ACI STRUCTURAL JOURNAL TECHNICAL PAPER

Title no. 84-S39

Proposed Design Rules for Load Distribution in Precast Concrete Decks

by John Stanton

Design rules are described for determining the lateral distribution of concentrated vertical loads in decks made from precast concrete members. The rules are derived from the results of an analytical parameter study, and design examples show how to use them. These new rules are put forward for possible inclusion in Chapter 16 of ACI 318.

Keywords: building codes; finite strip method; **floors**; hollow core slabs; **loads (forces)**; orthotropism; **precast concrete**; **roofs**; structural analysis; **structural design**.

Precast concrete decks, particularly those made from hollow-core units, are widely used for roofs and floors. Design of individual members within the deck is relatively straightforward when the load is uniformly distributed, and design aids are available both from manufacturers and in the *PCI Design Handbook*.[1] However, when point, line, or patch loads are applied, the extent to which individual members share in carrying them is less obvious. (A patch load is a uniform load applied over a limited area.) This paper concerns the development of simple analytical procedures for determining such interaction. The methods proposed are derived from the results of an extensive study[2] using both analytical and experimental methods. The analysis was based on the assumption of linear elastic behavior.

The behavior of thin plates in the linear elastic range is well understood.[3] However, the behavior of hollow-core decks is complicated by the lack of full-moment continuity at the longitudinal joints between members, by their slightly different flexural stiffnesses in the two orthogonal directions, and by the deflections due to distortions of the individual cells.

Several studies have addressed the problem of analysis. Many[4-6] have used orthotropic plate theory,[3] often modeling the assumed hinges between members by assigning zero transverse bending stiffness to the plate (articulated plate theory), whereas others have treated the members as separate line elements[7,8] connected into a gridwork. The field of bridge engineering has provided a wealth of data, well summarized in Reference 9, but most of it concerns determination of response to nominal truck loading rather than to an arbitrarily placed point load. Tests[2,4,6,10-12] have demonstrated that considerable load-sharing can be achieved, leading to significant economies if taken into account.

ANALYTICAL MODEL

Some transverse moment can be carried across the grout joint between members, particularly in wide decks in which the members are prevented to some extent from moving apart by friction forces at the supports. A poured-in-place structural topping or transverse ties provide similar restraint. However, the extent of this partial moment continuity is uncertain, and although empirical efforts have been made to quantify it,[4] it is ignored here in the interests of simplicity and safety.

The present analysis was performed using the finite strip method.[13] Each member was modeled as a plate made up from a number of interconnected strips, and the plates were connected together by rotational hinges with full-shear continuity.[14] The ends were simply supported, and the lateral edges were free. This modeling is an improvement over the orthotropic plate theory because the hinges are represented discretely rather than being smeared over the members, making it possible to distinguish between the responses of decks made from wide and narrow planks.

Shear flexibility of the plates was introduced in the transverse direction to describe approximately the deformations due to cell distortion. In the finite strip method, this can be achieved without great difficulty, requiring the addition of one degree of freedom per strip. This can be condensed out subsequently if desired at the expense of only minor restrictions on load

Received Sept. 11, 1986, and reviewed under Institute publication policies. Pertinent discussion will be published in the July-August 1988 *ACI Structural Journal* if received by Mar. 1, 1988.

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placement. The introduction of shear flexibility into orthotropic plate theory complicates the calculations significantly, and in the special case of articulated plate theory (the usual choice), it is impossible.

In any method based on Fourier series, the accuracy depends on the number of terms used. Convergence is fastest for deflection, followed by moments (flexural and torsional), and is slowest for shear. The inclusion of shear flexibility slows convergence and requires more strips per plank. More terms are needed for loads or responses that are near the supports or are close together. Thus shear response near the support when the load is nearby usually gives the critical condition. One hundred terms were used for the analyses, and critical results were found to differ by less than 5 percent when compared with calculations using 500 terms.

The design procedures were developed from analyses of a particular deck, chosen both to represent practical conditions and to possess properties that would lead to design rules safe for other decks of common geometries. Properties were selected in light of results from a wider parameter study,[2] in which many different deck geometries and member properties were investigated after first verifying the analytical model against experimental evidence. The deck that was used as a standard was 48 ft (4.5 m) square, made of 12 in. (0.305 m) deep by 4 ft (1.2 m) wide planks with four 10 in. (254 mm) diameter circular voids.

It can be shown[2] that the maximum moment per unit width in a deck is a function of $\sqrt{I/J}$, where I and J are the flexural and torsional stiffnesses of the members. Fortunately $\sqrt{I/J}$ varies only slightly with plank depth-to-width ratio and with void size, shape and spacing, allowing the use of one set of member properties to represent all plank types and a concomitant reduction in the number of parameters to be considered in the simplified design rules. The error introduced is small compared to those caused by disregarding many other effects and, furthermore, an accurate calculation of J requires a special finite element that is unlikely to be readily available to designers or manufacturers.

By contrast, the transverse shear flexibility of the member is strongly influenced by the void shape. Rectangular voids undergo relatively large cell distortions (similar to the deformations of a Vierendeel girder) compared to those of roughly circular voids,[15] and the result is a greater concentration of response under the load. Thus the design rules proposed here apply only to planks with voids that are roughly circular or are shaped to cause less shear distortion (e.g., tall narrow voids).

Patch loads were used in all cases. The use of a true point load in plate theory leads to finite deflections but locally infinite moments and shears at the load. This is a feature of thin plate theory and is not peculiar to the finite strip method of analysis. Using a patch load (i.e., a uniform load applied over a small area) resolves the problem. Dimensions of the load footprint were chosen to be 0.02ℓ wide by 0.03ℓ long. These dimensions reflect the approximate dimensions of the region over which the load will be spread at middepth of the member and the fact that the spreading effects of the webs exist in one direction only. The 0.02ℓ width is close to the member depth and the void spacing in the chosen deck, so it essentially places all of the load over one web. This is believed to be a reasonable procedure in view of the fact that the local complex geometry around the void, including the out-of-plane bending of the thin material above and below the voids, is modeled in the analysis by smearing out the properties into an equivalent plate. While this gives reasonable results at some distance from the load, the less representative modeling close to it demands that the loading be chosen so as not to be unsafe. The 0.03ℓ length reflects a real load about 4 in. (102 mm) long on the surface, spreading out at 45 deg through the thickness of the deck to give approximately a 0.03ℓ loaded length at the mid-depth of the member.

