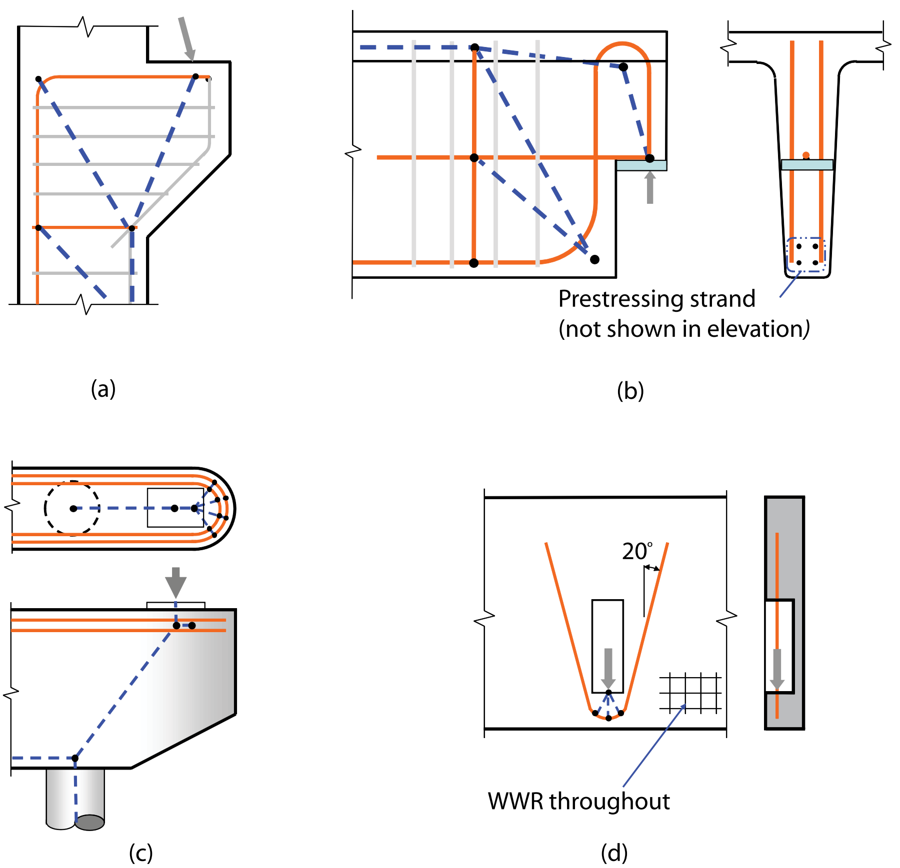

Fig. 2: Strut-and-tie models with curved-bar nodes: (a) column corbel; (b) dapped-end beam; (c) hammer-head bridge pier with rounded end; and (d) pocket in a spandrel beam

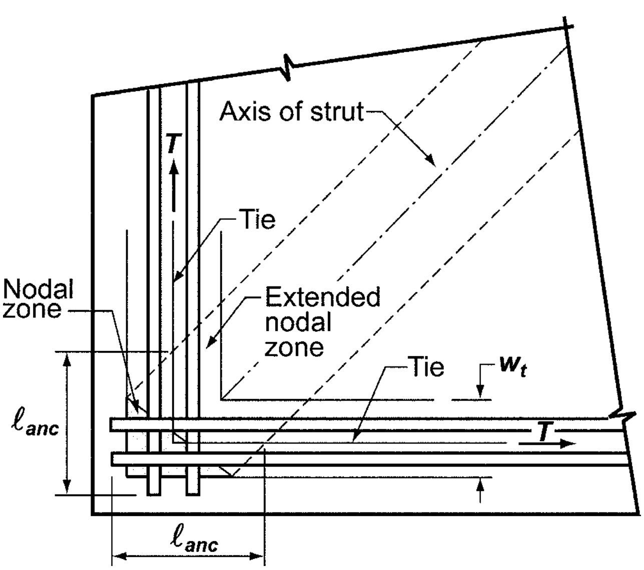

Fig. 3: Development of the tie reinforcement in nodal zones cannot rely on bond alone (from Reference 1)

NODE STRENGTH

ACI 318 requirements for C-T-T nodes

ACI 318 limits the compressive stress at nodes to $0.85\beta_n f'_c$. The β_n values reflect the degree of disruption of the nodal zone due to the incompatibility of tension strains in the ties and compression strains in the struts. For C-T-T nodes, where there is tension strain in two directions, β_n is 0.6, which is less than the 0.8 value for compression-compression-tension (C-C-T) nodes. Thus, the allowable compressive stress for a typical curved-bar node (a C-T-T node) is 0.85 × 0.6 = 51% of the specified compressive strength, f'_c.

Frame corners tests

To confirm that the ACI 318 limits are appropriate, the performance of frame-corner connections with curved-bar nodes was reviewed. Several researchers[2-6] have studied the flexural strength of reinforced concrete corners. In most cases, conventional corner reinforcement (Fig. 1) is sufficient to develop the full flexural strength of the adjacent members. One study[6] of the flexural strength of corners reinforced with two No. 6 (No. 19) bars spaced

3 in. (75 mm) apart in 6 in. (150 mm) square members, however, reported that all of the specimens (except for one specimen with a welded diagonal stiffener) failed at a moment below the nominal flexural strength. The report noted that, "It is likely that the failures were caused by bearing failure of the concrete in the diagonal compression zone between the bends in the tensile and compressive reinforcement."[6] In these tests, the calculated compressive stress at ultimate load given by Eq. (1) varied from 1.08 to 1.60 times the measured concrete compressive strength. Thus, the limiting strength given in ACI 318 of $0.51f_c'$ appears safe, if not overly conservative. Using the ACI 318 strength limit for C-T-T nodes (β_n = 0.6), the minimum radius becomes

$$r_b \geq \frac{2A_{ts}f_y}{bf_c'} \tag{3a}$$

DAPPED-END BEAM TESTS

Tests[7] have also been performed on a series of dapped-end tee beams to investigate several reinforcement schemes for the end region. A typical reinforcement scheme (Specimen 1B) is shown in Fig. 4(a). Specimen 1B failed at a load of 27 kips (120 kN) when a diagonal tension crack developed and extended through the lower corner of the full section. Although the end region was not designed using an STM, the cracking pattern clearly indicated a diagonal strut extending upward from the lower corner of the full section. The lower end of strut was equilibrated by the inclined and horizontal extensions of the continuous reinforcing bars [1 No. 4 and 1 No. 3 (1 No. 13 and 1 No. 10)] at the lower corner of the full section. Thus, the bend region is a curved-bar node. Shear reinforcement was not used inside the end region, so there were no ties at the upper end of the strut. The expected shear resistance of the full section, however, was developed, and the overall performance was considered satisfactory. Therefore, although the diagonal crack extended through the curved-bar node, specimens exhibiting this behavior should be considered to have failed in shear.

