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Shear Strength of Deep Beams


by K. N. Smith and A. S. Vantsiotis

Results of tests on 52 deep reinforced concrete beams under two equal symmetrically placed point loads are reported. The investigation's objectives were to study the effect of vertical and horizontal web reinforcement and shear span-to-effective depth ratio on inclined cracking shear, ultimate shear strength, midspan deflection, tension reinforcement strain, and crack width.

Test results indicated that web reinforcement produces no effect on formation of inclined cracks and seems to moderately affect ultimate shear strength. Addition of vertical web reinforcement (ϱ_v = 0.18 to 1.25 percent) improves ultimate shear strength of deep beams. However, addition of horizontal web reinforcement (ϱ_{vh} = 0.23 to 0.91 percent) had little or no influence on ultimate shear strength. Considerable increase in load-carrying capacity was observed with increasing concrete strength and decreasing shear span-to-effective depth ratio.

Keywords: beams (supports); compressive strength; cracking (fracturing); **deep beams**; failure; loads (forces); reinforced concrete; reinforcing steels; **shear strength**; span-depth ratio; **web reinforcement**.

Because of their proportions, the strength of deep beams is usually controlled by shear, rather than flexure, provided normal amounts of longitudinal reinforcement are used. On the other hand, deep beams' shear strength is significantly greater than that predicted using expressions developed for slender beams, because of their special capacity to redistribute internal forces before failure and develop mechanisms of force transfer quite different from beams of normal proportions. As reported in literature[1-3] for beams with ordinary shear span-to-effective depth ratios ($a/d > 2.5$), inclined cracking shear is essentially ultimate shear capacity of a beam without web reinforcement. However, in deep beams $a/d < 2.5$ without web reinforcement and loaded directly on the top or compression face, ultimate strength far exceeds inclined cracking shear.[4-9]

Still no accurate theory exists for predicting ultimate shear strength of deep reinforced concrete beams. The greater number of parameters affecting beam strength has led to a limited understanding of shear failure. These parameters include the proportions and shape of the beam, loading and support conditions, amount and arrangement of tensile, compressive, and web reinforcement, as well as the concrete and steel properties. This investigation was undertaken to provide more information on shear failure and study web reinforcement's effect on ultimate shear strength and behavior of deep reinforced concrete beams.

TEST PROGRAM

Fifty-two simply supported deep reinforced concrete beams were tested to failure in the laboratory investigation. They consisted of four series A, B, C, and D of 15, 16, 19, and 2 beams, respectively. Ratios of shear span to effective depth a/d of 0.77, 1.01, 1.34, and 2.01 were used for each series, respectively. All beams tested had a rectangular cross section. Each beam was loaded directly on the top compression face with two equal concentrated loads four inches from the midspan and supported at the bottom (Fig. 1). The ends of all beams extended 12 inches (305 mm) beyond the supports' centerline to provide adequate anchorage for the longitudinal steel in the concrete. The longitudinal steel consisted of straight bars without bent ends or hooks; different types of reinforcement used are shown in Fig. 2. Bearing plates of 4 × 4 × 1 in. (102 × 102 × 25 mm) were used at the supports and the two points of loading.

Five beams were without web reinforcement. The remaining 47 beams contained both horizontal and vertical reinforcement. Vertical web reinforcement was made up of closed U-shaped stirrups (#2 bars), while horizontal web reinforcement consisted of straight #2 bars. Vertical and horizontal web reinforcement was uniformly spaced in each beam, but different spacings were used for different beams. Physical properties of all beams tested are shown in Table 1. A constant rate of loading used for all beams was approximately 2 kip/min (8.90 kN/min).

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Designation of test specimens

Each test specimen is described by five characters. The first character is the number 0, 1, 2, 3, or 4 indicating the number of vertical stirrups in the shear span. The second character is the letter A, B, C, or D defining the four beam series of different a/d ratios of 0.77, 1.01, 1.34, and 2.01, respectively. The third character is the number 0, 1, 3, 4 or 6, representing the number of horizontal bars used for horizontal web reinforcement (bars used for longitudinal tension or compression reinforcement are not included). The last two characters are numbers from 01 to 52, indicating the sequence number in which the beam was tested.

Materials and testing details

The concrete mix contained high-early-strength portland cement and local river aggregates. The main properties (by weight) per batch were: cement 42 to 44 lb (19 to 20 kg), sand 192 lb (87.1 kg), gravel 192 lb (87.1 kg), water 32 to 40 lb (14.5 to 18.1 kg); maximum aggregate size ½ in. (12.7 mm). The water amount was varied to obtain approximately the same workability and slump for each batch. Concrete strengths are based on 6 × 12 in. (150 × 300 mm) cylindrical test specimens, cast and tested simultaneously with the beam. High strength deformed bars were used in all test beams. A summary of the properties of the reinforcing bars used is given in Table 2. Specially manufactured steel molds were used to cast the specimens, and the concrete was compacted by a vibrator. All beams were cured for 7 or 8 days before testing. The beams were painted white on one face to facilitate crack observation.

Measurements

All tests were performed by using a closed loop MTS testing system. The test setup is shown in Fig. 3. Beams were loaded in 10 kip (44.5 kN) increments corresponding to 5 kip (22.23 kN) increments of shear in the shear spans. At each load increment the total applied load on the beam, two end reactions, midspan deflection, maximum crack width, and longitudinal steel strains at midspan and points of load application were measured. The cracks were then plotted and marked. The procedure was repeated for a number of load stages, until failure of the specimen occurred. A test was terminated when the total load on the specimen started to significantly drop off. Total load input and midspan deflection were also recorded continuously through the test and plotted on an x-y plotter. A photograph showing the cracking pattern was taken for each beam at the end of each test.

TEST RESULTS

Behavior under load and failure mode

All 52 beams tested failed in shear. All beams were taken to failure, i.e., the span collapsed due to excessive destruction of concrete in the shear span. Photographs of test specimens that show typical observed cracking patterns and failure mode are shown in Fig. 4 and 5. The numbers written along the cracks on the photographs indicate the termination of cracks observed at the end of a particular load stage. The large numbers represent the specimen number. No cracking was observed in any beam up to about 20 percent of ultimate load. The first vertical flexural cracks were formed in the region of maximum bending moment. Between 40 and 50 percent of the ultimate load a sudden major inclined tension crack formed almost in the middle of the shear span. Inclined tension crack angle with respect to the horizontal plane was about 50 to 60 deg for series A, 45 to 50 deg for Series B, 40 to 45 deg for Series C, and 35 to 40 deg for Series D beams. In some beams these inclined cracks appear to initiate by flexural cracks that also originated at about the shear span's middle. With increasing load the in-