DEVELOPMENT OF DESIGN PROCEDURES

Results of the analyses are conveniently expressed in terms of distribution widths. The total response to the given load is calculated and is then distributed laterally over a nominal distribution width. The members falling within that width are then designed to resist that part of the distributed response that is tributary to them.

In previous uses of the concept, variation across the distribution width of either load or response has been treated as uniform. This has drawbacks. First, it might lead to the erroneous impression that the response really is distributed that way. Fig. 1 shows the variation of moment per unit width at midspan across the standard deck when a concentrated load is placed in the middle. It is clearly far from being uniform, but is more nearly described by a decaying exponential. A second problem arises with the common choice of an assumed rectangular variation in decks where the distribution width is about three planks wide. If two concentrated loads are placed with one unloaded plank between them, the assumption of a rectangular variation would lead to the unrealistic conclusion that the nominally unloaded plank would have to be designed to resist twice the moment in a loaded plank. Superposition of two of the true diagrams confirms what might be expected; namely, that the most intense response occurs in the loaded planks.

A better representation can be obtained by treating the response as if it were distributed in a triangular fashion across the deck. The distribution width must then be twice that used in the rectangular diagram so that a given concentrated load will still lead to the same maximum response per unit width. The triangular dia-

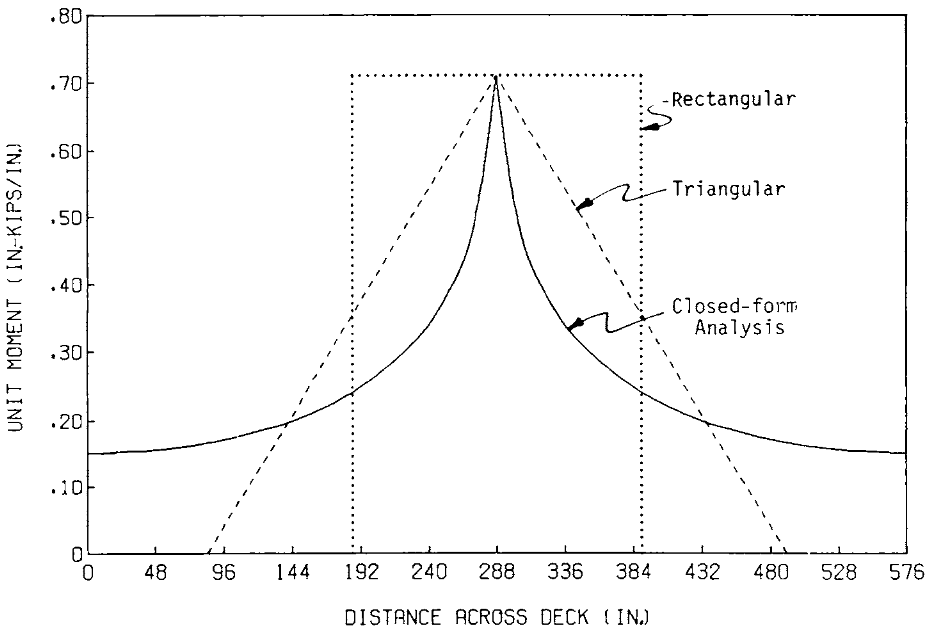

Fig. 1 — Lateral variation of unit longitudinal bending moment

gram alleviates the artificial problems caused by overlapping of the distribution widths of two adjacent loads. It also resembles more closely the true shape, and so is adopted here.

A second aspect of the problem is that the load, rather than the response, has traditionally been spread over the distribution width, leading unavoidably to the same distribution width for all response types (moment, shear, etc.). Inspection of the plate equations[3] shows that those response quantities for which the solution converges fastest are also spread out the most widely. Thus shear is the most concentrated and deflection is the most uniformly distributed response. This can be seen in Fig. 2, which shows lateral variation of deflection, longitudinal moment, and longitudinal shear. Each is nondimensionalized by dividing through by the maximum value of that particular response quantity. Because deflection readings are the easiest to obtain, test programs have tended to rely heavily on them in deriving estimates of response. Fig. 2 shows that a design for shear based on distribution factors derived from deflection data could be significantly unsafe. Separate expressions for each response type are used here.

It was also found that the maximum response intensity reduced (and so the distribution width increased) when the longitudinal distance between the load and response points was greater. The expression for the prescribed distribution width becomes slightly more complicated if this effect is included, but the penalty for ignoring it (a factor of 2 to 3 in flexure and much more in shear) is very large. For moment, and to a lesser extent for shear, the distribution width is also a function of the response location along the span.

For moment, both effects can be accounted for by defining the basic distribution width as

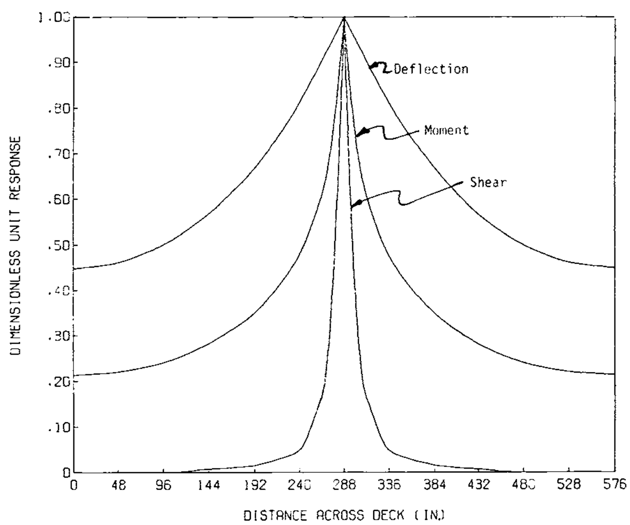

Fig. 2 — Lateral variation of dimensionless unit response

$$B_m = C_1\ell[0.14 + 2.25\xi_R(1 - \xi_R)] \quad (1)$$

where

$$C_1 = 1 + 6\xi_{LR} \qquad \xi_{LR} \leqslant 1/12 \quad (2a)$$

$$C_1 = 1.375 + 1.5\xi_{LR} \qquad \xi_{LR} > 1/12 \quad (2b)$$

$$\xi_{LR} = |\xi_L - \xi_R|$$

ξ = x/ℓ and subscripts L and R refer to load and response

x = longitudinal distance from one end of the span

Eq. (1) was obtained by fitting a parabola through the computer-generated results. In the common case in which load and response coincide, the coefficient C_1 is