An alternate scheme (Specimen 2B), in which a single inclined hanger bar was used, is shown in Fig. 4(b). This specimen did not exhibit the typical failure behavior exhibited by Specimen 1B. At a load of only 20 kips (89 kN), the web split at the lower corner of the full section. This failure at the curved-bar node is attributable to the difference in the reinforcement details, as the single No. 5 (No. 16) hanger bar had only 5/8 in. (16 mm) clear side cover. The eccentricity can be accounted for by considering an effective width for the curved-bar node equal to twice the distance from the center of the bar to the nearest concrete surface—about 2 in. (50 mm) in this case. Based on strain measurements, the stress in the No. 5 (No. 16) bar was about 30 ksi (200 MPa) just before failure. The corresponding compressive stress over the reduced effective web width was 2700 psi (1806 MPa), or about 48% of the measured concrete compressive strength. Thus, the limiting strength given in ACI 318 of $0.51f_c'$ may be unconservative for bars with shallow cover.

Two other tests[8] on dapped-end beams with curved-bar nodes contained a reinforcement scheme similar to that shown in Fig. 2(b). Before reaching ultimate load, a diagonal crack extended downward to the lower corner of the beam. The calculated ultimate compressive stress at the curved-bar node was 1.50 and 1.89 times the measured concrete compressive strength.

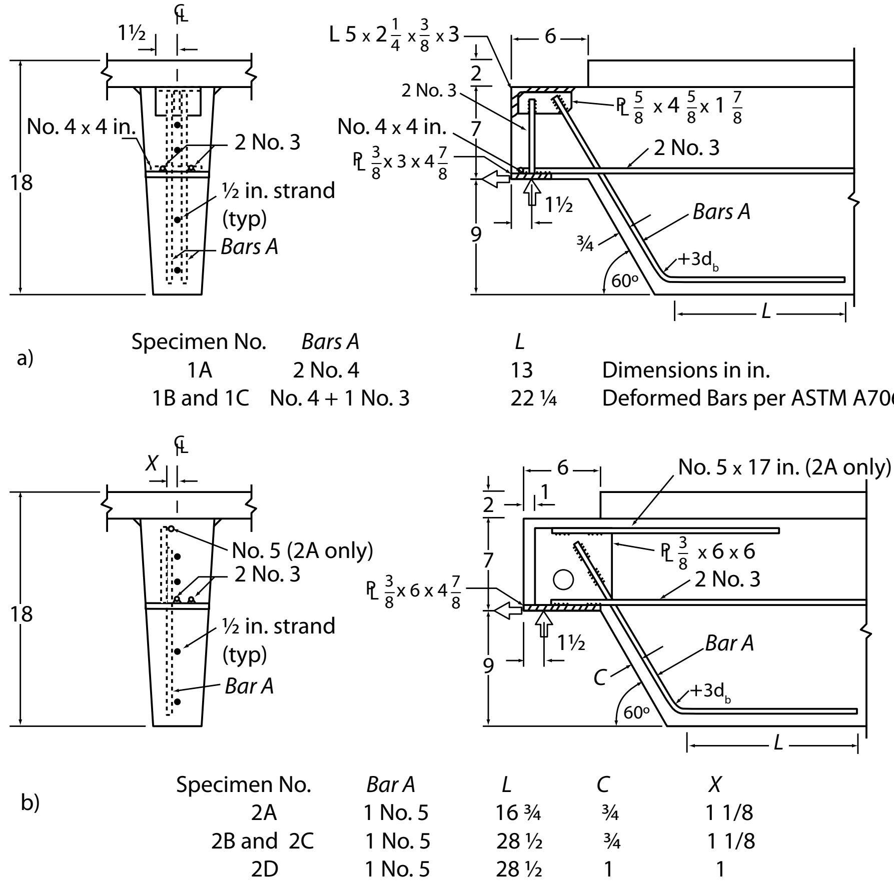

Fig. 4: Reinforcement for double-tee specimens with dapped ends (based on Reference 7) (note: 1 in. = 25.4 mm)