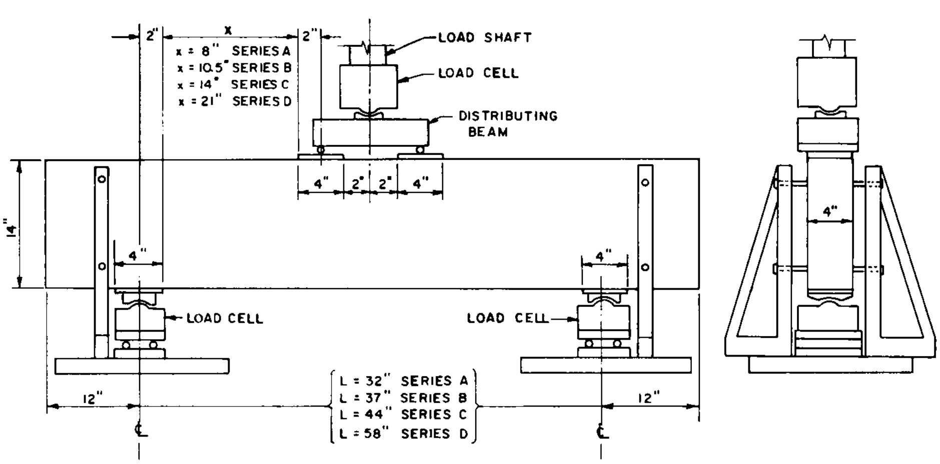

Fig. 1 — Loading and supporting conditions for test beams

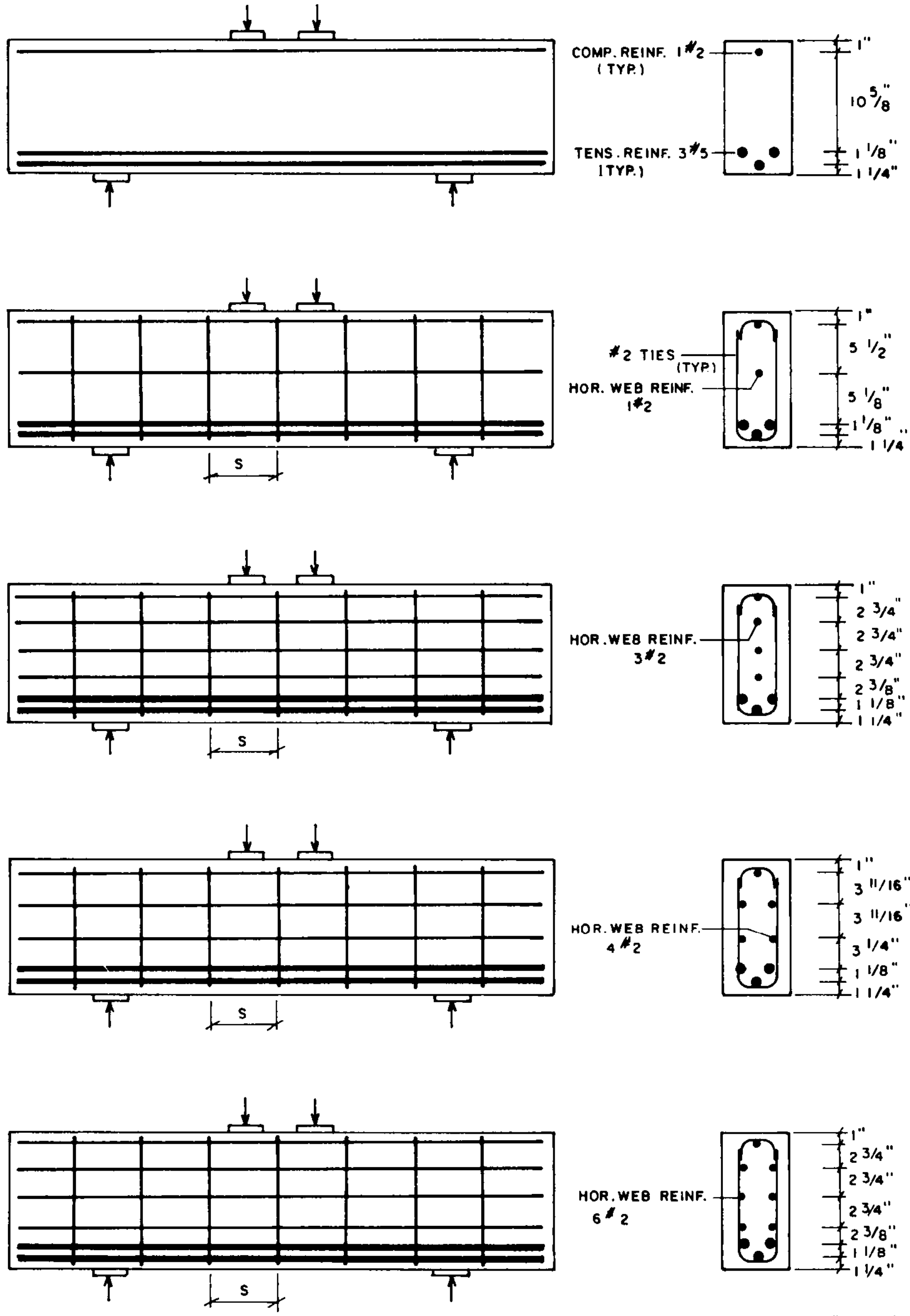

Fig. 2 — Types of web reinforcement used

clined crack propagated backwards until it reached the beam bottom at the support block's edge. In the meantime, the crack propagated forwards until it reached a distance equal to about 0.20 of the total depth from the top of compression zone, underneath the loading point, and stopped there. With further increase in the applied load, the existing vertical flexural and inclined shear cracks propagated very slowly while new inclined cracks were formed parallel to the original inclined cracks in the shear span. An almost stable position of all existing cracks was observed at approximately 60 to 70 percent of ultimate load. The stable position was identified by sudden shortening of the load increment marks on the cracks that were easy to identify on the beams. At about 85 to 90 percent of ultimate load, new inclined cracks were formed parallel to the line joining the load edge and support blocks. Also at about the same load level a tension vertical crack appeared over the supports. This is the result of the thrust's eccentricity which essentially acts along the inclined crack. Finally, beam failure occurred by concrete destroyed in either the reduced compression zone at the head of the inclined crack and the region adjacent to the loading block, or by fracture of the concrete along the inclined

Table 1 — Physical properties of beams tested

Beam	a/d	ℓ_n/d	f'_c, psi	Vertical web reinforcement: Bar size	A_v, in.^{2}	s, in.	$\varrho_v = \frac{A_v}{sb}$	Horizontal web reinforcement: Bar size	A_{vh}, in.^{2}	s_2, in.	$\varrho_{vh} = \frac{A_{vh}}{s_2 b}$
Series A beams											
0A0-44	0.77	2.33	2970	—		—	—	—	—	—	—
0A0-48	0.77	2.33	3035	—		—	—	—	—	—	—
1A1-10	0.77	2.33	2710	#2	0.10	9	0.0028	#2	0.05	5.50	0.0023
1A3-11	0.77	2.33	2615	#2	0.10	9	0.0028	#2	0.05	2.75	0.0045
1A4-12	0.77	2.33	2330	#2	0.10	9	0.0028	#2	0.10	3.70	0.0068
1A4-51	0.77	2.33	2980	#2	0.10	9	0.0028	#2	0.10	3.70	0.0068
1A6-37	0.77	2.33	3055	#2	0.10	9	0.0028	#2	0.10	2.75	0.0091
2A1-38	0.77	2.33	3145	#2	0.10	4	0.0063	#2	0.05	5.50	0.0023
2A3-39	0.77	2.33	2865	#2	0.10	4	0.0063	#2	0.05	2.75	0.0045
2A4-40	0.77	2.33	2950	#2	0.10	4	0.0063	#2	0.10	3.70	0.0068
2A6-41	0.77	2.33	2775	#2	0.10	4	0.0063	#2	0.10	2.75	0.0091
3A1-42	0.77	2.33	2670	#2	0.10	2	0.0125	#2	0.05	5.50	0.0023
3A3-43	0.77	2.33	2790	#2	0.10	2	0.0125	#2	0.05	2.75	0.0045
3A4-45	0.77	2.33	3020	#2	0.10	2	0.0125	#2	0.10	3.70	0.0068
3A6-46	0.77	2.33	2890	#2	0.10	2	0.0125	#2	0.10	2.75	0.0091
Series B beams											
0B0-49	1.01	2.75	3145	—	—	—	—	—	—	—	—
1B1-01	1.01	2.75	3200	#2	0.10	10.50	0.0024	#2	0.05	5.50	0.0023
1B3-29	1.01	2.75	2915	#2	0.10	10.50	0.0024	#2	0.05	2.75	0.0045
1B4-30	1.01	2.75	3020	#2	0.10	10.50	0.0024	#2	0.10	3.70	0.0068
1B6-31	1.01	2.75	2830	#2	0.10	10.50	0.0024	#2	0.10	2.75	0.0091
2B1-05	1.01	2.75	2780	#2	0.10	6	0.0042	#2	0.05	5.50	0.0023
2B3-06	1.01	2.75	2755	#2	0.10	6	0.0042	#2	0.05	2.75	0.0045
2B4-07	1.01	2.75	2535	#2	0.10	6	0.0042	#2	0.10	3.70	0.0068
2B4-52	1.01	2.75	3160	#2	0.10	6	0.0042	#2	0.10	3.70	0.0068
2B6-32	1.01	2.75	2865	#2	0.10	6	0.0042	#2	0.10	2.75	0.0091
3B1-08	1.01	2.75	2355	#2	0.10	4	0.0063	#2	0.05	5.50	0.0023
3B1-36	1.01	2.75	2960	#2	0.10	3.25	0.0077	#2	0.05	5.50	0.0023
3B3-33	1.01	2.75	2755	#2	0.10	3.25	0.0077	#2	0.05	2.75	0.0045
3B4-34	1.01	2.75	2790	#2	0.10	3.25	0.0077	#2	0.10	3.70	0.0068
3B6-35	1.01	2.75	2995	#2	0.10	3.25	0.0077	#2	0.10	2.75	0.0091
4B1-09	1.01	2.75	2480	#2	0.10	2	0.0125	#2	0.05	5.50	0.0023
Series C beams											
0C0-50	1.34	3.33	3000	—	—	—	—	—	—	—	—
1C1-14	1.34	3.33	2790	#2	0.10	14	0.0018	#2	0.05	5.50	0.0023
1C3-02	1.34	3.33	3175	#2	0.10	14	0.0018	#2	0.05	2.75	0.0045
1C4-15	1.34	3.33	3290	#2	0.10	14	0.0018	#2	0.10	3.70	0.0068
1C6-16	1.34	3.33	3160	#2	0.10	14	0.0018	#2	0.10	2.75	0.0091
2C1-17	1.34	3.33	2880	#2	0.10	8	0.0031	#2	0.05	5.50	0.0023
2C3-03	1.34	3.33	2790	#2	0.10	8	0.0031	#2	0.05	2.75	0.0045
2C3-27	1.34	3.33	2800	#2	0.10	8	0.0031	#2	0.05	2.75	0.0045
2C4-18	1.34	3.33	2965	#2	0.10	8	0.0031	#2	0.10	3.70	0.0068
2C6-19	1.34	3.33	3010	#2	0.10	8	0.0031	#2	0.10	2.75	0.0091
3C1-20	1.34	3.33	3050	#2	0.10	4.5	0.0056	#2	0.05	5.50	0.0023
3C3-21	1.34	3.33	2400	#2	0.10	4.5	0.0056	#2	0.05	2.75	0.0045
3C4-22	1.34	3.33	2650	#2	0.10	4.5	0.0056	#2	0.10	3.70	0.0068
3C6-23	1.34	3.33	2755	#2	0.10	4.5	0.0056	#2	0.10	2.75	0.0091
4C1-24	1.34	3.33	2840	#2	0.10	3.25	0.0077	#2	0.05	5.50	0.0023
4C3-04	1.34	3.33	2690	#2	0.10	4	0.0063	#2	0.05	2.75	0.0045
4C3-28	1.34	3.33	2790	#2	0.10	3.25	0.0077	#2	0.05	2.75	0.0045
4C4-25	1.34	3.33	2685	#2	0.10	3.25	0.0077	#2	0.10	3.70	0.0068
4C6-26	1.34	3.33	3080	#2	0.10	3.25	0.0077	#2	0.10	2.75	0.0091
Series D beams											
0D0-47	2.01	4.50	2830	—	—	—	—	—	—	—	—
4D1-13	2.01	4.50	2330	#2	0.10	6	0.0042	#2	0.05	5.50	0.0023