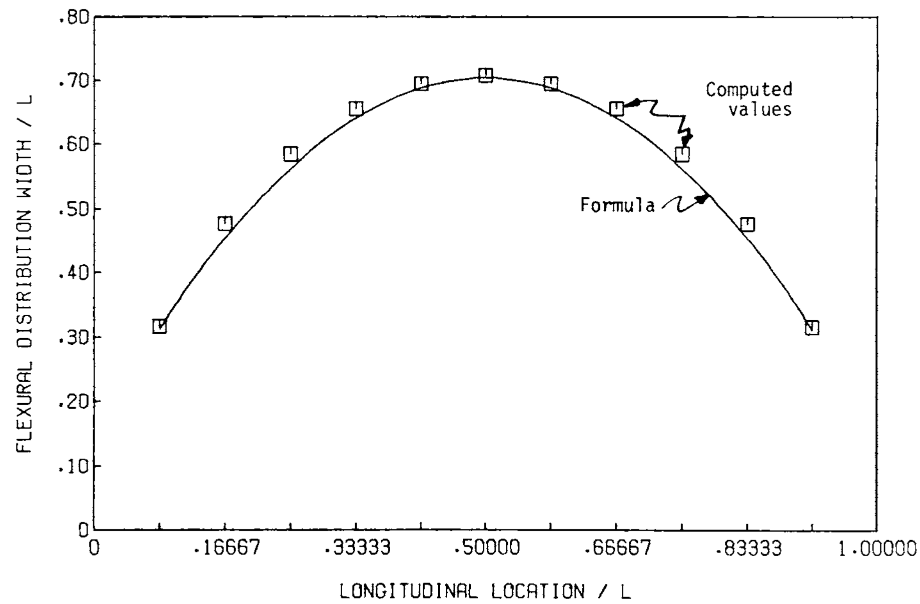

Fig. 3 — Varation of distribution width for longitudinal bending moment with longitudinal response location (response at load location)

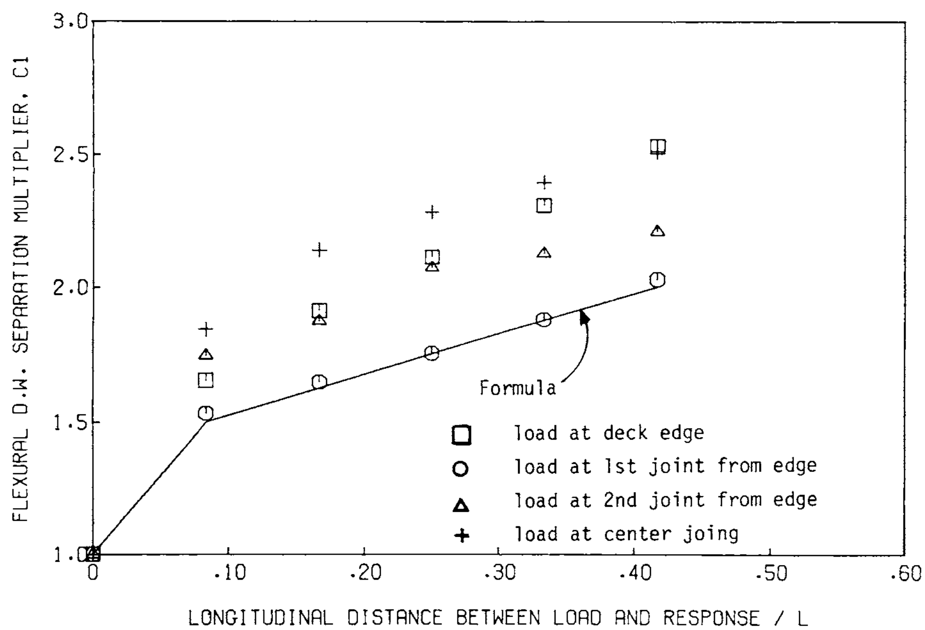

Fig. 4 — Variation of distribution width for longitudinal bending moment with distance between load and response

equal to 1.0. Further, if they are both at midspan, the distribution width is 0.70ℓ (this corresponds to a width of 0.35ℓ if the variation across the deck is assumed to be rectangular rather than triangular). That the distribution width should be a linear function of ℓ can be demonstrated theoretically[2] using orthotropic plate theory. This is true for moment and deflection but not for shear.

The predictions of these equations are compared with the true distribution widths in Fig. 3 and 4, and the agreement can be seen to be good. They err on the side of conservatism at large separations of load and response locations, but this is not a serious penalty because such load cases are unlikely to prove critical.

In the interior of the deck the moments are highest when the load is applied at a joint. For a very wide deck in which the joints act like hinges, symmetry dictates that the response will double if the load is placed at the very edge of the deck instead of at an interior joint. For loads placed near the edge but not at it, the response will increase to some intermediate value. It is conceptually convenient to assume that the interior response value is valid for all load locations for which the assumed triangular distribution width lies entirely within the confines of the deck. A simple rule for dealing with the case where the distribution width partially overhangs the edge is to disregard the overhanging part of the response and to increase the intensity of the re-

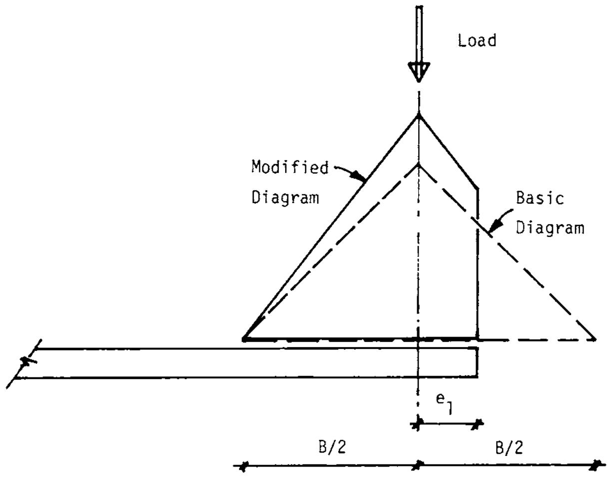

Fig. 5 — Method for edge correction

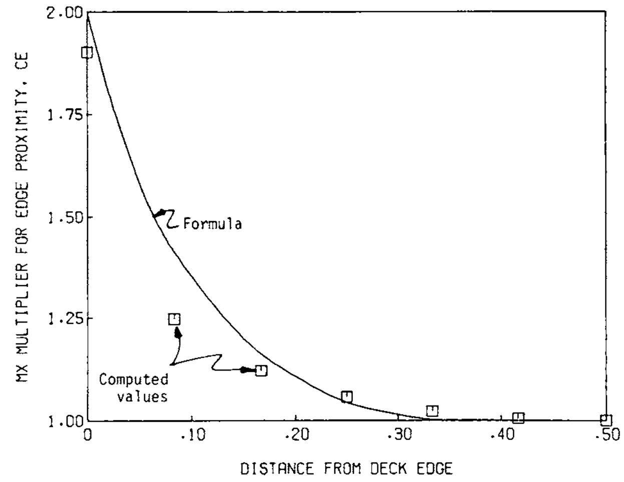

Fig. 6 — Edge coefficient C_e *for longitudinal bending moment*

mainder of the diagram so that equilibrium is maintained (i.e., the area under the diagram remains the same). This is illustrated in Fig. 5 and is achieved by multiplying the ordinates by a factor