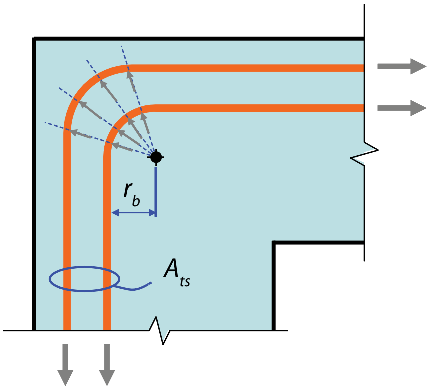

Fig. 5: Frame corner with two layers of reinforcement

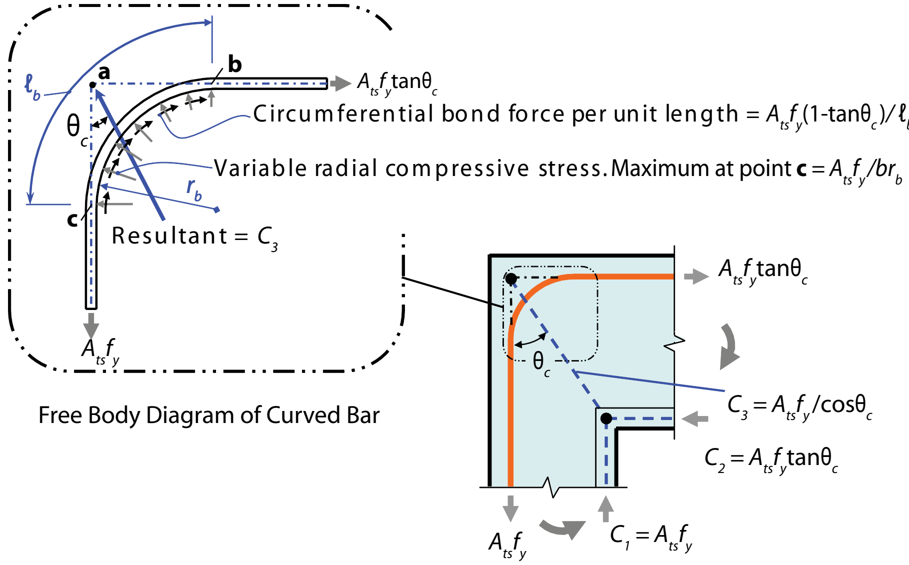

Fig. 6: Unequal tie forces in a frame corner result in bond stress along the circumference of the bend. The radius of the bend must produce a bend length l_c adequate to develop the required bond force

C-C-T curved bar nodes

A C-C-T node is formed by a curved-bar node with a 180-degree bend. The upper right nodes shown in Fig. 2(b) and (c) are examples of such C-C-T nodes. Unlike the general case, where ties exiting the curved bar node create disruptive tensile strains in two directions, parallel ties provide confinement. As such, Eq. (3a) is too conservative for curved-bar nodes with 180-degree bends. Using the ACI 318 strength limit for C-C-T nodes (β_n = 0.8) the minimum radius from Eq. (2) becomes

$$r_b \geq \frac{1.5A_{ts}f_y}{bf_c'} \quad (3b)$$

where A_{ts} is the area of nonprestressed tie reinforcement at one end of the 180-degree bend.

Multi-layer curved-bar nodes

Where more than one layer of reinforcement is used in the plane of the STM, nodal zone stresses are increased in proportion to the number of layers. Figure 5 illustrates the use of two layers of reinforcement at a frame corner. In these cases, Eq. (3a) or (3b) may be used provided A_{ts} is taken as the area of tie reinforcement in all layers, and r_b is taken as the bend radius at the inside layer.

BOND STRESS AT CURVED-BAR NODES

In some cases, for example a junction between a wall and a slab with different effective depths (Fig. 6), the tie forces are not equal. The compressive stress on the inside radius of the bar must therefore vary, and circumferential bond stress develops along the bar.[9]

The maximum nodal compressive stress occurs at the point of tangency for the tie carrying the greater force. Assuming this tie yields, the compressive stress is given by Eq. (1). At 90-degree corners, the resultant force in the strut is $A_{ts}f_y/\cos\theta_c$, where θ_c is the smaller of the two angles between the strut (or the resultant of two or more struts) and ties extending from a curved-bar node. The bond force that must be developed along bend length l_b is $A_{ts}f_y(1 - \tan\theta_c)$. In view of the high contact stress on the inside of the bend, it would seem that the development length l_d for straight bars can be conservatively applied to the bend region at a curved-bar node. Accordingly, at 90-degree corners, the ratio of curve length to development length l_d should be at least $(1 - \tan\theta_c)$; that is

$$l_b \geq l_d(1 - \tan\theta_c) \quad (4a)$$

In terms of r_b, Eq. (4a) becomes

$$r_b \geq \frac{2l_d(1 - \tan\theta_c)}{\pi} - \frac{d_b}{2} \quad (4b)$$

EFFECTIVE WIDTH OF CURVED-BAR NODES

The effective width of a node is usually taken as the width of the member transverse to the plane of the STM. This assumption is usually valid for curved-bar nodes, but there are three potential concerns: 1) excessive compressive stress under the bend region of the bar; 2) transverse eccentricity of the bars relative to the member; and 3) side splitting of bars with shallow side cover.

The minimum bend radii provided in ACI 318 are sufficient to avoid crushing under the bend region of the bar. Assuming the minimum bend radius provisions are met, this condition need not be checked, even when large, widely-spaced bars are used.

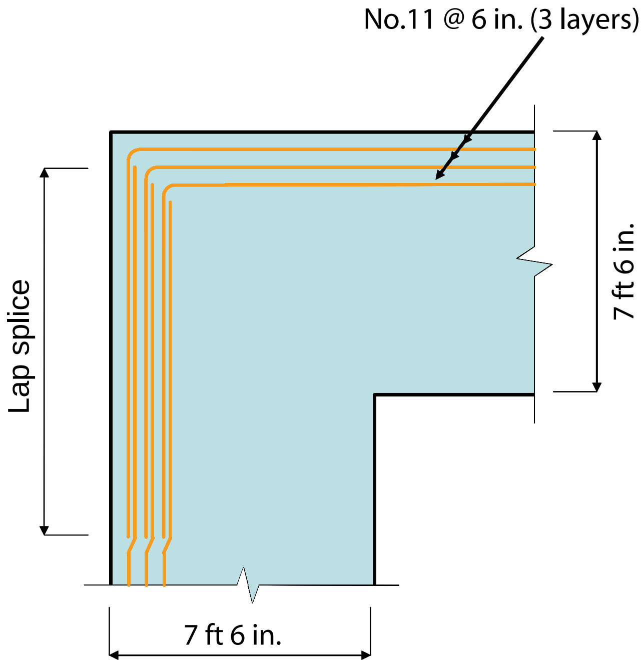

Fig. 7: Primary flexural reinforcement adapted from the design of a wall/roof slab junction of a tunnel (note: 1 in. = 25.4 mm; 1 ft = 304.8 mm)

The second concern, transverse eccentricity, can be addressed by limiting the effective width of the node to twice the distance from the centroid of the tie to the nearest face of the member. In this way, the struts and ties are coplanar.

Side splitting is the most difficult concern. In the dapped-end beam tests[7] previously discussed, the web split at a No. 5 (No. 16) bar with 5/8 in. (16 mm) cover. Others[5] have observed side-splitting failures at frame corners with 0.87 in. (22 mm) diameter bars and 1.3 in. (33 mm) clear side cover. The commentary to the development length equations in Section 12.2.2 of ACI 318 states that when the cover to the center of the bar is greater than 2.5 bar diameters, pullout failures are expected rather than a splitting failure. Although more research is required to definitively assess the minimum side cover requirements for curved-bar nodes, a clear side cover of at least 2 bar diameters appears conservative. Where this side cover cannot be provided, the bend radius required by Eq. (3a) or (3b) should be increased by multiplying the calculated bend radius by a factor of 2 bar diameters divided by the specified clear cover.