All beams contained 3 #5 bars as longitudinal tension reinforcement; $A_s = 0.93$ in.^{2}, $\varrho = \frac{A_s}{bd} 100 = 1.94$ percent.

All beams contained 1 #2 bar as longitudinal compression reinforcement; $A_s' = 0.05$ in.^{2}, $\varrho' = \frac{A_s'}{bd} 100 = 0.10$ percent.

All beams had an overall depth h = 14 in., an effective depth d = 12 in., and a width b = 4 in.
1 psi = 6.895×10^{-3} MPa; 1 in. = 25.4mm.

crack. Crushing always occurred at a position other than the region of maximum moment.

The failure mode of all beams was similar. No significant modes of failure changes were observed between beams with or without web reinforcement. Observed patterns of the inclined cracks gave the appearance of a tied-arch system to the beams, with the tension reinforcement acting as the tie rod and portions of the beams outside the inclined cracks as compression struts. This arch-type behavior was apparent in all test specimens with or without web reinforcement. Beams with web reinforcement exhibited considerably

Table 2 — Properties of reinforcing bars

Bar size	#2		#5		#5*	
Cross-sectional area A in.^{2} (mm^2)	0.05	(32.2)	0.31	(200)	0.31	(200)
Upper yield strength, ksi (MPa)	70.12	(483.5)	—		—	
Lower yield strength, ksi (MPa)	63.43	(437.4)	62.50	(431)	61.13	(421.5)
Ultimate tensile strength, ksi (MPa)	84.28	(581.1)	93.02	(641.4)	106.50	(734.3)
Strain at yield, percent	0.2349		0.2140		0.2170	
Strain at Ultimate, percent	16.30		11.14		10.66	

*Bars used only in beams 0A0-48, 0B0-49, 0C0-50, and 0D0-47.

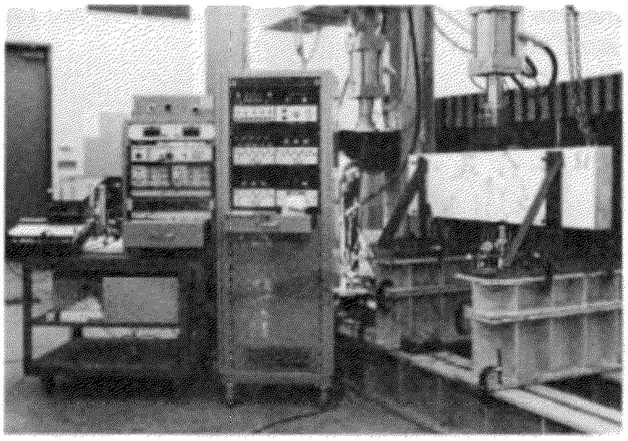

Fig. 3 — Test setup

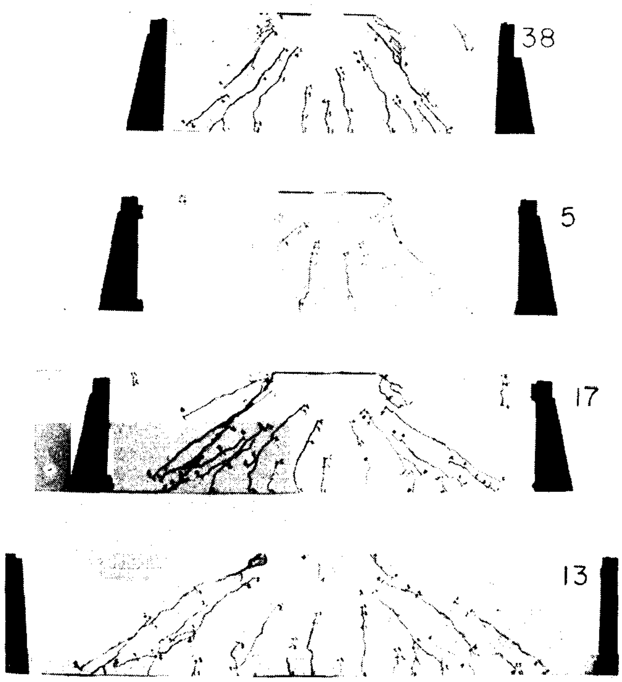

Fig. 5 — Beams with web reinforcement after failure

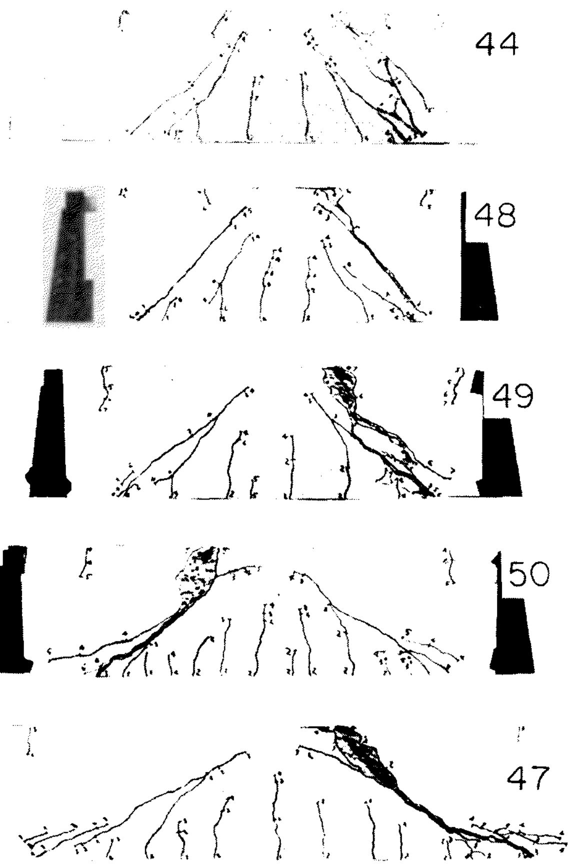

Fig. 4 — Beams without web reinforcement after failure

less damage at failure than beams without web reinforcement. Beams with web reinforcement exhibited more uniform cracking and smaller crack widths at corresponding load levels and failure. The longitudinal bars were properly anchored at the beam ends, preventing the bars from pulling out of the supports. All end anchorages functioned properly during testing and did not affect the mode of beam failure.