$$C_e = 1/[1 - 0.5(1 - 2e_1/B)^2] \qquad (3a)$$

if the edge distance is less than $B/2$ on one side only, and

$$C_e = 1/\{1 - 0.5\,[(1 - 2e_1/B)^2 + (1 - 2e_2/B)^2]\} \qquad (3b)$$

if it is on both sides (i.e., the deck is narrower than the distribution width B). Here e_1 and e_2 are distances from the center of the load to the free edges of the deck, and B is the basic distribution width derived for the interior of the deck. The subscript to B is omitted here because Eq. (3a) and (3b) are later used for deflection and shear response as well.

The results of this approximation are shown in dimensionless form in Fig. 6, and they can be seen to give reasonable agreement with the true values. For the small region in which they are unconservative, the error is on the order of 2 percent.

Displacement response is somewhat simpler to evaluate. Using the same standard deck as for moments, and approximating the response as triangularly distributed across the deck, the basic distribution width B_d for midspan load and response is 1.25ℓ. For a load at quarter-span, the basic distribution width was found to be 1.12ℓ and 1.37ℓ for response at quarter-span and midspan, respectively. The dependence on response location and separation between load and response is thus slight, and the use of 1.25ℓ is recommended for all interior cases. It will underpredict deflections for coincident load and response when they are not at midspan, but by an amount that is probably less than that caused by other uncertainties. Furthermore deflections at locations other than midspan are seldom calculated or important. Thus

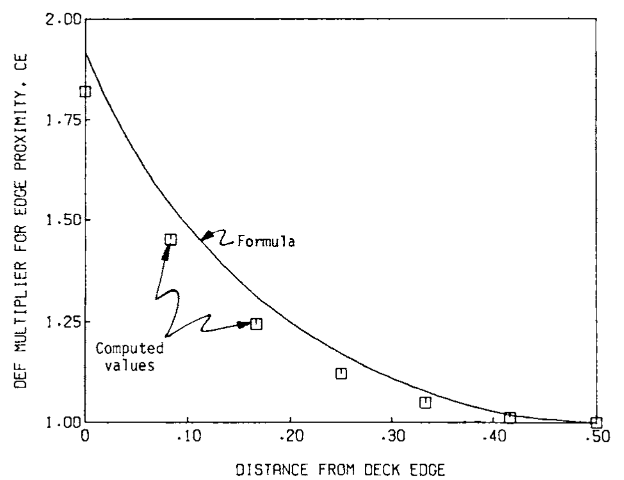

Fig. 7 — Edge coefficient C_e *for midspan deflection*

$$B_d = 1.25\ell \qquad (4)$$

Application of the concept of a distribution width to a deflection calculation is less intuitively obvious than for a moment calculation. One simple way is to calculate the deflection of one isolated member subjected to a concentrated load equal to $2b/B_d$ times the real value, where b is the member width. This is then the maximum deflection under the real load in the real deck, and deflections in the adjacent members can be calculated on the basis of the triangular variation across the distribution width.

Eq. (3a) and (3b) may be used to correct for cases where the basic distribution width overhangs the edge of the deck. (These will be quite common, at least involving all decks narrower than 1.25ℓ) Fig. 7 shows that they give a reasonable and slightly conservative result in all cases.

Shear is perhaps the most complex response. This is partly because the ACI Building Code[16] advocates evaluation of shear effects using forces rather than stresses,

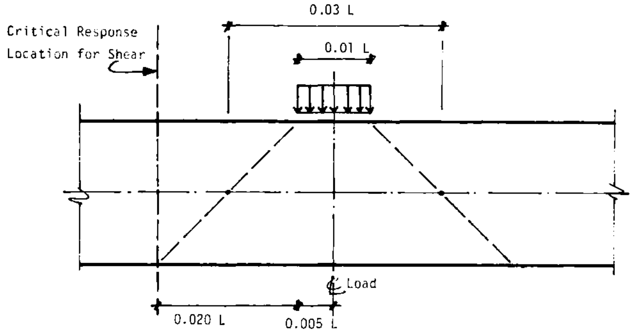

Fig. 8 — Critical location for shear

so that any response quantity such as torsion that causes shear stress needs to be turned into an equivalent shear. Torsion in reinforced concrete is often ignored if it arises from compatibility rather than equilibrium requirements and if the system is reasonably ductile. Hollow-core planks seldom contain shear reinforcement, so their shear ductility is limited and the inclusion of all effects causing shear stress is important. Even if the exact distribution of all shear stresses in the members was known, the question of evaluating the member's resistance to them is still not well resolved. However, it is a behavioral question that lies outside the scope of this study.

Questions of accuracy and interpretation also arise even in the elastic analysis for shear. Convergence of the Fourier series is slower for shear than any other quantity, particularly if the load and response are close together. The use of thin plate theory, on which the finite strip program used here was based, also poses its own problems of interpretation of shear results close to the load. The use of patch loads rather than true point loads is helpful in both cases.

In all cases, shear response was obtained at least 0.025ℓ from the center of the load. This corresponds approximately to the response being evaluated at a distance from the end of the load equal to the member depth. This is illustrated in Fig. 8, which is based on a span-to-depth ratio of 50. For deeper members this is closer than considered necessary by the ACI Building Code and leads to conservative estimates of imposed shear stress.

Fig. 9 shows calculated distribution widths for shear (ignoring torsion). The abscissa is the response location, and each curve shows the result for a different distance between load and response. It can be seen that the results are much more dependent on the separation between load and response than on the response location. For a given separation, the distribution width generally increases for response locations nearer the support. The local reduction right at the support is believed to be caused by slow convergence of the Fourier series, although a check using 500 rather than the standard 100 terms showed only a 3.5 percent change. The values obtained for response at midspan and at the support were nearly the same, so B_v was chosen to be independent of the response location.

