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Joints between Reinforced Concrete Members of Similar Depth

by A. H. Mattock and J. F. Shen

The treatment in design codes of joints between reinforced concrete members of similar depth, and the background to that treatment are reviewed. Tests designed to study the effectiveness of hanger reinforcement at the intersection of beams of similar depth are reported. It is concluded that provisions for the design of hanger reinforcement should be included in the ACI Building Code.

Keywords: beams (supports); connections; **joints (junctions)**; **reinforced concrete**; standards; **strength**; structural design.

Indirectly supported and/or loaded members frequently occur in reinforced concrete structures. Such a situation occurs when floor beams frame into the side faces of supporting beams. In such a case, the floor beam is supported indirectly by internal forces rather than directly by a reaction acting through its bottom face. If appropriate reinforcement is not provided in the region of the joint between such beams, premature shear failure or premature yield of the flexural reinforcement may occur.

Ferguson[1] in 1956 was probably the first to draw attention to the possible detrimental effect of indirect support or loading on the shear strength of reinforced concrete members. Leonhardt[2] discussed Ferguson's tests with reference to the "truss analogy" model for shear resistance. Such a model for the case of the intersection of two beams of equal depth is shown in Fig. 1(a). It is clear that a vertical tension member is necessary at the intersection of the two trusses to transfer the end shear from the bottom of the supported member to the top of the supporting member. The force in this vertical tie would be equal to the end shear in the supported member.

Fig. 1(b) shows a truss analogy model for the intersection of two beams of unequal depth, which have their top faces at the same level. In this case, part of the end shear is supported by direct strutting action in the supporting beam below the bottom of the supported beam. The remainder of the end shear is carried by the vertical tie at the intersection of the beams. Leonhardt[3] suggested that stirrups be placed in the supporting beam at the intersection of the beams to act as the vertical tie member in the truss analogy model. He called these stirrups "hanger reinforcement" and proposed that they be designed to carry $(h_1/h_2) \times$ (end shear in supported beam), where h_1 and h_2 are, respectively, the overall depths of the supported and supporting beams. The total cross-sectional area of all legs of the stirrups located within the joint region and constituting the hanger reinforcement was to be taken into account when calculating its yield strength. This reinforcement was to be additional to the web reinforcement that would normally be provided in the intersecting members. This approach was adopted by CEB in 1973.[3]

Instances of premature yield of flexural reinforcement in indirectly supported members were reported by Gesund, Mills, and Martin[4] in 1968 and Elfgren[5] in 1972. In both cases, cantilever beams projecting from the side faces of a primary member were being used to load that member in combined flexure, torsion, and shear. The cantilevers were designed to resist flexure and shear in the normal manner, but no special reinforcement was provided in the joint. Yield of the flexural reinforcement occurred at moments significantly less than the calculated flexural strengths of the members.

The premature yielding of the cantilever flexural tension reinforcement in the preceding tests was probably caused by the flexural tension reinforcement being subject to an additional tension due to shear, as well as the tension due to flexure. This additional tension is caused by the horizontal component of the forces in the inclined concrete struts between the diagonal tension cracks. Adjacent to an indirect support, the diagonal tension cracks will be at a constant slope and the additional tension will occur at the face of support. In the case of a direct support acting on the compression face

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of the beam, cracks will radiate from the support, forming a fan. In this case, the additional force due to shear will decrease as the slope of the cracks increases, and will be zero at the face of support.

CODE PROVISIONS

The first code to require the provision of hanger reinforcement at the intersection of reinforced concrete beams was the CEB-FIP Model Code for Concrete Structures in 1978.[6] Section 18.2.4 requires that:

> . . . hanging or transmission reinforcement shall be calculated for the total reaction acting at the support and can be reduced by the ratio h_1/h_2 if the height h_1 of the supported beam is smaller than the height h_2 of the bearing beam, provided that the top surfaces of the two beams are at the same level. Transmission reinforcement should be composed preferably of stirrups surrounding the main flexural reinforcement of the bearing beam. For high loads, some of these stirrups may be distributed outside the space common to both beams. The main reinforcement of the supported beam shall be placed above that of the bearing beam.