Inclined cracking and ultimate loads

Inclined cracking load is defined as the load at which the first major inclined tension crack appears in the shear span. This was a sudden crack that usually originated in the middle of the shear span and propagated toward the support and loading point from a subsequent increase in applied load. Results of all beams tested are summarized in Table 3. The load values are live load only; the beam's weight was less than 400 lb (181.5 kg). Inclined tension cracking load was observed to be considerably less than ultimate load for tested beams, with or without web reinforcement. Inclined cracking and ultimate loads (twice the shear) for the beams are plotted in Fig. 6 versus shear span-depth ratio a/d. Fig. 6 indicates a definite decrease in inclined cracking and ultimate loads with increasing a/d ratio.

Test results reported in literature[4,10-13] also show a large increase in shear capacity beyond the inclined cracking shear for $a/d < 2.5$. This increase in ultimate shear strength observed in deep beams $a/d < 2.5$ is mainly attributed to the arch action that seems to decrease with increasing a/d ratio.

Deflections

Total applied load (twice the shear) versus midspan deflection curves for beams without web reinforcement and typical curves for beams with web reinforcement are shown in Fig. 7. Inclined cracks had the greatest influence on the beam behavior. Formation of the first major inclined cracks significantly reduced beam stiffness, with this effect more evident in beams with higher shear span-depth ratios *(a/d)*.

In general there was little difference in deflection manner between beams of the same series. Deflections of beams with higher shear span-depth ratios were generally higher for corresponding load levels. However, midspan deflection at failure was less than $\ell n/200$ for tested beams.

Table 3 — Test results

Beam	Total load (twice the shear) Inclined crack, kips	Total load (twice the shear) Ultimate load, (kips)	Midspan deflection at failure, in.	Maximum crack width of failure ($\times 10^{-3}$ in.)	Maximum tensile steel strain ($\times 10^{-6}$ in./in.)	$\frac{(M_u)_T}{M_n}$
			Series A beams			
0A0-44	33.20	62.74	0.115	32	1878	0.69
0A0-48	29.20	61.20	0.113	32	1848	0.67
1A1-10	26.00	72.50	0.154	29	2072	0.83
1A3-11	25.20	66.70	0.132	28	1980	0.77
1A4-12	26.50	63.50	0.133	27	1840	0.77
1A4-51	35.00	76.86	0.140	30	2181	0.85
1A6-37	33.60	82.77	0.140	27	2339	0.91
2A1-38	32.20	78.46	0.145	30	2368	0.85
2A3-39	32.80	76.70	0.123	24	2267	0.86
2A4-40	32.20	77.30	0.119	27	2218	0.86
2A6-41	28.80	72.80	0.121	26	2135	0.82
3A1-42	27.00	72.40	0.125	28	2361	0.83
3A3-43	32.70	77.66	0.122	26	2272	0.88
3A4-45	25.85	80.28	0.137	25	2246	0.88
3A6-46	32.10	75.60	0.133	27	2108	0.84
			Series B beams			
0B0-49	25.50	67.00	0.173	59	2440	0.88
1B1-01	24.20	66.30	0.139	24	2258	0.87
1B3-29	24.70	64.55	0.148	29	1967	0.87
1B4-30	25.60	63.10	0.143	28	1979	0.84
1B6-31	25.80	68.95	0.142	27	2019	0.94
2B1-05	25.20	58.00	0.139	29	1968	0.79
2B3-06	27.40	59.00	0.127	27	1925	0.81
2B4-07	24.10	56.70	0.144	24	1915	0.80
2B4-52	25.80	67.40	0.145	37	2080	0.88
2B6-32	28.50	65.30	0.146	22	2070	0.88
3B1-08	23.90	58.80	0.155	22	2316	0.86
3B1-36	26.50	71.47	0.148	27	2387	0.96
3B3-33	25.40	71.20	0.153	25	2789	0.97
3B4-34	23.90	69.70	0.159	25	2560	0.95
3B6-35	26.10	74.70	0.163	25	2691	0.97
4B1-09	25.20	69.00	0.172	20	2506	0.96
			Series C beams			
0C0-50	22.25	52.00	0.207	60	2041	0.87
1C1-14	19.40	53.50	0.184	28	2740	0.91
1C3-02	26.90	55.50	0.169	28	2390	0.90
1C4-15	25.20	58.90	0.164	27	2780	0.95
1C6-16	24.50	55.00	0.168	28	2140	0.90
2C1-17	23.10	55.80	0.177	26	—	0.94
2C3-03	22.10	46.60	0.165	28	1804	0.79
2C3-27	19.00	51.85	0.157	30	2058	0.88
2C4-18	26.20	56.00	0.165	26	2190	0.93
2C6-19	22.10	55.80	0.175	24	—	0.92
3C1-20	23.40	63.30	0.210	27	2016	1.04
3C3-21	19.90	56.20	0.180	29	2280	0.99
3C4-22	23.80	57.40	0.183	18	2430	0.98
3C6-23	23.80	61.70	0.193	27	2170	1.04
4C1-24	24.60	65.90	0.220	28	2620	1.10
4C3-04	23.70	57.80	0.203	22	2380	0.98
4C3-28	23.10	68.50	0.218	27	—	1.15
4C4-25	23.00	68.60	0.210	24	2416	1.14
4C6-26	25.10	71.70	0.215	26	2446	1.17
			Series D beams			
0D0-47	17.00	33.00	0.225	60	1765	0.78
4D1-13	17.85	39.30	0.236	24	2364	0.94

1 kip = 4.448 kN; 1 in. = 25.4mm.