DESIGN OF FRAME CORNERS

Traditionally, frame corners subject to closing moments are designed based on the flexural strength at the face of the support. As required by Section 12.12 of ACI 318, the hooked-bar development length is checked at the face of the support. Design using an STM with a curved-bar node has two distinct advantages over this traditional approach. First, by assuring nodal stresses are within ACI 318 limits, the possibility of a diagonal flexural failure at less than the flexural strength of the connected members is avoided. Second, the design is not limited by the available development length to the face of the support.

In most cases, standard bend radii will exceed the minimum required by Eq. (3a). There are, however, exceptions such as very deep members with multiple layers of reinforcement. For example, the wall-to-roof junction for a tunnel shown in Fig. 7 has flexural reinforcement consisting of three layers of No. 11 (No. 36) bars spaced 6 in. (150 mm) on center. In this case, all three layers contribute to the force in the diagonal strut. According to Eq. (3a), the bend radius should be at least 23 in. (580 mm), based on an f_c' of 4000 psi (28 MPa). The standard bend radius is only 6 in. (150 mm). Thus, the nodal stress would greatly exceed the ACI 318 limit. The ACI 318 limit on C-T-T nodal stress, however, is very conservative relative to actual performance of frame corners in tests, and the as-built detail would be expected to perform satisfactorily under normal loading. Nonetheless, the ability of the as-built corner to develop the nominal moment capacity is questionable. Note that the 23 in. (580 mm) radius required by Eq. (3a) could have easily been accommodated within the dimensions of the corner, enhancing the structural integrity of the wall-to-slab connection.

If standard hooks at the corners had been used to develop the bars, construction would have been significantly simplified. The effectiveness of the hooks in developing the nominal moment capacity, however, is highly questionable. In addition to the concerns noted with respect to Fig. 7, the straight extensions on the end of the standard hooks would likely not be long enough to develop the needed tensile force if a crack parallel to the diagonal strut extends to the outside corner.

DESIGN OF DAPPED-END BEAMS

Designs using curved-bar nodes can simplify the detailing of dapped-end connections. Designs that do not employ curved-bar nodes require separately anchored ties at the lower corner of the full section. A continuous bar or series of parallel bars through the lower corner region is a much simpler and more economical alternative. For members with narrow webs, such as precast double tees, there may not be enough room to provide separately anchored ties. In such locations, a continuous bent bar may be a better choice.

As shown in Fig. 2(b), a continuous bent bar (or pair of bars) can be used for the C-C-T node above the reaction point, as well as the C-T-T node at the lower corner of the full section. The curved-bar C-C-T node avoids congestion and is more economical than welded embedments, such as those shown in Fig. 4.

CLOSING REMARKS

The new tool presented in this article for designing connections in reinforced concrete structures provides design guidelines where none currently exist in U.S. practice. The design guidelines and equations for curved-bar nodes are not intended as accurate predictors of capacity; rather, they provide conservative designs consistent with current limits for nodal stress in STMs. Available test data, although very limited, indicate that the design recommendations are conservative, but further research and testing that examine specific performance parameters such as bend radius and clear cover would allow even more diverse and reliable use of STMs with curved-bar nodes. Nonetheless, if used in accordance with the conservative design guidelines provided herein, curved-bar nodes are a powerful and economical tool for detailing connection regions in reinforced concrete.

Design Examples

To provide additional practical guidance to designers, design guidelines and five example designs using curved bar nodes are provided in an appendix to this article. The appendix can be viewed at the end of the electronic version of this article available at **www.concreteinternational.com**.

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DESIGN GUIDELINES

Strut-and-tie models with curved-bar nodes are effective tools for designing D-regions that include frame corners, corbels, dapped-end beams, pier cap ends, and spandrel pockets. The following paragraphs summarize design guidelines for the use of curved-bar nodes. Design examples are provided following the design guidelines.

Bend radius and node stress

Bend radii should be within the limits of Eq. (3a) and (3b) (repeated here for convenience):

for C-T-T nodes: $$r_b \geq \frac{2A_{ts}f_y}{bf'_c}$$ Eq. (3a)

for C-C-T nodes: $$r_b \geq \frac{1.5A_{ts}f_y}{bf'_c}$$ Eq. (3b)

Equations (3a) and (3b) ensure that node stresses are within the limits prescribed in Appendix A of ACI 318-08. Equation (3a) applies to a C-T-T node formed by a bar bent over an included angle of less than 180 degrees, and Eq. (3b) applies to what is effectively a C-C-T node formed by a bar with a 180-degree bend. As a general rule, a larger bend radius will result in lower nodal stress. For optimum performance, the bend radius should be as large as possible, but the center of curvature must fall within the limits of the member or joint as defined by the geometry of the truss. Figure A.1 illustrates the region in which the center of curvature must fall for a typical frame corner.

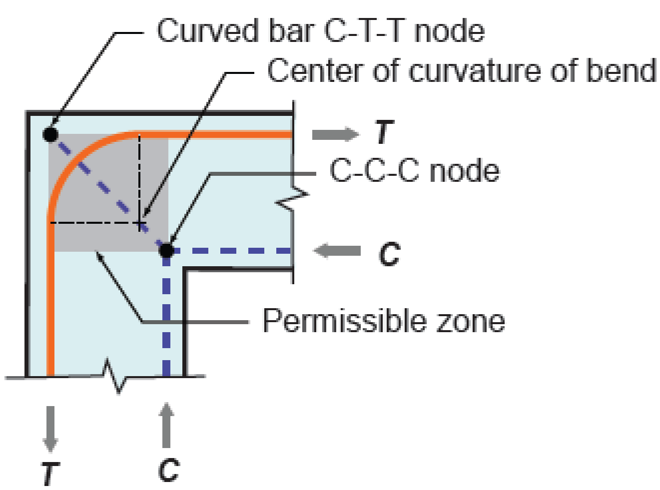


Fig. A.1: Permissible zone for the center of curvature of a curved-bar node at a frame corner

Multi-layer curved-bar nodes

Where more than one layer of reinforcement is used in the plane of the STM, nodal zone stresses are increased in proportion to the number of layers. Equations (3a) and (3b) may be used provided A_{ts} is taken as the area of tie reinforcement in all layers, and r_b is taken as the bend radius at the inside layer.