Fig. 10 shows the variation of the distribution width with separation between load and response. A lower bound to these results is given by

$$B_v = 6.67\ S \text{ but not less than } 0.125\ell \tag{5}$$

where S is the distance from the face of the load to the point at which the shear is being evaluated. B_v is then taken as the basic distribution width, and the maximum shear force per unit width Q_v is given by

$$Q_v = 2V/B_v \tag{6}$$

where V is the total shear force at the reponse point. Eq. (3a) and (3b) were once again found to describe

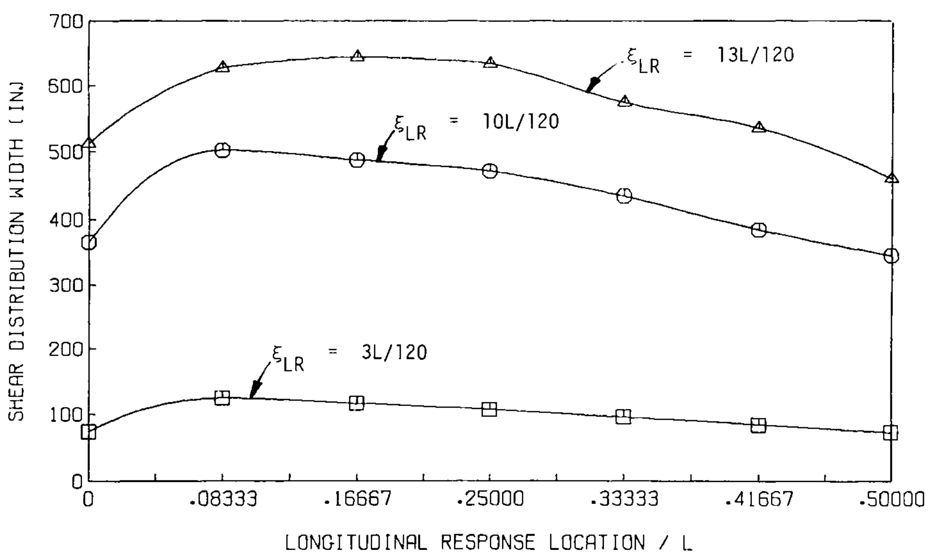

Fig. 9 — Variation of distribution width for longitudinal shear with longitudinal response location

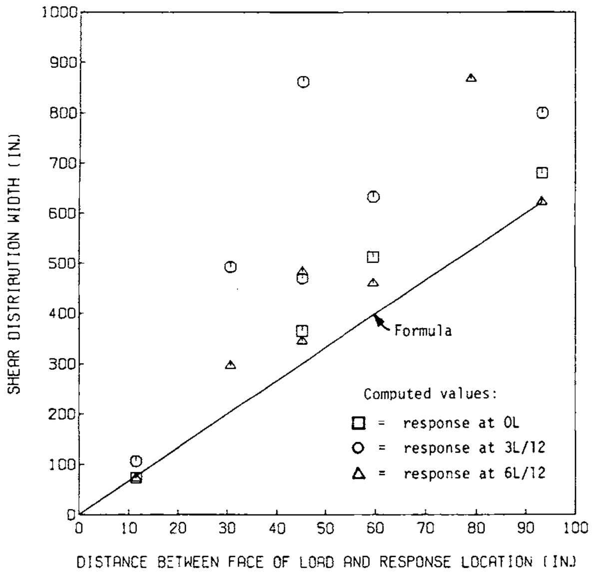

Fig. 10 — Variation of distribution width for longitudinal shear with distance between the response location and the face of the load

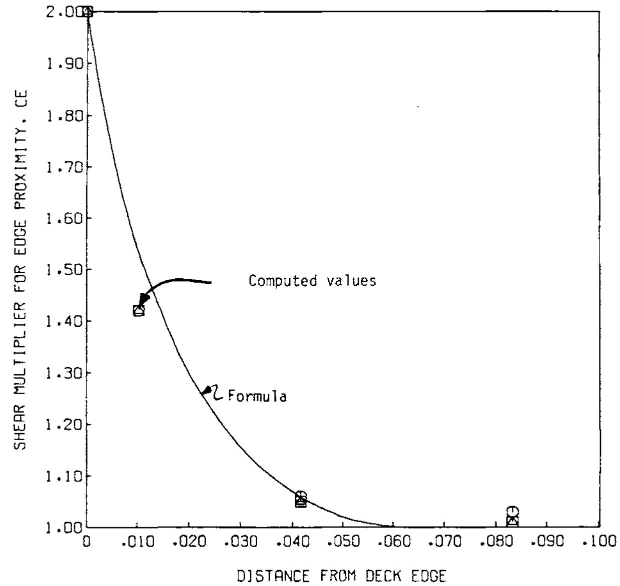

Fig. 11 — Edge coefficient C_e for longitudinal shear

adequately the correction for loads placed near the edge of the deck, as shown in Fig. 11.

Shear forces cause the maximum shear flow at the neutral axis, which is probably close to middepth where the webs are thinnest, so on both counts that location leads to the highest shear stress. It is given approximately by

$$\tau = V_1/h\Sigma t \tag{7}$$

where V_1 is the shear force applied to one member and Σt is the sum of all the web widths in the member.

Torsion causes shear stresses in the member webs, which may add to those caused by shear forces. They may be calculated by ignoring the inner webs. (This is conservative for members with four or more webs, but not very much since the inner webs have lower shear stresses and shorter lever arms and therefore carry a relatively small proportion of the total torque.) The torsional shear stress at any point around the perimeter is then given by

$$\tau = T/2At \tag{8}$$

where

T = torque
A = area enclosed by a line drawn along the mid-thickness of the flanges and outer webs
t = thickness of web or flange at the point of interest

The flanges of hollow-core planks are often thinner than the outer webs, so the maximum torsional shear stresses will occur over (or under) the innermost void. However, it is assumed that the latter will be exceeded by the combination of shear and torsional shear stresses in the outer web, which will therefore control the shear design. Assuming for a typical section

$$A = (0.95b) \times (0.85h) = 0.8\ bh \tag{9}$$

then

$$\tau = T/1.6bht \tag{10}$$

However

$$T = b\ m_{xy} \tag{11}$$

where m_{xy} = torsional moment per unit width and the maximum m_{xy} was found to be closely approximated by $0.22V$, so

$$\tau \doteq V/7.5\ ht \tag{12}$$

The shear force per unit width that would lead to the same shear stress in the outer web is $h\Sigma t/b$ times this value. Thus the effects of the torque could be replaced by a shear force per unit width Q_t which causes the same maximum shear stress

$$Q_t = 2V\,\Sigma t/15bt_1 \tag{13a}$$

where t_1 is the thickness of the outer web. A similar calculation for solid slabs yields

$$Q_t = 2V/[h\,(6-3h/b)] \tag{13b}$$

The torsion response differs from the others in that the total of all the resisting torsional couples is not a statically determinate quantity as is the case for moment and shear. In those cases a knowledge of the total static response and the maximum response per unit width leads to the definition of the distribution width. A distribution width for torsion B_t is still needed for use in Eq. (3a) and (3b) to account for the larger values en-