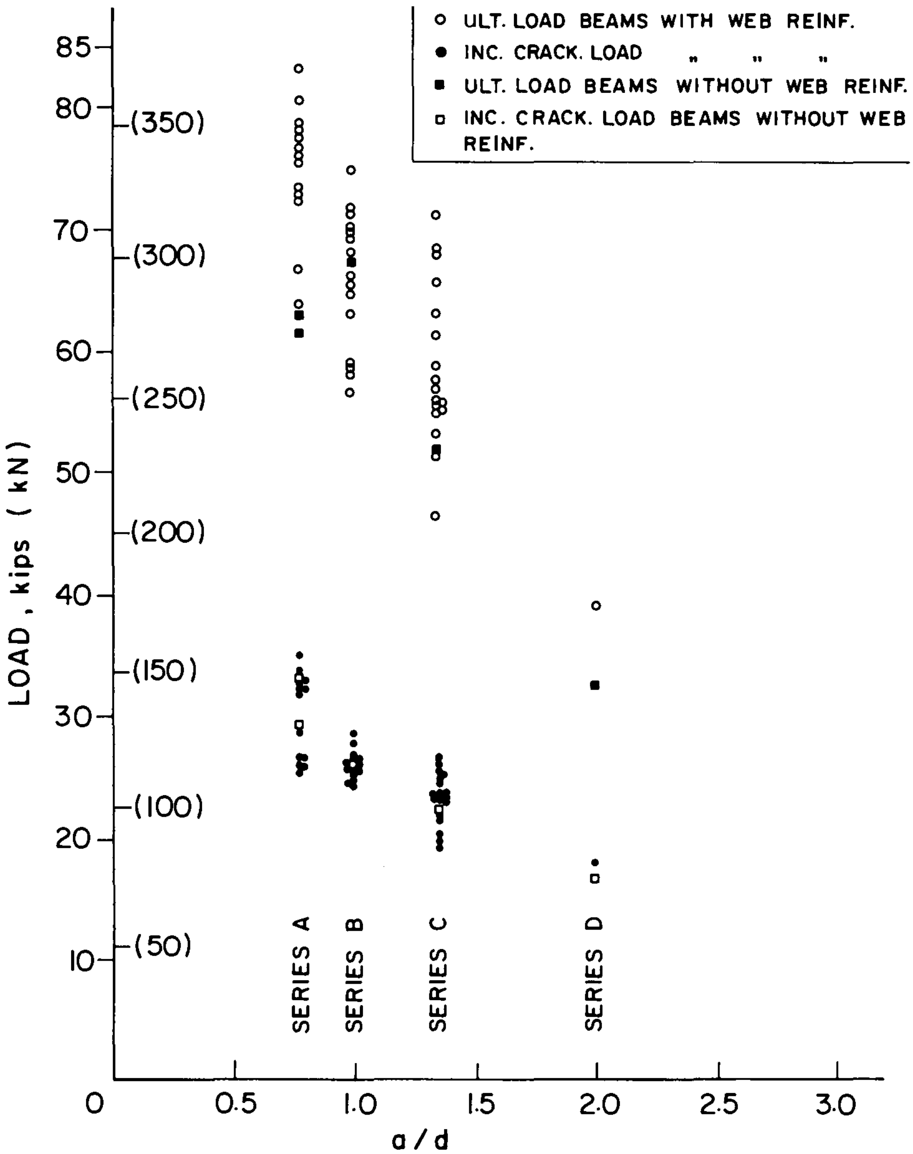

Fig. 6 — Ultimate and inclined cracking loads (twice the shear) versus a/d

Longitudinal steel strains

Test results indicate that strains in the region of maximum bending moment are almost uniform at every load level, and failure of test beams occurred near yielding of the main longitudinal bars. Tensile steel strains increased at almost a constant rate with no sudden increase observed at or after major inclined crack formation. No attempt was made to measure steel strains near the supports, the location of the major inclined cracks.

Crack width

Curves of the total applied load (twice the shear) versus the observed maximum crack width for beams without web reinforcement and typical curves for beams with web reinforcement are presented in Fig. 8. Tests indicate that crack widths tend to increase with load, especially at loads higher than inclined cracking loads. Cracks are almost uniform on both sides of the beam. Maximum crack widths along the major inclined crack in the shear span occurred almost at middepth of the beam. Beams without web reinforcement exhibited considerably larger crack widths at failure. Web reinforcement was effective in reducing crack widths at all corresponding load levels and particularly in beams with $a/d > 1.0$.

ANALYSIS OF TEST RESULTS

Effect of concrete strength

Studying concrete strength's effect on ultimate shear strength of deep reinforced concrete beams was not the intent of the investigation. However, during testing it was noted that within the same series, beams with higher amounts of web reinforcement but lower concrete strengths failed at lower loads than beams with lower amounts of web reinforcement and higher concrete strengths. Examples are beams 1A1-10 and 1A3-11; 2A1-38 and 2A3-39; 2A4-40 and 2A6-41; 3A4-45

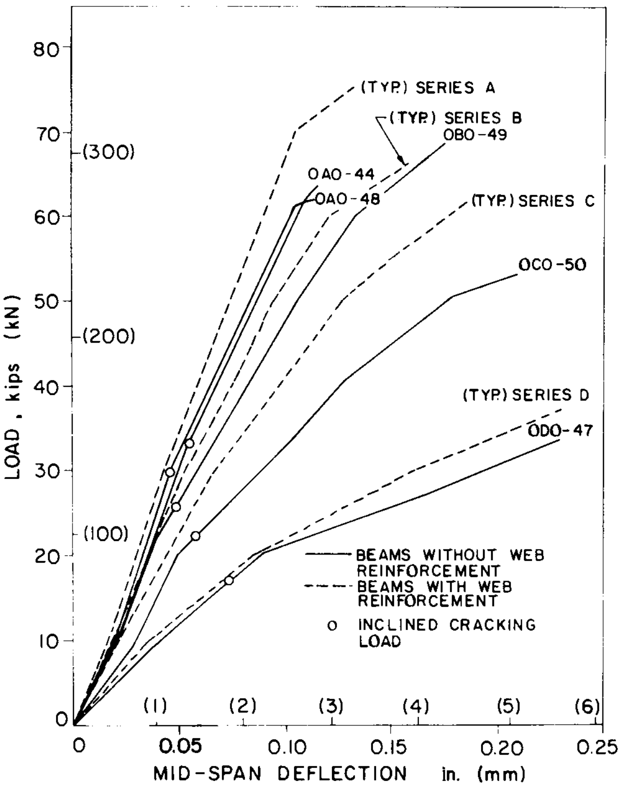

Fig. 7 — Load (twice the shear) versus deflection

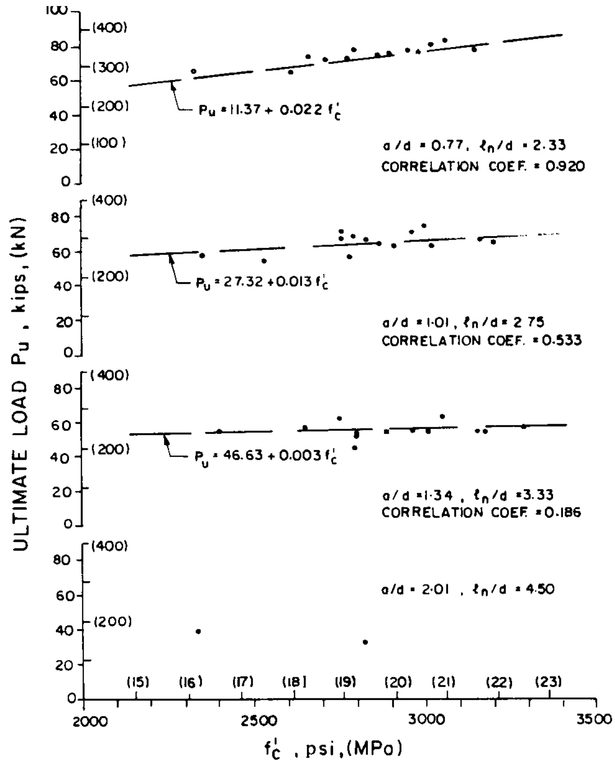

Fig. 9 — Ultimate load P_u *versus concrete compressive strength* f'_c

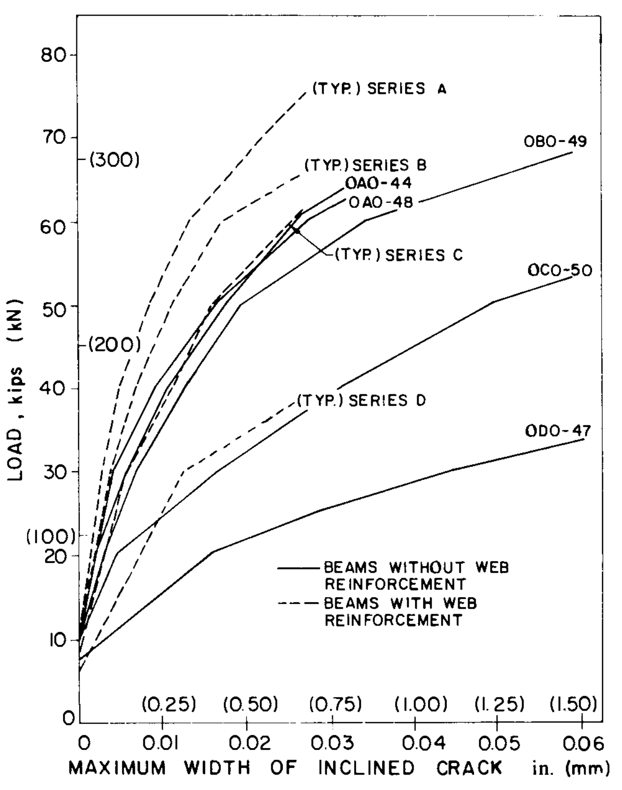

Fig. 8 — Load (twice the shear) versus maximum crack width

and 3A6-46; 2B4-07 and 2B4-52; 3C1-20 and 3C3-21; and 4C1-24 and 4C3-04 (Table 3). This observation indicates that concrete strength could have considerable influence on ultimate shear strength of deep reinforced concrete beams.