There is no requirement in the ACI Building Code[7] for the provision of hanger reinforcement at locations of indirect support. However, ACI-ASCE Committee 426 on shear and diagonal tension included a provision for hanger reinforcement in their suggested revisions to shear provisions for building codes,* in 1979. Their proposal was as follows:

> When a beam or beams are framed monolithically into a girder, stirrups having a total capacity $\phi A_v f_y$ equal to or greater than the total shear force transferred from the beams to the girder should be placed in the girder within 0.5 times the depth of the beams on either side of the beams. This requirement may be waived if the shear stress in the beams is less than $3\sqrt{f'_c}$, if the lower face of the girder is more than 0.5 times the depth of the beam below the bottom of the beam, or if the girder is supported on its lower face at the joint.

Waiving of the requirement when the shear stress in the beams is less than 3 $\sqrt{f'_c}$ psi (0.25 $\sqrt{f'_c}$ MPa) was based on the belief that if significant diagonal tension cracking has not developed in the supported beam, then truss-like action will not occur and the end shear in the supported beam will be transferred to the girder over its entire depth. Hence, hanger reinforcement would not be needed.

This proposal assumes a crack surface in the supporting girder as shown in Fig. 2(a), which is taken from a preceding reference.* Such a crack surface

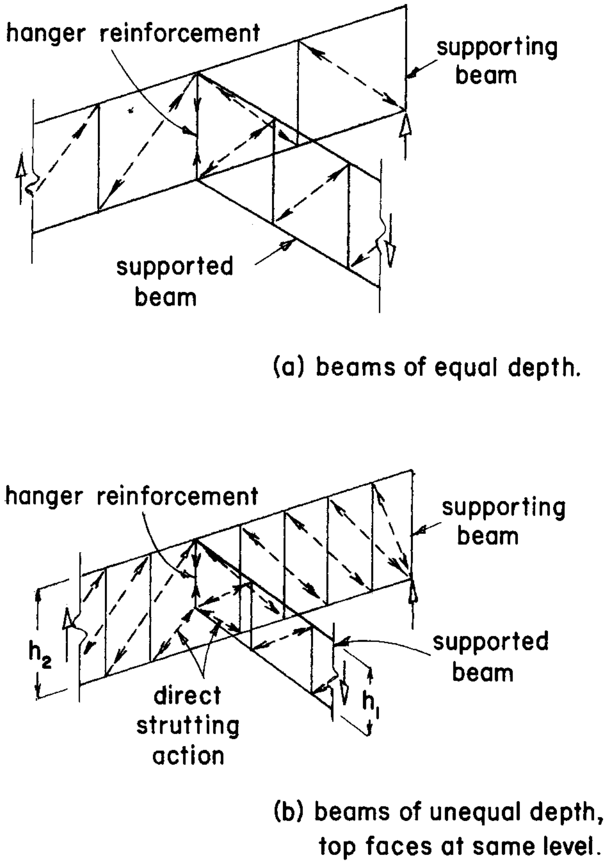

Fig. 1 — Truss analogy models for the intersection of two beams

would intersect all the legs of the stirrups placed in the girder at the intersection with the supported beam. Therefore, as in Reference 6, it is assumed that all legs of the hanger reinforcement will act to "hang up" the reaction from the beam in the girder. This assumption stems from the fact that the tests of Leonhardt,[2,3] on which these provisions[6,]* were based, involved situations in which little or no moment was transferred from the supported beam to the supporting beam, i.e., little or no torsional stresses were induced in the supporting beam. In such cases, the crack surface in the supporting beam would be similar to that shown in Fig. 2(a), and all stirrup legs crossing this surface would resist the reaction from the supported beam.