Bond stress in curved-bar nodes

In cases where the compression strut (or the resultant of multiple struts) does not bisect the angle formed by the ties extending from the curved-bar node, the length of the bend should be proportioned to develop the difference in tie force. For 90-degree bends, the minimum radius required for sufficient development length through the curve may be checked using Eq. (4b) (repeated here for convenience):

$$r_b \geq \frac{2l_d(1-tan\,\theta_c)}{\pi} - \frac{d_b}{2} \qquad \text{Eq. (4b)}$$

Effective width of curved-bar nodes

The effective width of nodes transverse to the plane of the STM truss is usually taken as the width of the member. The minimum bend radii provided by ACI 318-08 are sufficient to avoid crushing under the bend region of the bar itself. Where the centroid of the tie is not located in the middle of the member, the effective width of the node should be taken as twice the distance from the centroid of the tie to the nearest face of the member. To avoid side splitting, clear side cover of at least 2 d_b is suggested. Where this side cover cannot be provided, the bend radius should be increased in inverse proportion to the decrease in clear cover.

Frame corners

Design using an STM with a curved-bar node has two distinct advantages over traditional approaches. First, by assuring node stresses with ACI limits, the possibility of a diagonal flexural failure at less than the full potential flexural strength of the connected members is avoided. Second, the design is not limited by the available development length to the face of the support.

Dapped-end beams

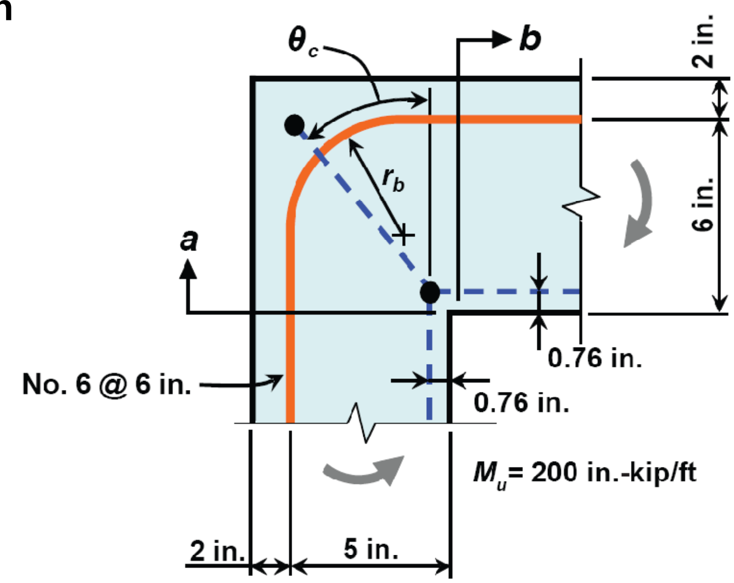
Designs using curved-bar nodes can simplify the detailing of dapped-end beams. A continuous bent bar (or series of parallel bars) can be used for two nodes: the C-C-T node above the reaction point and the C-T-T at the lower corner of the full section. Designs that do not employ curved-bar nodes require separately anchored ties at these locations. The details of the curved bar nodes are, however, very important for a successful design. Eccentricity and side cover must be considered.

Crack control

In addition to the primary tie reinforcement, confinement or skin reinforcement may be needed to control cracking in the connection region, especially for connection of large members (see Design Example 4).

DESIGN EXAMPLES

Example 1: Wall/Slab Junction



M_u = 200 in.-kip/ft

f_y = 60 ksi; f_c' = 4000 psi; d = 5 in.

A_{ts} = No. 6 @ 6 in. = 0.88 in.^{2}/ft

Design Method

Design a wall/slab junction using an STM with a curved-bar node.

First, flexural strengths at Sections a and b are checked using traditional approaches. The locations of the compression struts are based on the depth of the compression block, a. Next, the minimum bend radius is checked using Eq. (3a) for C-T-T nodes. Then, the radius required to assure development of the difference in tie forces is check using Eq. (4b). Finally, it is shown that the traditional approach of checking hooked bar development would not work for this example.

Flexural Design

Flexure at Section a:

$$a = \frac{A_{ts}\, f_y}{0.85\, \beta_1\, f_c'\, b} = \frac{0.88 \times 60}{0.85 \times 0.85 \times 4 \times 12} = 1.52 \text{ in.}$$

$$c = a/\beta_1 = \frac{1.52}{0.85} = 1.79 \text{ in.}$$

$$\varepsilon_s = \frac{\varepsilon_c(d-c)}{c} = \frac{0.003\,(5.0 - 1.79)}{1.79} = 0.0054 > 0.005 \Rightarrow \Phi = 0.9$$

$$\Phi M_n = \Phi A_s f_y (d - a/2) = 0.90 \times 0.88 \times 60\,(5.0 - 1.52/2) = 201 \text{ in.-kip/ft}$$

The stresses at Section b will be different than at Section a. For simplicity, however, the centroid of the compression strut is also assumed to be at *a*/2 = 0.76 in. from the inside face of the wall at Section b.

C-T-T Node Stress

Check bend radius per Eq. (3a):

$$r_b \geq \frac{2\,A_{ts}\,f_y}{b\,f_c'} = \frac{2 \times 0.88 \times 60}{12 \times 4} = 2.20 \text{ in. OK (STD } r_b = 3\,d_b = 2.25 \text{ in.)}$$

Development of Difference in Tie Force

θ_c = Arctan(4.24/5.24) = 39.0 degrees

$$l_d = \frac{f_y\,\psi_t\,\psi_e}{25\lambda\sqrt{f_c'}} \times d_b = \frac{60{,}000 \times 1.0 \times 1.0}{25 \times 1.0\sqrt{4000}} \times 0.75 = 28.5 \text{ in.}$$

Check minimum radius required for development using Eq. (4b):

$$r_b \geq \frac{2l_d(1 - \tan\theta_c)}{\pi} - \frac{d_b}{2} = \frac{2 \times 28.5\,(1 - \tan 39.0)}{\pi} - \frac{0.75}{2} = 3.1 \text{ in.; use } r_b = 4 \text{ in.}$$

Note that required r_b is greater than the standard radius.