Plotting measured failure loads P_u (twice the shear) versus f'_c and $\sqrt{f'_c}$ for beams with constant a/d ratio (Fig. 9 and 10) shows good correlation exists between the test data. Results from a linear regression analysis indicate that plots of P_u versus f'_c result in higher correlation coefficients than plots of P_u versus $\sqrt{f'_c}$, especially at lower a/d ratios. Also, slopes of the plotted lines seemed to decrease significantly with increasing a/d ratios. This indicates that the influence of concrete strength on ultimate load capacity of deep reinforced concrete beams is more noticeable at lower a/d ratios and diminishes as the a/d ratio increases. This may be explained by the presence of dominant arching action in beams with low a/d ratios. The beam behaves as a short strut and tie system. Assuming that anchorage failure will not occur if adequate anchorage is provided at the supports for the main longitudinal steel, failure of the strut and tie system will be governed by concrete compressive strength f'_c and/or by yield strength of the main longitudinal steel. Therefore, before substantial yielding of the main longitudinal steel occurs in deep reinforced concrete beams failing in shear, ultimate capacity of such beams is proportional to concrete compresive strength f'_c . However, because this study was directed at web reinforcement's effect, the range of f'_c

in Fig. 9 and 10 is too restricted to be absolutely definite regarding concrete strength's effect.

Effect of web reinforcement

The problem is calculating web reinforcement's actual contribution when a member is required to resist a collapse load greater than the load that will cause collapse of the same member without web reinforcement. Use was made of ultimate shear strength of beams without web reinforcement to predict the actual contribution of vertical and horizontal web reinforcement to ultimate shear strength of deep beams with web reinforcement.

The following is assumed to be true for beams with web reinforcement

$$(V_u)_T = (V_c)_{CALC} + (V_s) \qquad \text{Eq. (1)}$$

where $(V_u)_T$ = measured ultimate shear strength of beam with web reinforcement
$(V_c)_{CALC}$ = shear strength due to concrete
(V_s) = shear strength due to web reinforcement

The term $(V_c)_{CALC}$ in Eq. (1) was calculated by using the following expression

$$(V_c)_{CALC} = \frac{(f'_c)_1}{(f'_c)_2} (V_u)'_T \qquad \text{Eq. (2)}$$

where $(f'_c)_1$ = concrete compressive strength of beam with web reinforcement
$(f'_c)_2$ = concrete compressive strength of beam without web reinforcement at same *a/d* ratio
$(V_u)_T'$ = measured ultimate shear strength of beam without web reinforcement at same *a/d* ratio

Rearranging Eq. (1) we can express web reinforcement's contribution V_s to the ultimate shear strength of deep beams as

$$V_s = (V_u)_T - (V_c)_{CALC} \qquad \text{Eq. (3)}$$

Eq. (3) was used to evaluate the shear carried by web reinforcement. Test results indicate that the ratio of web reinforcement's total contribution to ultimate shear strength of beams with web reinforcement $V_s/(V_u)_T$ varied from 0.0 to 0.30. This shows that web reinforcement increases ultimate shear strength of deep reinforced concrete beams. Maximum observed increase in ultimate shear strength was not more than 30 percent, even in beams with ϱ_v = 1.25 percent and ϱ_{vh} = 0.91 percent. Contribution of web reinforcement (vertical and horizontal) to ultimate shear strength never exceeded the limiting value of $4\sqrt{f'_c}\ bd$.

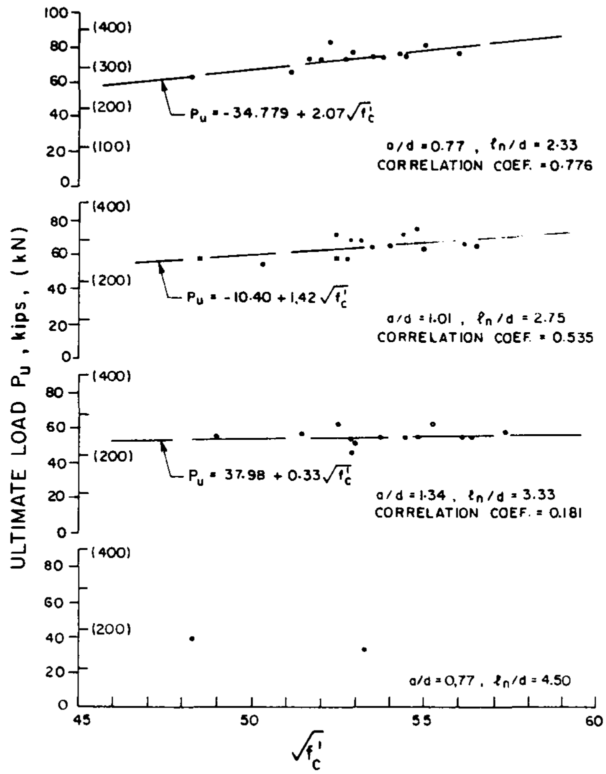

Fig. 10 — Ultimate load P_u *versus concrete compressive strength* $\sqrt{f'_c}$

Effect of vertical web reinforcement

Plots of V_s versus ϱ_v for constant values of ϱ_{vh} are shown in Fig. 11 to 13 for beams tested. Results of linear regression analysis are also shown on the figures. Correlation coefficients were generally high. The low correlation coefficients observed in some cases are mainly due to experimental error in the test data.

The plots in Fig. 11 to 13 indicate that:

1. In Series A beams, increasing vertical web reinforcement amount from ϱ_v = 0.28 to 1.25 percent seems to increase slightly total shear strength provided by web reinforcement V_s. This increase was more noticeable in beams with lower amounts of horizontal web reinforcement (ϱ_{vh} = 0.23 to 0.45 percent). In beams with higher amounts of horizontal web reinforcement (ϱ_{vh} = 0.68 to 0.91 percent), little or no increase was observed in V_s with increasing vertical web reinforcement.

2. In Series B beams vertical web reinforcement's effect on V_s was more pronounced than in Series A beams. Increasing vertical web reinforcement amount from ϱ_v = 0.23 to 0.77 percent considerably increased V_s for beams with horizontal shear ratios of up to ϱ_{vh} = 0.68 percent). A lower increase of V_s with increasing ϱ_v was seen in beams with a higher percentage of horizontal web reinforcement (ϱ_v = 0.91 percent).

3. In Series C beams, the influence of ϱ_v on V_s was more evident than either Series A or B beams. The increase of V_s with increasing values of ϱ_v was about the same for all beams with low (ϱ_{vh} = 0.23 percent) and