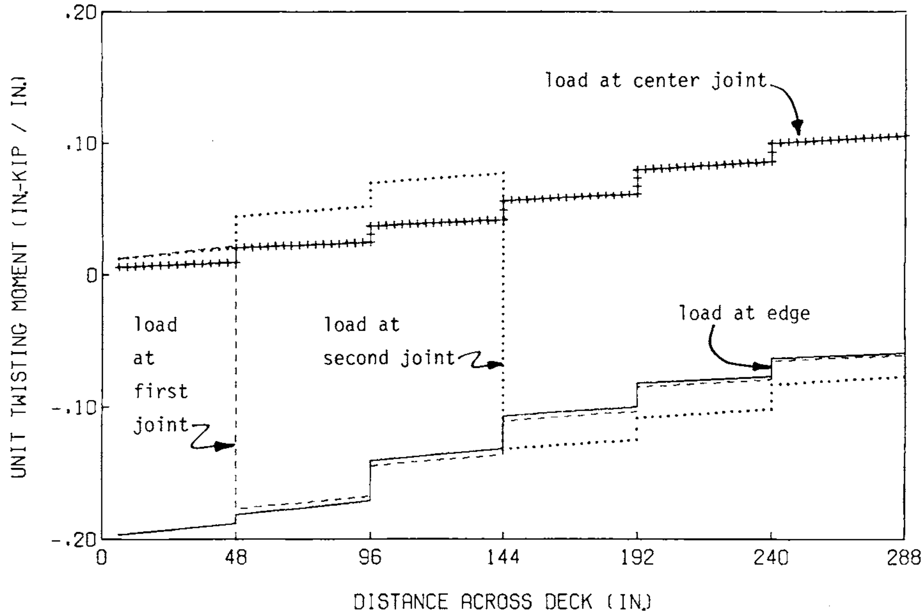

Fig. 12 — Lateral variation of distribution width for torsion for various load locations

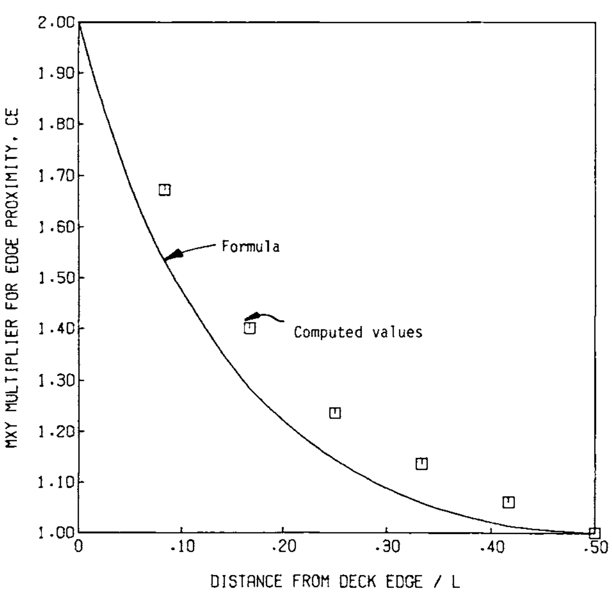

Fig. 13 — Edge coefficient C_e *for torsion*

countered near an edge. Fig. 12 shows the lateral variation of m_{xy} for various load locations, and the increase in $m_{xy,max}$ near the deck edge is clear. (The figure also shows that m_{xy} is nearly constant across any one member but is discontinuous across a joint.) A distribution width was derived by fitting Eq. (3a) to the $m_{xy,max}$ values. A perfect fit was not possible at all points, but taking B_t equal to 1.04ℓ gave a good match nearest the edge. It was rounded to 1.0ℓ and is shown in dimensionless form in Fig. 13.

The combined effects of shear and torsion may be calculated by

$$Q_{eq} = Q_v + Q_t$$

where Q_v and Q_t are as defined in Eq. (6) and (13), each modified by the appropriate edge correction factor C_e. For typical situations in which the shear is evaluated one member depth from the face of the load, Q_t was found to be on the order of one half of Q_v.

In the interior of the deck the outer web of each member will adjoin a grout key. For the grout key to work compositely with the web, it must be full and the interface must be sufficiently rough that shear stresses can be transferred across it without any slip. These conditions depend on very high standards of workmanship, so it would be prudent to ignore the thickness of the grout key when computing that of the outer web in calculations for shear and torsion.

The foregoing concepts are presented as design rules, expressed in language suitable for consideration for inclusion in a code or design handbook. A brief commentary accompanies them.

PROPOSED DESIGN RULES AND COMMENTARY

Design rules

1. In lieu of a rational analysis, response to concentrated forces perpendicular to the plane of the members may be determined in accordance with the procedures of following Paragraphs 2 through 7, for slabs that are solid or hollow core with approximately circular voids, have a span-to-depth ratio of 10 or greater and have no openings in the distribution width.

2. The total shear or bending response in any member shall be computed as the member width multiplied by the maximum response per unit width in that member.

3. The response per unit width shall be computed by adding the individual responses per unit width caused by the different loads. Response to a line load may be treated by considering it as a series of point loads. The effects of shear and torsion shall be combined and treated as an equivalent shear in accordance with the procedure of Paragraph No. 5.