However, if the supporting beam or girder is torsionally restrained so that a significant moment is transferred from the supported beam to the girder, torsional stresses in the girder will change the shape of the crack surface, as shown in Fig. 2(b). The crack surface is then crossed only by the hanger reinforcement stirrup legs close to the interface between the supported beam and the girder, i.e., only these stirrup legs will actually act as hanger reinforcement. This was demonstrated accidentally by Collins and Lampert[8] in tests of T-shaped horizontal reinforced concrete frames. The

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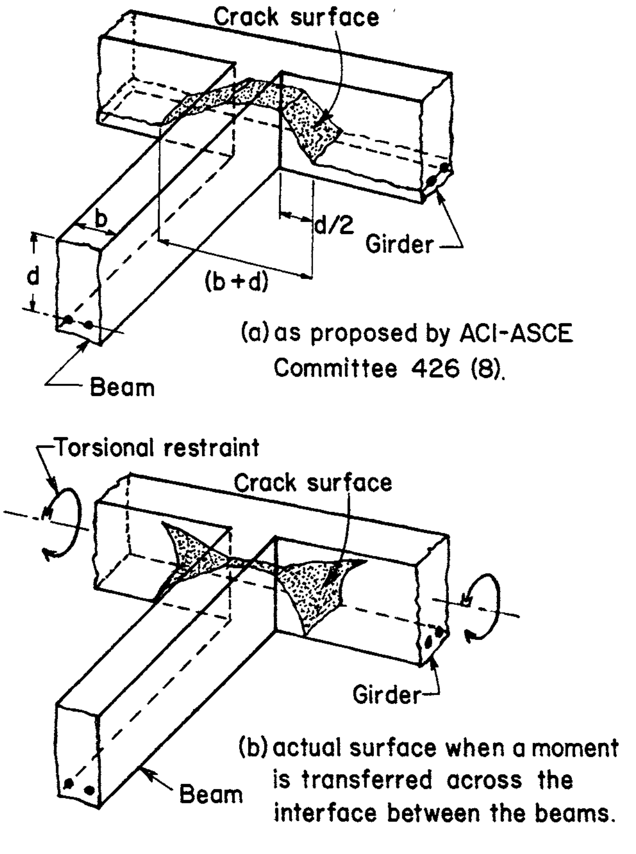

Fig. 2 — Crack surface at the intersection of two beams

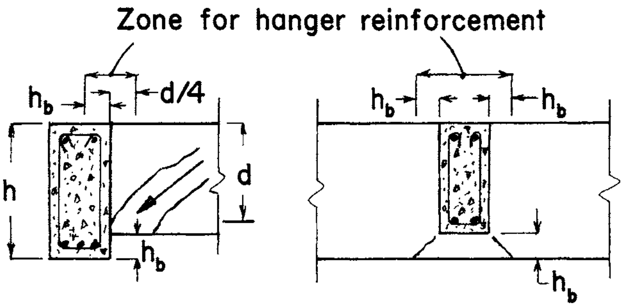

Fig. 3 — Fig. D1 from Appendix D to Canadian design code,[10] showing region within which hanger reinforcement is to be placed

leg of the T was loaded at its midlength, vertical reactions were provided at the ends of the beams, and torsional restraint was provided at both ends of the cross beam. The frame simulated a floor beam and supporting spandrel beam in a beam and girder floor system. Hanger reinforcement as recommended by Leonhardt[2] was provided in the joint between the beams. In spite of this, some of the joints failed prematurely in the region of the interface between the floor beam and the spandrel beam.

Tests conducted at the University of Washington in 1982 and summarized later in this paper demonstrated that when a significant moment is transferred from the supported beam to the supporting beam, only that hanger reinforcement adjacent to the interface between the supported and supporting beams is effective.

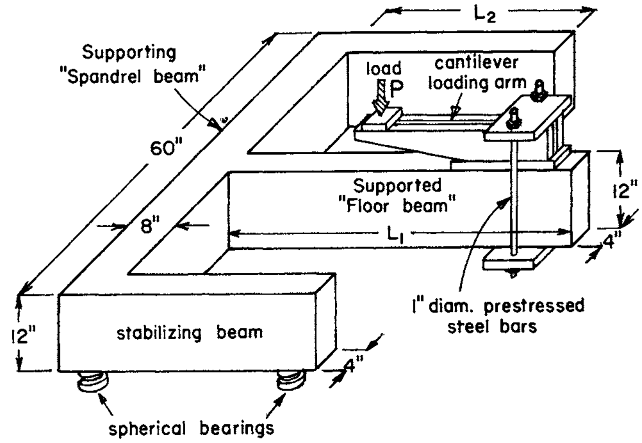

Fig. 4 — Typical test specimen, showing arrangements for test

In 1984, the Canadian standard for design of concrete structures for buildings[9] was published. It contained the following requirement for hanger reinforcement at beam-girder intersections in Section 11.3.9.2:

> When a load is applied to a side face of a member, additional transverse reinforcement capable of transmitting a tensile force of $(1 - h_b/h)$ times the applied factored load shall be provided. In the supporting member, only the additional full depth transverse reinforcement in a region within a distance h_b from the shear interface may be assumed effective. In the supported member, only the additional full depth transverse reinforcement within a distance of one quarter of the effective depth of the supported member on each side of the shear interface may be assumed effective. This requirement may be waived if the interface transmitting the load extends to the top of the supporting member and if the average shear stress on this interface is no greater than $0.25\phi\sqrt{f_c'}$ MPa ($3.0\phi\sqrt{f_c'}$ psi).

In Section D11.3.9 of Appendix D, published with the preceding standard, it is stated "that in determining the zone of effective reinforcement, h_b need not be taken as being less than 75 mm." h and h_b are as shown in Fig. 3, which is Fig. D1 in Appendix D.

EXPERIMENTAL STUDIES

Tests were carried out to evaluate the effectiveness of hanger reinforcement in the case of a floor beam framing into one side of a spandrel beam of equal depth, and to check the joint ACI-ASCE Committee 426 design proposals for hanger reinforcement.*

The test specimens consisted of a horizontal E-shaped frame, as shown in Fig. 4. The frames were supported on spherical bearings at the ends of the supporting spandrel beam, and at the ends of the stabilizing beams which provided torsional restraint to the spandrel beam. The load was applied to the supported floor beam through a 30-in. (760-mm) long cantilever loading arm, which was prestressed to the end of the floor beam. The load was applied to the tip of the loading arm by a hydraulic testing machine. This testing arrangement results in a distribution of bending

*Suggested revisions to shear provisions for building codes submitted by joint ACI-ASCE Committee 426 on shear and diagonal tension.

moments and pattern of cracking in the floor beam similar to that which would occur in an actual floor beam in the vicinity of a supporting spandrel beam. A point of inflection occurs in the floor beam immediately below the center of action of the load, distance a from the face of the spandrel beam. (This loading arrangement prevents direct strutting from occurring between the point of application of the load and the bottom of the interface, which would probably occur if the load were applied directly to the floor beam.)

Details of the test specimens are set out in Table 1. The concrete was made from Type III cement, natural sand, 3/8-in. maximum size gravel and water. No. 3 and larger reinforcing bars were deformed and conformed to ASTM A 615. The No. 2 bars and 3/16-in. diameter rods were smooth.

Design of test specimens

The nominal a/d ratio for the floor beam in Specimens 1 and 4 was 2.0, for Specimens 2 and 5 it was 0.5, and for Specimen 3 it was 1.0. The design flexural strength at the face of support of the floor beams in Specimens 1 and 4 was such that the corresponding shear in the floor beam was about $2.5b_w\, d\sqrt{f_c'}$ psi ($0.21b_w\, d\sqrt{f_c'}$ MPa). The same flexural strength was provided in Specimen 2, but because of the smaller a/d ratio, the corresponding shear was about the maximum permitted by the ACI Building Code,[7] i.e., about $10b_w\, d\sqrt{f_c'}$ psi ($0.83b_w\, d\sqrt{f_c'}$ MPa). The flexural strength of Specimen 3 was double that of Specimen 2, so that the shear at flexural failure would be the same as in Specimen 2. The shear corresponding to the design flexural strength of the floor beam in Specimen 5 was about $8b_w d\sqrt{f_c'}$ psi ($0.66b_w\, d\sqrt{f_c'}$ MPa). In each case, sufficient stirrup reinforcement was provided in the floor beam to satisfy the ACI Building Code.[7] In Specimens 1 and 4, the minimum reinforcement requirement governed, so that the calculated shear strength of the beam was somewhat greater than the calculated shear at flexural failure. To insure that failure would occur in the junction region between the two beams or in the floor beam, the spandrel beam in each specimen was designed to carry 120 percent of the moments, shears, and torsional moments induced in it by the floor beam when it developed its design strength.