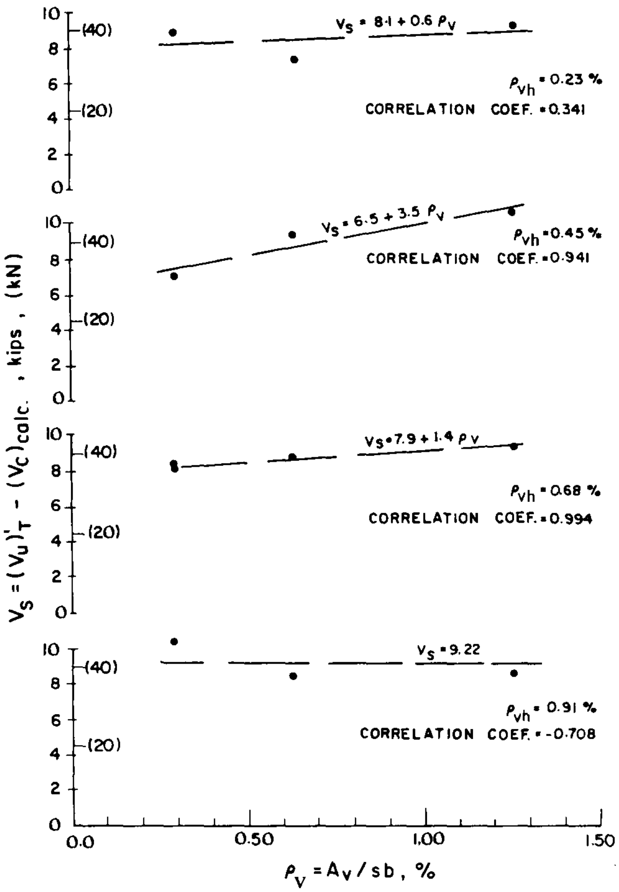

Fig. 11 — Influence of vertical web reinforcement, Series A (a/d = *0.77)*

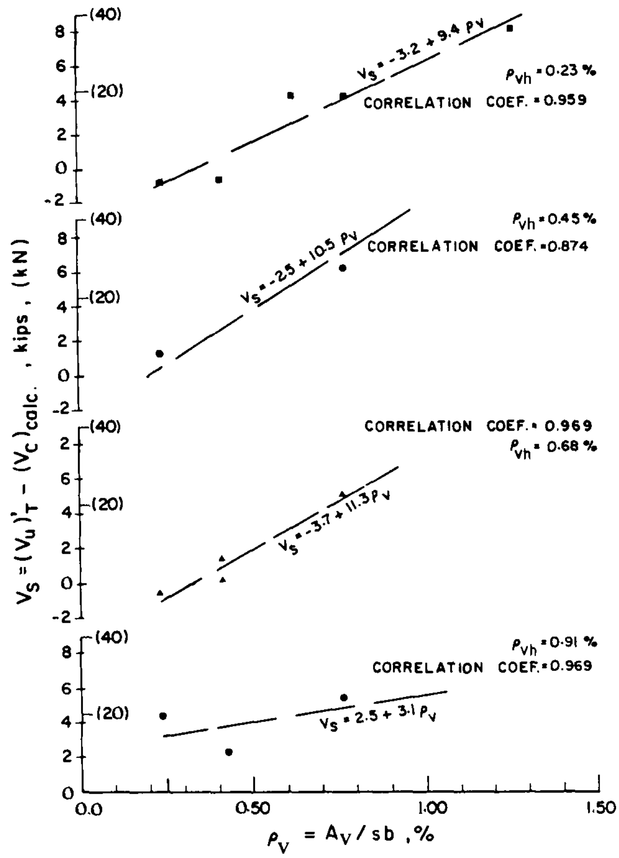

Fig. 12 — Influence of vertical web reinforcement, Series B (a/d = *1.01)*

high (ϱ_{vh} = 0.91 percent) amounts of horizontal web reinforcement.

In summary, Fig. 11 to 13 indicate that increasing vertical web reinforcement from ϱ_v = 0.18 to 1.25 percent increases total shear strength provided by web reinforcement V_s. The average increase observed in V_s with increasing ϱ_v was 1.40 ϱ_v, 8.60 ϱ_v, and 15.20 ϱ_v for Series A, B, and C, respectively. These values indicate that increase in V_s was small in Series A beams, but considerably higher in Series B and C beams. Therefore, it can be concluded that effectiveness of vertical stirrups diminishes as shear span-depth ratio *a/d* decreases. The *y* intercept in Fig. 11 to 13 indicates horizontal web reinforcement's contribution to V_s. By observing the change in value of the *y* intercept, horizontal web reinforcement's influence on V_s is shown to be more pronounced in Series A beams with $a/d < 1.0$.

Effect of horizontal web reinforcement

Plots of total shear strength provided by the web reinforcement V_s, calculated using Eq. (3) versus horizontal shear ratio percentage ϱ_{vh} for constant values of ϱ_v, are shown in Fig. 14 to 16 for beams tested. Results of a linear regression analysis are also shown. Correlation coefficients in general were significantly lower than coefficients obtained for plots of V_s versus ϱ_v.

The plots in Fig. 14 to 16 indicate that:

1. In Series A beams, a small increase in V_s was observed with increasing ϱ_{vh} and particularly in beams with low amounts of vertical web reinforcement ($\varrho_v \leqslant$ 0.63 percent). In beams with high amounts of vertical reinforcement (ϱ_v = 0.91 percent) no increase was observed.

2. In Series B beams, horizontal web reinforcement's influence on V_s was slightly higher than observed in Series A beams. Increasing horizontal web reinforcement amount from ϱ_{vh} = 0.23 to 0.91 percent slightly increased the V_s for beams with vertical shear ratios up to ϱ_v = 0.42 percent. A lower increase of V_s with increasing ϱ_{vh} was seen in beams with higher percentages of vertical web reinforcement (ϱ_v = 0.77 percent).

3. In Series C beams, little or no increase was observed in V_s with increasing amounts of horizontal web reinforcement from 0.23 to 0.91 percent.

Fig. 14 to 16 indicate that increasing horizontal web reinforcement ϱ_{vh} = 0.23 to 0.91 percent slightly increases total shear strength provided by web reinforcement V_s. The average increase observed in V_s with increasing ϱ_{vh} was 1.10 ϱ_{vh}, 3.80 ϱ_{vh}, and 1.10 ϱ_{vh} for Series A, B, and C, respectively. These values indicate that the increase in V_s with increasing ϱ_{vh} was considerably less than that observed with increasing ϱ_v.

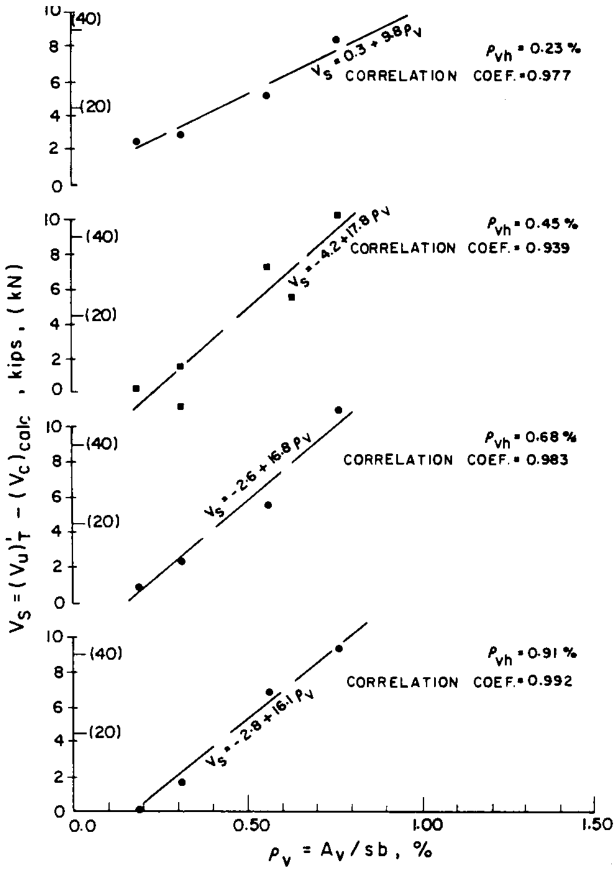

Fig. 13 — Influence of vertical web reinforcement, Series C, (a/d = 1.34)

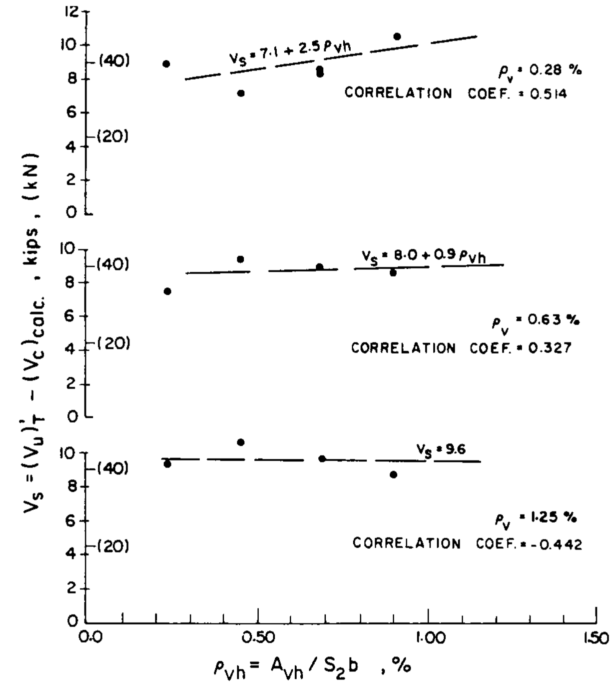

Fig. 14 — Influence of horizontal web reinforcement, Series A (a/d = 0.77)

Shear strength versus flexural strength

$(M_u)_T/M_n$ was computed for beams tested and is shown in Table 3. $(M_u)_T$ is the computed moment at failure based on the measured load input, i.e.,

$(M_u)_T = (V_u)_T$ (Distance from centerline of support to centerline of load)

M_n is the flexural strength computed according to conventional ultimate strength theory. The computed ratio $(M_u)_T/M_n$ varied from 0.67 to 1.17. The average ratio was 0.78 for beams without web reinforcement, 0.82 0.86 for Series D beams. These values show that all beams tested had not reached their full flexural capacity at failure. However, for a few beams in Series C with high amounts of vertical and horizontal web reinforcement, $(M_u)_T/M_n$ was approximately 1.0 or higher. This indicates that although these beams failed in a mode similar to all other beams, these beams had reached, for all practical purposes, their full flexural capacity.