4. Basic distribution widths B_d, B_m, and B_v for deflection, bending moment, and shear shall be computed as

$$B_d = 1.25\,\ell$$
$$B_m = C_1\ell\,[0.14 + 2.25\xi_R(1 - \xi_R)]$$
$$B_v = 6.67\,S \text{ but not less than } 0.125\,\ell$$
$$B_t = \ell$$

where

$C_1 = 1 + 6\xi_{LR}$ if $\xi_{LR} \leq 1/12$
$C_1 = 1.375 + 1.5\xi_{LR}$ if $\xi_{LR} > 1/12$
$\xi_{LR} = |\xi_L - \xi_R|$
ξ_L = distance of load from one support divided by span length
ξ_R = distance of response from the same support divided by span length
ℓ = span length
S = the distance between the face of the load and the section at which the shear is being calculated

5. If the center of the concentrated load is further than one half of the appropriate basic distribution width from the nearest free edge, the distribution of that response shall be taken as triangular, with the apex of the triangle at the load and extending half a distribution width each side. The maximum response per unit width shall be given by $2M/B_m$ for moment and $2V(\,1/B_v + \Sigma t\,/\,15\,b\,t_1\,)$ for shear, except that the expression $\Sigma t/(15bt_1)$ shall be replaced by $1/[h(6-3h/b)]$ for solid slabs. The maximum deflection shall be computed as that caused by a concentrated load $2b/B_d$ times the true value applied to an isolated member. In the foregoing

M = total static moment at the response location caused by the concentrated load
V = total static shear at the response location caused by the concentrated load
b = member width
Σt = sum of all web widths in one member
t_1 = width of outermost web

6. If the center of the concentrated load is located a distance e from a free edge, where e is less than one-half of the basic distribution width for that response type, then the triangular diagram prescribed in Paragraph 5 shall be truncated at the free edge and the intensity of the remainder of the diagram shall be divided by $[1 - 0.5(1 - 2e/B)^2]$ if $e <$ B/2 on one side only, and by $\{1 - 0.5[(1 - 2e_1/B)^2 + (1 - 2e_2/B)^2]\}$ if $e < B/2$ on both sides.

Here e, e_1 and e_2 are distances from the center of the load to the free edges of the deck and B is the basic distribution width for that response type.

7. The basic distribution widths for deflection and bending moment, B_d and B_m, may be increased by 10 percent if the deck width is greater than twice the span. For deck widths between ℓ and 2ℓ the increase may be calculated using linear interpolation.

Commentary

Precast concrete decks have uncertain transverse moment continuity at the joints between members, so safe design requires that the joints be treated as hinges. The structural action is relatively complex because of this articulation.

Studies[2] have shown that in any deck subject to concentrated load, deflections are more uniformly distributed than moments, which are more uniformly distributed than shears. However, for shear and bending the distribution width increases significantly when response is considered at some longitudinal distance from the load. This is reflected in the equations for distribution widths.

If a concentrated load is applied at a joint between members, twisting occurs, inducing torsional shear stresses. For simplicity of calculation, these are converted to an equivalent unit shear component that is to be added to the true unit shear. They must be calculated separately and then added because the shear distribution width is directly proportional to the distance between load and response, whereas the torsional response is essentially independent of it.

The use of an assumed triangular transverse variation of response instead of the traditional rectangular one represents more closely the true shape and avoids the artificial problems that arise when two loads are placed on alternate members and their distribution widths overlap on the unloaded member between. However, for a single concentrated load, the maximum unit response is the same for both shapes.

The simplified rules presented here may be unsafe in cases where the transverse shearing deformations (caused by cell distorsions) become significant. The limitations on span/depth and void shape prevent their use in such cases.

The correction factors of Paragraph 6 are approximate and are based on the requirement that the area under the unit response diagram must always be equal to the total response.

DESIGN EXAMPLES

Example 1 (General)

A 12-in. hollow-core floor is shown in Fig. 14. Find the contribution of the concentrated load to the moment under the center of the load, the shear 12 in. from the face of the load and the midspan deflection. 30 kip service load. Appropriate load factors for strength design to be applied separately.

Moment at 14 ft from support:

$\xi_{LR} = 0$
$C_1 = 1.0$
$\xi_R = 14/42 = 0.333$
Deck width/length = 60/42 = 1.428
Wide deck correction = (1.428 − 1) × 0.1 + 1 = 1.0428
$B_m = 1.0 \times 1.0428 \times (0.14 + 2.25 \times 0.333 \times 0.667) \times 42$ ft = 28.0 ft
$M = 30$ kip × 14 × 28/42 = 280 ft-kips
$m_{max} = 2M/B_m = 2 \times 280/28 = 20$ ft-kips/ft
Moment per plank = 4 ft × 20 ft-kips/ft = 80 ft-kips

Shear 12 in. from face of load:

$B_v = 0.125\,\ell = 5.25$ ft
$V = 2/3 \times 30$ kips = 20 kips
$Q_v = 2V/B_v = 2 \times 20/5.25 = 7.619$ kips/ft

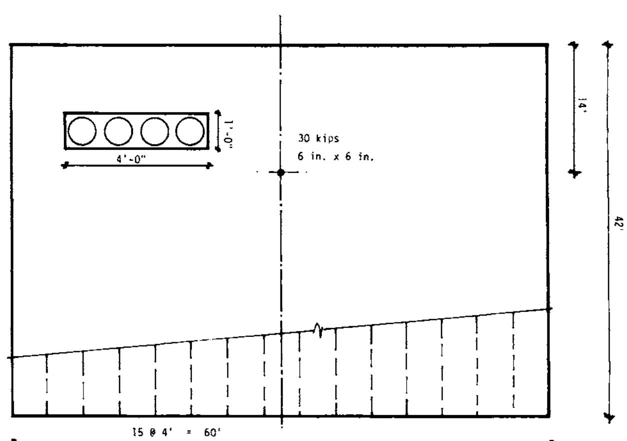

Fig. 14 — Deck for Example 1

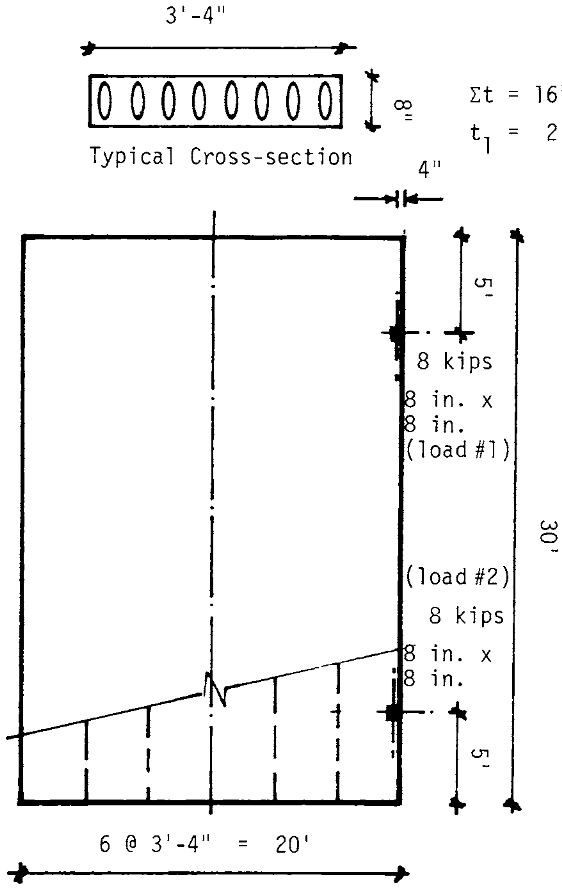

Fig. 15 — Deck for Example 2

$Q_t = 2V\Sigma t/(15bt_1) = 40 \times 8/(15 \times 4 \times 1.75) = 3.048$ kips/ft