Also note that r_b< d; therefore, the center of curvature is inside the C-C-C node, as it must be.

Development Using Traditional Approach

Traditional approach is to check development length of hook, l_{dh}, at face of support.

$$l_{dh} = (0.02\,\psi_e f_y/\lambda\sqrt{f_c'})d_b = (0.02 \times 1.0 \times 60{,}000 / 1.0 \times \sqrt{4000}) \times 0.75 = 14.2 \text{ in.}$$

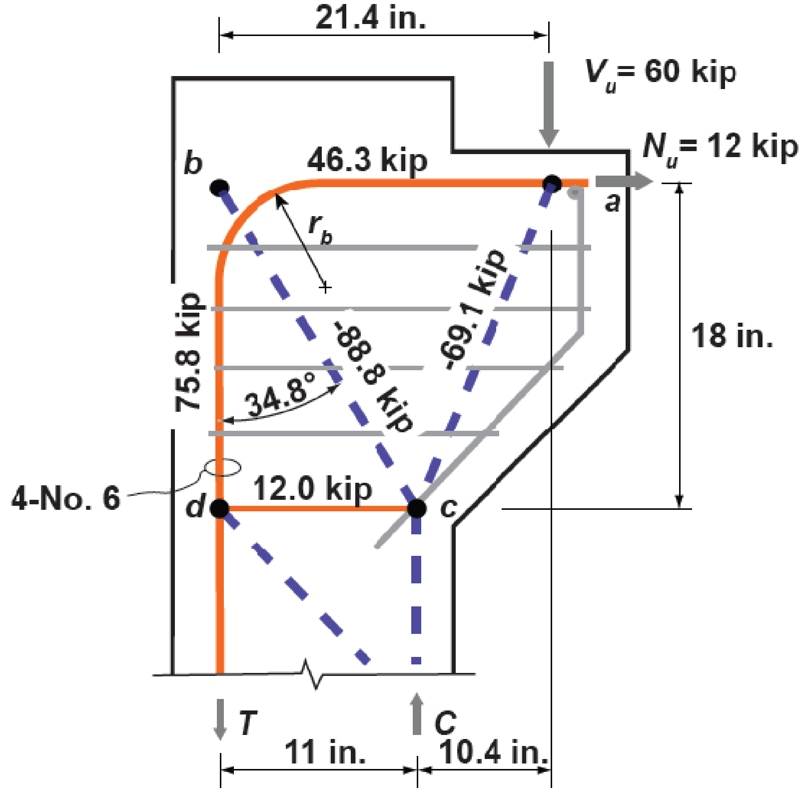
Note that the required development length far exceeds that available.

Example 2: Column Corbel

Use an STM with a curved-bar node to design a corbel. The example is based on example 17.2 from the "Notes on ACI 318-05" by the Portland Cement Association (PCA), but the column reinforcement above the corbel is not continuous.

f_y = 60 ksi; f'_c = 5000 psi

16 x 16 in. column



Check Ties

Tie bd: F_u = 75.8 kip

4 – No. 6 bars; A_{ts} = 1.76 in.^{2}

$\Phi F_n = \Phi A_{ts} f_y$ = 0.75 x 1.76 x 60 = 79.2 kip

C-T-T Node Stress

Check bend radius per Eq. (3a):

$$r_b \geq \frac{2A_{ts}f_y}{bf'_c} = \frac{2 \times 1.76 \times 60}{16 \times 5} = 2.64 \text{ in.}$$

Standard bend = $3d_b$ = 2.25 in. N.G.

Development of Difference in Tie Forces

θ_c = Arctan(11/18) = 31.4 degrees

Check basic bar development for No. 6 bars, which should be considered top bars.

$$l_d = \frac{f_y \psi_t \psi_e}{25\lambda\sqrt{f_c'}} = \frac{60{,}000 \times 1.3 \times 1.0}{25 \times 1.0 \sqrt{5000}} \times 0.75 = 33.1 \text{ in.}$$

Check minimum radius required for development using Eq. (4b):

$$r_b \geq \frac{2l_d(1 - \tan\theta_c)}{\pi} - \frac{d_b}{2} = \frac{2 \times 33.1\,(1 - \tan 31.4)}{\pi} - \frac{0.75}{2} = 7.8 \text{ in.; use } r_b = 8 \text{ in.}$$

Note that required r_b is much greater than the standard radius.

Development Using Traditional Approach

Traditional approach is to check development length of hooked, l_{dh}, at face of support.

$$l_{dh} = (0.02\,\psi_e f_y/\lambda\sqrt{f_c'})d_b = (0.02 \times 1.0 \times 60{,}000 / 1.0 \times \sqrt{5000}) \times 0.75 = 12.7 \text{ in.}$$