Hanger reinforcement with a yield strength equal to the shear at flexural failure of the floor beam was provided adjacent to the interface between the floor and spandrel beams in Specimens 1, 2, and 3. The hanger reinforcement was considered to consist of those legs of the stirrups in the supporting spandrel beam that were adjacent to the interface between the floor and spandrel beams, together with additional single-leg stirrups placed in the same location when appropriate.

The hanger reinforcement in Specimens 4 and 5 was designed according to the recommendations of joint ACI-ASCE Committee 426.* In the case of Specimen 4, no hanger reinforcement was provided, since the calculated maximum shear in the floor beam at flexural ultimate was less than $3b_w\, d\sqrt{f_c'}$ psi ($0.25b_w d\sqrt{f_c'}$ MPa). In Specimen 5, all stirrup legs in the spandrel beam within a distance $d/2$ either side of the floor beam were considered to act as hanger reinforcement, as proposed by joint ACI-ASCE Committee 426.

*Suggested revisions to shear provisions for building codes submitted by joint ACI-ASCE Committee 426 on shear and diagonal tension.

Table 1 — Details of test specimens

Specimen	1	2	3	4	5
L_1, in.	52.5	31.5	43.0	52.5	37.5
L_2, in	32.0	17.0	17.0	32.0	17.0
a, in.	22.0	5.5	11.0	22.0	5.5
f_c', psi	4590	4340	4350	4700	4600
A_s	2 #3	2 #3	4 #3	2 #3	2 #3
f_y, ksi	70.56	71.53	72.43	71.53	54.60
d, in.	11.13	11.06	10.38	11.06	11.06
A_{vh}	2 #2	1 #4 + 2 #3	2 #4	—	4 #3 + 2 #2
f_{vhy}, ksi	58.67	68.59 (#4) 71.53 (#3)	68.61	—	54.60 (#3) 57.28 (#2)
A_v	3/16 in.	#2	#2	#2	#2
f_{vy}, ksi	59.76	57.04	57.65	57.14	57.28
s, in.	5.5	2.5	2.25	5.5	2.5

1 in. = 25.4 mm; 1 psi = 6.89 kPa; 1 ksi = 6.89 MPa.
A_s = flexural tension reinforcement in top of floor beam; f_y = yield strength of flexural tension reinforcement; d = effective depth of floor beams; A_{vh} = total hanger reinforcement provided in spandrel beam adjacent to interface between floor beam and spandrel beam (except for Specimen 5 — see text); f_{vhy} = yield strength of hanger reinforcement; A_v = bar size of two-legged stirrups used in floor beam; f_{vy} = yield strength of stirrups used in floor beam; s = spacing of stirrups in floor beam.

Test procedures and instrumentation

An incrementally increasing load was applied to the tip of the cantilever loading arm by a hydraulic testing machine acting through a spherical bearing. At each load stage, measurements were taken of the strains in the hanger reinforcement and the floor beam flexural reinforcement at the interface of the floor and spandrel beams. These strains were measured using electrical resistance strain gages coupled to a strip chart recorder. Also at each load stage, any growth of cracks was marked and the maximum crack widths in the top and side faces of the floor beam were measured.

Strength and behavior of specimens

Data from the tests are presented in Table 2. Typical patterns of cracking in the floor beams of the test specimens are shown in Fig. 5.

Specimens 1 and 4, for which $V_n(test)$ was about $3b_w\, d\sqrt{f_c'}$ psi ($0.25b_w\, d\sqrt{f_c'}$ MPa), both behaved in a similar manner, despite the fact that hanger reinforcement was provided adjacent to the interface between the floor beam and spandrel beam in Specimen 1 but not in Specimen 4. This similarity in behavior validates the waiver of the requirement to provide hanger reinforcement when V_n in the supported beam is less than $3b_w d\sqrt{f_c'}$ psi ($0.25b_w d\sqrt{f_c'}$ MPa), contained in the joint ACI-ASCE Committee 426 proposals* and also in the Canadian Code.[9] Diagonal tension cracking was not well developed in these beams and, hence, truss-like ac-