Test results indicate that $(M_u)_T/M_n$ was lower for beams without web reinforcement. Increasing amount of vertical and horizontal web reinforcement increased $(M_u)_T/M_n$, showing that presence of web reinforcement seems to restore normal flexural action (beam action)

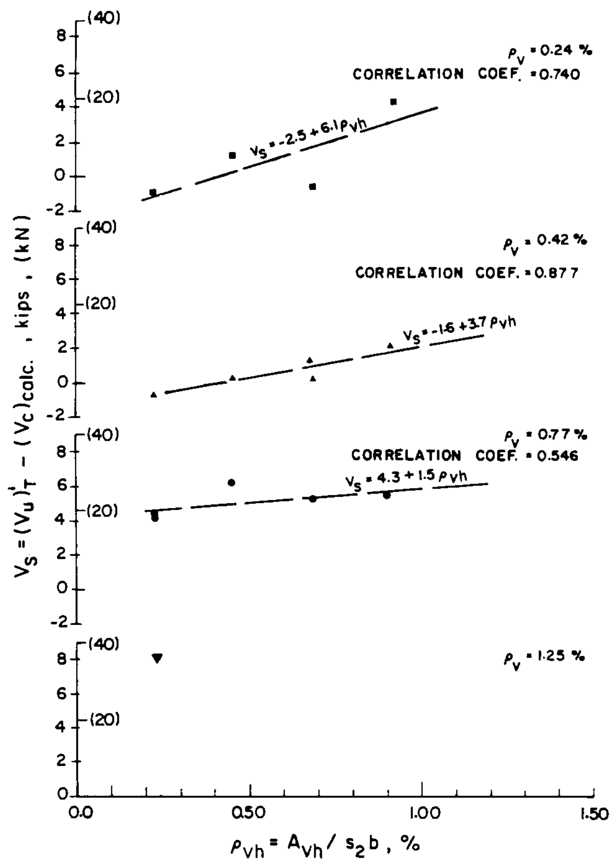

Fig. 15 — Influence of horizontal web reinforcement, Series B (a/d = 1.01)

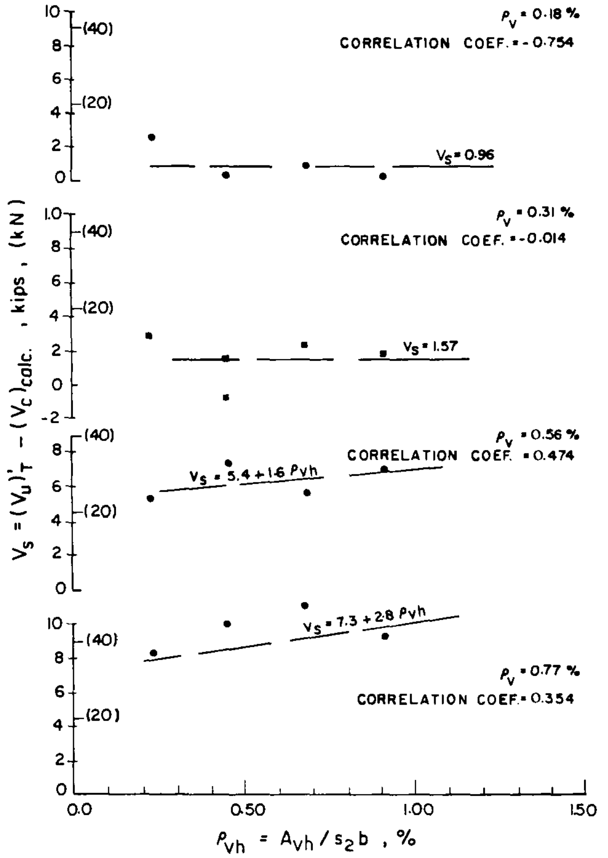

Fig. 16 — Influence of horizontal web reinforcement, Series C (a/d = 1.34)

and reduce arch action observed in deep reinforced concrete beams.

CONCLUSIONS

The following conclusions can be drawn based on the test results:

1. Beams generally failed in shear. Measured ultimate loads were considerably lower than the ultimate loads predicted by the flexural beam theory.

2. No significant change in failure mode was observed between different series of beams tested. Cracking patterns were essentially the same for beams with or without web reinforcement. However, less damage at falure was observed in beams with web reinforcement.

3. A significant decrease in beam stiffness was observed with the major inclined crack formation in the shear span. But presence of a minimum amount of vertical and horizontal web reinforcement ϱ_v = 0.18 percent and ϱ_{vh} = 0.23 percent) was effective to considerably reduce crack widths and deflections after inclined cracking. Therefore, a minimum percentage of web reinforcement should be used for crack control.

4. Inclined cracking loads were considerably lower than ultimate loads for beams with or without web reinforcement. Test results show that inclined cracking loads vary between 40 and 50 percent of the ultimate loads.

5. Presence of vertical (ϱ_v = 0.18 to 1.25 percent) and horizontal (ϱ_{vh} = 0.23 to 0.91 percent) web reinforcement had no effect on inclined cracking load.

6. In general, web reinforcement increased ultimate shear strength for all beams tested. Addition of up to 1.25 percent and 0.91 percent horizontal web reinforcement increased ultimate shear strength by not more than about 30 percent. Web reinforcement's contribution to ultimate shear strength never exceeded the limiting value of $4\sqrt{f_c'}\,bd$.

7. Presence of vertical web reinforcement increases ultimate shear strength of deep beams. However, vertical stirrups' effectiveness seems to diminish for beams with $a/d < 1.0$.

8. Horizontal web reinforcement appears to have little influence on the ultimate shear strength. Its influence is more noticeable in beams with $a/d < 1.0$.

9. Increasing concrete strength increased the beam's ultimate load (twice the shear) capacity. This increase was more pronounced in beams with low a/d ratios and seems to diminish as a/d ratio increases.

A subsequent paper will compare the test results with present building code provisions for design of deep reinforced concrete beams.

ACKNOWLEDGMENT

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NOTATION

A_s = area of longitudinal tension reinforcement
A'_s = area of longitudinal compression reinforcement
A_v = area of vertical shear reinforcement within distance s
A_{vh} = area of horizontal shear reinforcement within distance s_2
a = shear span, distance between concentrated load and face of the support[1] (distance between the center of concentrated load and center of support[4]), but not more than 1.15 the clear distance between the face of the support and loading blocks
b = width of member's compression face
d = distance from extreme compression fiber to centroid of tension reinforcement
f_c' = compressive strength of concrete measured using 6 in. by 12 in. cylinders, in psi
$\sqrt{f_c'}$ = square root of measured compressive strength of concrete, psi
f_y = reinforcement's yield strength
h = overall depth,
ℓ_n = clear span measured face to face of supports
M_n = nominal moment capacity
$(M_u)_T$ = computed moment at failure based on measured load input
P_{cr} = actual recorded inclined cracking load on beams
P_u = actual recorded ultimate load on beam (twice the shear)
s = spacing of vertical web reinforcement in direction parallel to longitudinal reinforcement
s_2 = spacing of horizontal web reinforcement in direction perpendicular to longitudinal reinforcement
V_{cr} = cracking shear
$(V_c)_{CALC}$ = shear strength provided by the concrete, calculated in accordance with Eq. (2)

V_s = shear strength provided by web reinforcement (vertical and horizontal) calculated in accordance with Eq. (3)
$(V_u)_T$ = measured ultimate shear strength of a beam with web reinforcement
$(V_u)_T'$ = measured ultimate shear strength of a beam without web reinforcement
ϱ = A_s/bd = longitudinal tension reinforcement ratio
ϱ' = A_s'/bd = longitudinal compression reinforcement ratio
ϱ_v = A_v/sb = vertical web reinforcement ratio
ϱ_{vh} = A_{vh}/s_2b = horizontal web reinforcement ratio

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