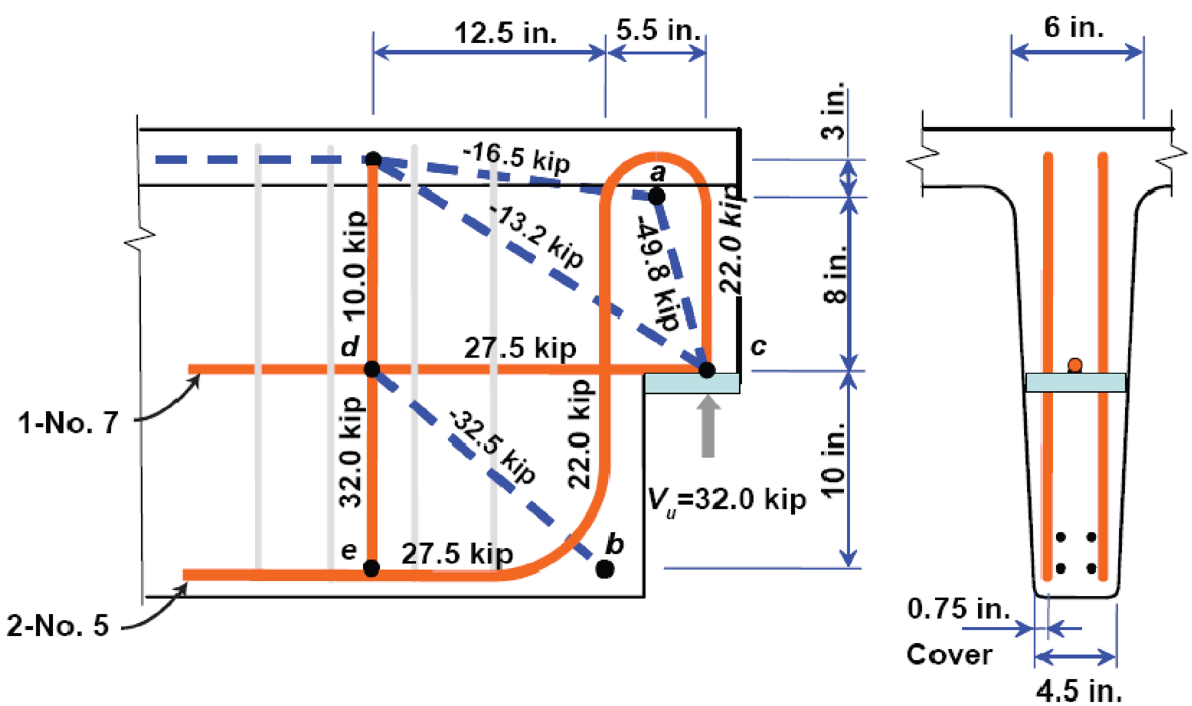
In this case, a standard hook could have been developed in the available length. See example 17.2 in "Notes on ACI 318-05" by PCA for additional design considerations.

Example 3: Dapped-End Beam

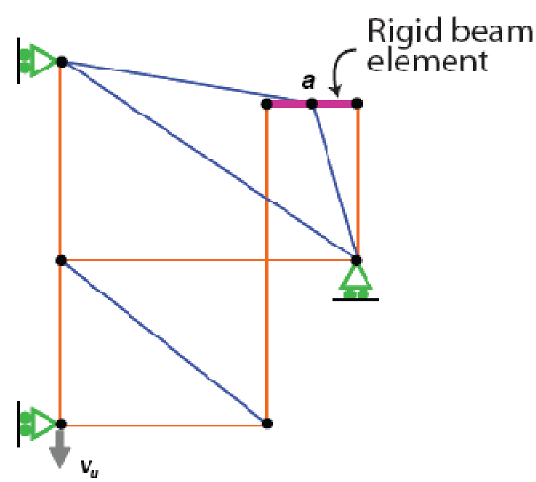
Design a dapped T-beam support using an STM with curved-bar nodes.

The reinforcement layout is shown below. The truss model used to calculate strut and tie forces is also shown. A group of stirrups is modeled as a single vertical tie at the left side of the model. To simulate the 180-degree bend at Node a, a rigid beam element is used to connect the tie extending from each end of the 180-degree bend. The beam element is hinged at Node a to assure the forces on each side of Node a are equal. The resulting model is determinate.

$f_y = 60$ ksi; $f'_c = 6000$ psi



Check Ties



Tie be: $F_u = 27.5$ kip

2 – No. 5 bars; $A_{ts} = 0.62$ in.^{2}

$\Phi F_n = \Phi A_{ts} f_y = 0.75 \times 0.62 \times 60 = 27.9$ kip

Tie cd: $F_u = 27.5$ kip

1 – No. 7 bar; $A_{ts} = 0.60$ in.^{2}

$\Phi F_n = 0.75 \times 0.60 \times 60 = 27.0$ kip; say OK

Node Stresses

Check bend radius at C-C-T Node *a* per Eq. (3b):

$$r_b \geq \frac{1.5A_{ts}f_y}{b\,f_c'} = \frac{1.5 \times 0.62 \times 60}{6.0 \times 6} = 1.55 \text{ in.}$$

Standard bend = *3d_b = 1.88 in. OK; use r_b = 2.5 in. to match reinforcement layout geometry.*

Check bend radius at C-T-T Node *b* per Eq. (3a):

$$r_b \geq \frac{2A_{ts}f_y}{b\,f_c'} = \frac{2 \times 0.62 \times 60}{4.5 \times 6} = 2.8 \text{ in.}$$

Increase minimum radius in inverse proportion to the decrease in side cover. Actual side cover at Node *b* is *0.75 in.; $2d_b$ = 2 x 0.625 = 1.25 in*.

Therefore, $r_b \geq$ *2.8 x 1.25/0.75 = 4.7 in.* Use r_b = 6 in. Note that r_b = 6 in. is significantly greater than the standard $3d_b$ bend of 1.88 in.