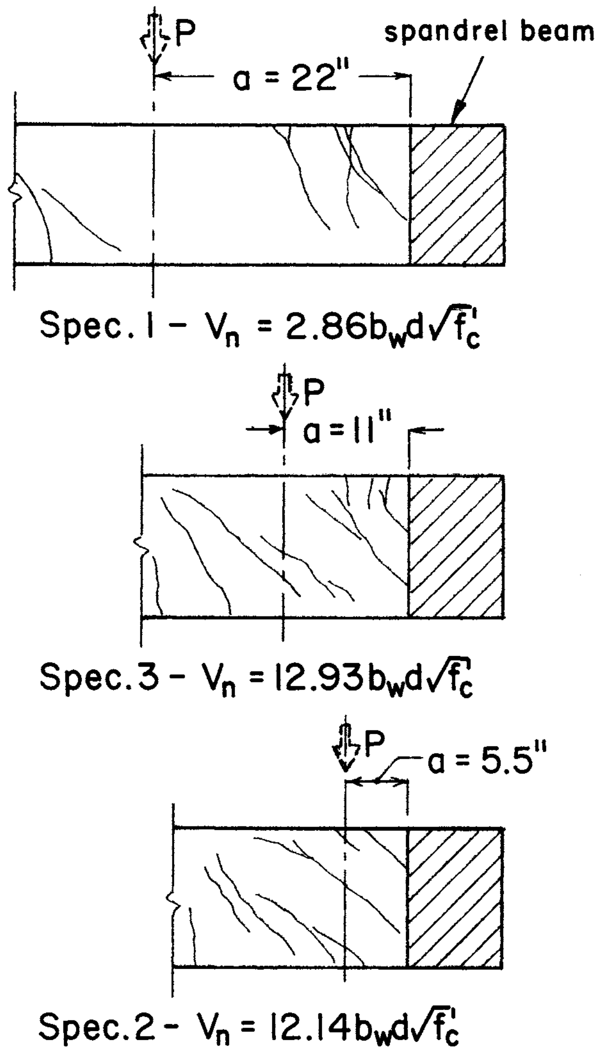

Fig. 5 — Typical floor beam cracking patterns in test specimens

tion was probably not significant. The hanger reinforcement in Specimen 1 did not yield and carried only 35 percent of the shear at ultimate. No diagonal tension cracks occurred in the vicinity of the point of inflection in either of these specimens, as seen in Fig. 5.

The floor beam of Specimen 2 was designed to have the same flexural strength as that in Specimen 1, but the shear at the interface was to be four times that in Specimen 1, and hanger reinforcement sufficient to carry this shear was provided in the spandrel beam adjacent to the interface. The desired behavior was achieved. In the floor beams of both specimens, the flexural strength was about 20 percent greater than the calculated strength $M_n(calc)$, and the moment at yield of the flexural reinforcement was about 10 percent greater than $M_n(calc)$. The flexural cracks widened rapidly after yield, reaching about 0.1 in. just before failure.

The pattern of diagonal tension cracks was well developed at failure, as seen in Fig. 5, clearly defining the diagonal compression struts of truss action. Their maximum width just before failure was 0.034 in. The hanger reinforcement yielded before failure and at failure carried 83 percent of the shear. The maximum force in the hanger reinforcement was close to the shear at yield of the flexural reinforcement.

Table 2 — Test results*

Specimen	1	2	3	4	5
P_{max}†	8.18	35.00	35.00	8.90	25.80
$V_n(test)$‡	8.64	35.37	35.41	9.36	26.20
$M_n(test)$‡	194.3	199.2	394.8	210.1	149.5
$V_y(test)$‡	8.04	31.88	29.30	7.46	14.40
$M_y(test)$‡	181.1	180.0	317.8	168.3	84.6
$M_n(calc)$	165.0	165.7	296.4	166.4	128.4
$\frac{V_n(test)}{b_w d/\sqrt{f'_c}}$	2.86	12.14	12.93	3.09	8.73
$\frac{M_n(test)}{M_n(calc)}$	1.18	1.20	1.33	1.26	1.16
$\frac{M_y(test)}{M_n(calc)}$	1.10	1.09	1.07	1.01	0.66
$A_{vh} f_{vs}$§	3.04	30.32	29.06	—	15.54
$A_{vh} f_{vhy}$	5.87	29.45	27.44	—	29.75
$\frac{A_{vh} f_{vs}}{V_n(test)}$	0.35	0.83	0.77	—	0.57

*Moments in kip.in.; shears and forces in kips.
†P_{max} = maximum applied load.
‡Includes effects of weight of loading equipment and floor beam.
§$A_{vh} f_{vs}$ = force in hanger reinforcement at ultimate.
1 kip = 4.45 kN; 1 kip. in. = 0.113 kN.m.

The floor beam of Specimen 3 was designed to carry the same shear as the floor beam in Specimen 2, but the corresponding moment at the interface was to be twice that in Specimen 2. Hanger reinforcement sufficient to carry the ultimate shear was provided in the spandrel beam adjacent to the interface. The behavior of Specimens 3 and 2 was very similar. Again, the diagonal tension cracking pattern was well defined, indicating truss-like action in the floor beam (see Fig. 5.) The hanger reinforcement yielded before failure and carried a maximum load approximately equal to the shear at yield of the flexural reinforcement.

The *a/d* ratio for Specimen 5 was the same as that for Specimen 2, but it was designed to carry a shear of about $8 b_w d \sqrt{f'_c}$ psi ($0.66 b_w d \sqrt{f'_c}$ MPa) through the interface, together with the corresponding moment. In Specimen 5, the hanger reinforcement was designed according to the proposals of joint ACI-ASCE Committee 426.* All stirrup legs in the spandrel beam within a distance $d/2$ either side of the floor beam were counted as hanger reinforcement. The behavior of this specimen was unsatisfactory. Yield of the floor beam flexural reinforcement occurred at 55 percent of the ultimate load and 66 percent of the calculated flexural strength. The width of the flexural cracks increased rapidly after yield of the flexural reinforcement, reaching about 0.1 in. just below the calculated flexural strength.

The pattern of diagonal tension cracking in the floor beam was similar to that which occurred in Specimen 2, indicating truss-like action. However, in Specimen 5, the hanger reinforcement did not provide effective sup-

*Suggested revisions to shear provisions for building codes submitted by joint ACI-ASCE Committee 426 on shear and diagonal tension.