Development of Difference in Tie Force

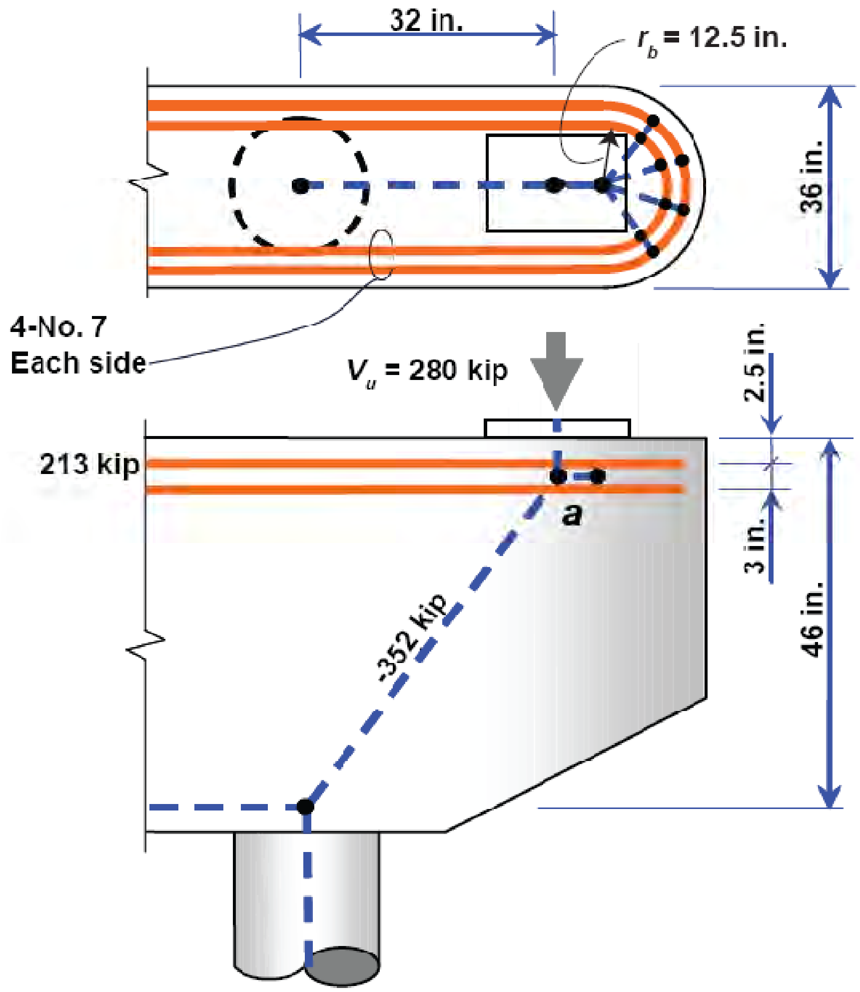
Θ_c = Arctan (10/12.5) = 38.7 degrees

$$l_d = \frac{f_y\,\psi_t\,\psi_e}{25\lambda\sqrt{f_c'}} \times d_b = \frac{60{,}000 \times 1.0 \times 1.0}{25 \times 1.0\sqrt{6000}} \times 0.625 = 19.4 \text{ in.}$$

$$r_b \geq \frac{2l_d(1 - \tan\theta_c)}{\pi} - \frac{d_b}{2} = \frac{2 \times 19.4\,(1 - \tan 38.7)}{\pi} - \frac{0.625}{2} = 2.2 \text{ in.; OK, } r_b = 6 \text{ in.}$$

Example 4: Pier Cap Cantilever

f_y = 60 ksi; f_c' = 4000 psi



Design a pier cap cantilever with rounded ends using an STM with curved-bar nodes. Node a is a C-C-T node.

Check Ties

Top Ties $F_u = 213$ *kip*

$A_{ts} = 2.4$ in.^{2}; $2A_{ts} = 4.8$ in.^{2}

$\Phi F_n = \Phi A_{ts} f_y = 0.75 \times 4.8 \times 60 = 216$ *kips OK*

Node Stresses

Check bend radius at C-C-T Node *a* per Eq. (3b). This is a multi-layer node. A_{ts} is taken as the area of reinforcement in both layers, and r_b is bend radius at the inside layer.

$b \approx 3+2\frac{1}{2}+2\frac{1}{2} = 8$ in.

$$r_b \geq \frac{1.5\, A_{ts} f_y}{b\, f_c'} = \frac{1.5 \times 2.4 \times 60}{8 \times 4} = 6.8 \text{ in.}$$

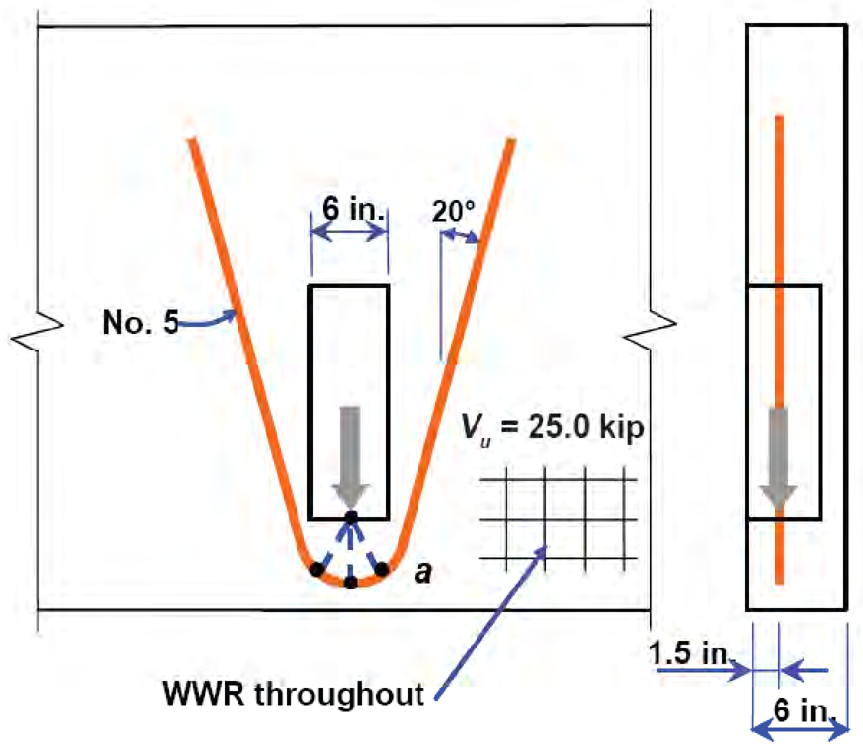
OK, $r_b = 12.5$ in.

In addition to the primary ties, nominal stirrups and longitudinal skin reinforcement should be provided for crack control.

Example 5: Spandrel Pocket

Design hanger reinforcement for a pocket spandrel using a curved-bar node. The bend angle is less than 180°, so the node will be considered a C-T-T node.

f_y = 60 ksi; f_c' = 6000 psi



Check Ties

$F_u = 25.0/(2\cos 20) = 13.3$ kip

No. 5 bar, $A_{ts} = 0.31$ in.^{2}

$\Phi F_n = \Phi A_{ts} f_y = 0.75 \times 0.31 \times 60 = 14.0$ kip OK

Node Stresses

Check bend radius at C-T-T node per Eq. (3a):

b ≈ 1½+ 1½ = 3 in.

$$r_b \geq \frac{2\,A_{ts} f_y}{b\,f_c'} = \frac{2 \times 0.31 \times 60}{3 \times 6} = 2.1 \text{ in.}$$

Use r_b = 4.0 in. to clear pocket.