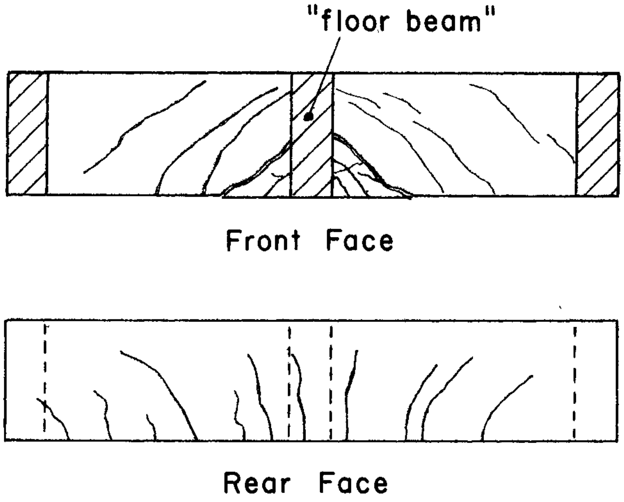

Fig. 6 — Cracking of the spandrel beam in Specimen 5

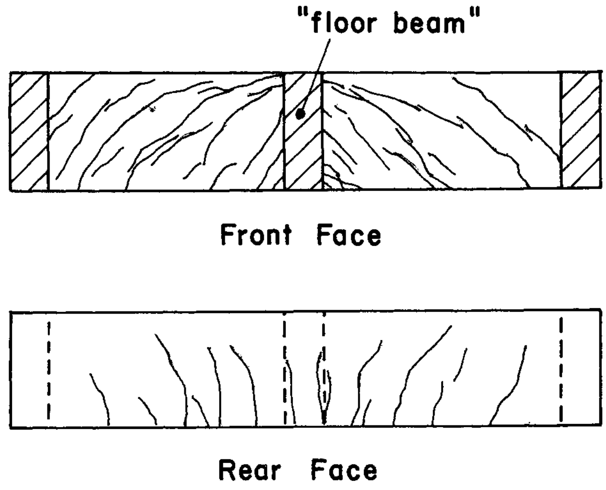

Fig. 7 — Cracking of the spandrel beam in Specimen 2

port for the bottom of the spandrel beam at the intersection of the beams. Because of this, the truss action caused an increase in the force in the floor beam flexural reinforcement at the face of support, leading to early yielding. The pattern of cracking in the spandrel beam of Specimen 5 is shown in Fig. 6. The force in the hanger reinforcement at failure was approximately equal to the yield strength of the legs of the hanger reinforcement stirrups adjacent to this face of the beam, and was only 57 percent of the shear at failure. Large yield strains in the hanger reinforcement resulted in very wide diagonal tension cracks in the spandrel beam adjacent to the floor beam, as the bottom of the spandrel beam was pushed downward (see Fig. 6).

Fig. 7 shows the pattern of cracking in the spandrel beam of Specimen 2, which is typical of that occurring in the remaining specimens. Note that inclined cracks that would cross the hanger reinforcement and mobilize it to resist the reaction from the floor beam occurred only on the face of the beam intersected by the floor beam. The observed cracking pattern in the spandrel beams of all the specimens is consistent with the crack surface shown in Fig. 2(b).

CONCLUSIONS FOR DESIGN

The following conclusions are drawn from these and other tests:[2,4,5,8]

1. Hanger reinforcement provided adjacent to the interface between intersecting beams of similar depth is effective in preventing premature yield of the flexural reinforcement in the indirectly supported beam. The hanger reinforcement location specified in the joint ACI-ASCE Committee 426 proposals* is incorrect.

2. Hanger reinforcement is unnecessary if the shear at ultimate in the indirectly supported beam is not greater than $3b_w d\sqrt{f'_c}$ psi ($0.25b_w d\sqrt{f'_c}$ MPa).

3. When the supported and supporting beams are of equal depth, or the beams are of unequal depth but their bottom faces are at the same level, then the yield strength of the hanger reinforcement should be equal to the shear being transferred across the interface between the two beams.

4. Because the absence of hanger reinforcement can result in premature yield of the flexural reinforcement in indirectly supported reinforced concrete beams, provisions for the design of hanger reinforcement should be included in the ACI Building Code.[7]

*Suggested revisions to shear provisions for building codes submitted by joint ACI-ASCE Committee 426 on shear and diagonal tension.

REFERENCES

1. Ferguson, P. M., "Some Implications of Recent Diagonal Tension Tests," ACI JOURNAL, *Proceedings* V. 53, No. 2, Aug. 1956, pp. 209-229.

2. Leonhardt, F., "La reduction de la couture d'effort tranchant dans les structures en béton armé (Interprétation de résultats d'essais à l'aide d'une généralisation de l'analogie du treillis)," Comité Européen du Béton, *Bulletin d'Information* No. 49, May 1965, pp. 86-158.

3. "Manuel de Calcul (Effort Tranchant — Torsion)," Comité Européen du Béton, *Bulletin d'Information* No. 92, (Paris), June 1973.

4. Gesund, H.; Mills, D. G.; and Martin, V. M.; "Ultimate Strength Tests of Reinforced Concrete Beams in Combined Torsion, Bending and Shear," *Publications,* International Association for Bridge and Structural Engineering, V. 28/II, 1968, pp. 31-48.

5. Elfgren, L., "Reinforced Concrete Beams Loaded in Combined Torsion, Bending and Shear (A Study of Ultimate Load-Carrying Capacity)," *Publication* 71:3, Chalmers University of Technology, Division of Concrete Structures, Goteborg, Sweden, 1972.

6. "CEB-FIP Model Code for Concrete Structures," Comité Euro-International du Béton, Paris, 1978.

7. ACI Committee 318, "Building Code Requirements for Reinforced Concrete and Commentary (ACI 318-89/ACI 318R-89)," American Concrete Institute, Detroit, 1989, 353 pp.

8. Collins, M. P. and Lampert, P., "Redistribution of Moments at Cracking — The Key to Simpler Torsion Design?" *Publication* 71-21, Department of Civil Engineering, University of Toronto, Feb. 1971, 49 pp.

9. "Design of Concrete Structures for Buildings," CAN3-A23.3-M84, Canadian Standards Association, Rexdale, 1984.