$Q_{eq} = Q_v + Q_t = 7.619 + 3.048 = 10.667$ kips/ft

Shear per plank $= b\,Q_{eq} = 4 \times 10.667 = 42.666$ kips

(This is larger than the load, indicating that the shear is distributed locally over less than one plank width.)

Midspan deflection:

For an isolated beam $\Delta = \dfrac{23\,P\ell^3}{1296\,EI} = 2.272$ in.

$B_d = 1.25\,\ell = 52.5$ ft < 60 ft, so no edge correction needed

$\Delta = 2.272 \times 2 \times 4$ ft$/52.5$ ft $= 0.35$ in.

Example 2 (Edge loads and combined loading)

Eight in. hollow-core floor as shown in Fig. 15. Find the (unfactored) shear per plank 8 in. from the face of load No 1

$$e_1 = 4 \text{ in.} \qquad e_2 = 236 \text{ in.}$$

For load No. 1:

Shear: $S = 8$ in., $V = 6.67$ kip

$B_v = 6.67\,S = 6.67 \times 8 = 53.3$ in.

$C_e = 1/[1-0.5(1-2e/B_v)^2] = 1/[1-0.5(1-8/53.3)^2] = 1.565$

$Q_v = C_e 2V/B_v = 1.565 \times 2 \times 6.67/53.3 = 0.391$ kips/in.

Torsion: $B_t = \ell = 360$ in.

$C_e = 1/[1-0.5(1-2e/B_t)^2] = 1\ \ell\ [1-0.5(1-8/360)^2] = 1.916$

$Q_t = C_e\,2V\Sigma t/15bt_1 = 1.916 \times 2 \times 6.67 \times 16/(15 \times 40 \times 2) = 0.340$ kips/in.

$Q_{eq} = Q_v + Q_t = 0.391 + 0.340 = 0.731$ kips/in.

For load No. 2:

Shear: $S = 252$ in., $V = 1.33$ kips

$B_v = 1680$ in.

$C_e = 1/\{1-0.5[(1-2e_1/B_v)^2 + (1-2e_2/B_v)^2]\} = 1/\{1-0.5[(1-8/1680)^2 + (1-236/1680)^2]\} = 7.387$

$Q_v = C_e\,2V/B_v = 7.387 \times 2 \times 1.33/1680 = 0.012$ kips/in.

Torsion: $B_t = \ell = 360$ in.

$C_e = 1/\{1-0.5[(1-2e_1/B_v)^2 + (1-2e_2/B_v)^2]\} = 1.916$

$Q_t = C_e\,2V\Sigma t/15bt_1 = 1.916 \times 2 \times 1.33 \times 16/(15 \times 40 \times 2) = 0.068$ kips/in.

$Q_{eq} = Q_v + Q_t = 0.012 + 0.068 = 0.080$ kips/in.

For both loads combined:

$Q_{eq} = 0.731 + 0.080 = 0.811$ kips/in.

Shear per plank $= b\,Q_{eq} = 40 \times 0.811 = 32.5$ kips

Example 3 (Line loads)

Same deck as Example 1, but loading is a 1 kip/ft line load 42 ft long in the span direction in the middle of the deck. Find the midspan (unfactored) moment for which the most heavily loaded plank must be designed.

Simulate the line load with eight concentrated loads of 5.25 kips each, the first placed $\ell/16$ from the support. The m_x values for half of the loads are obtained in the table below; the others can be obtained using symmetry. For a typical load at ξ_L

$$M(\ell/2) = 0.5 \times \xi_L \times 5.25 \times \ell$$

$$m_x = 2M/B_m = 5.25 \times \xi_L/(B_m/\ell)$$

$16\xi_L$	$16\xi_{LR}$	C_1	B_m/ℓ	m_x
1	7	2.031	1.427	0.230
3	5	1.844	1.295	0.760
5	3	1.656	1.163	1.411
7	1	1.375	0.966	2.378
				4.779

$$m_x = 2 \times 4.779 = 9.557 \text{ ft-kips/ft}$$

$$\text{moment per plank} = bm_x = 4 \times 9.557 = 38.23 \text{ ft-kips}$$

But

$$M(\ell/2) = w\ell^2/8 = 220.5 \text{ ft-kips}$$

So the average distribution width (based on triangular moment variation) is

$B_{av} = 2 \times 220.5/9.557 = 46.144 \text{ ft} = 1.10\, \ell$

This is 56 percent larger than the value for a single concentrated load at midspan.

Example 4 (Loads in different locations)

Deck as shown in Fig. 16. Find the midspan moment for each plank in terms of P.

For each load:

$\xi_L = 0.4167$

$\xi_R = 0.5$

$\xi_{LR} = 0.0833$

$e_1 = 20$ ft

$e_2 = 28$ ft

$M(\ell/2) = 48 \text{ ft} \times 0.4167 \times (1-0.5)\, P = 10\, P$ ft-kips

$C_1 = 1 + 6/12 = 1.5$

$B_m = 1.5(0.14 + 2.25 \times 0.5 \times 0.5) \times 48 \text{ ft} = 50.88$ ft

$C_e = 1/[1 - 0.5(1 - 2e_1/B_m)^2] = 1.022$

$m_{x,max} = 2M/B_m = 2 \times 10P \times 1.022/50.88 = 0.402\, P$ ft-kips/ft

Plank number	1	2	3	4	5	6
$m_{x,max}/P$ (left)	0.148	0.212	0.276	0.340	0.404	0.340
$m_{x,max}/P$ (right)	0.020	0.084	0.148	0.212	0.276	0.340
$m_{x,max}/P$ (total)	0.168	0.296	0.424	0.552	0.680	0.680
M per plank/P	0.672	1.184	1.696	2.208	2.720	2.720

Transverse variation of unit moment is illustrated in Fig. 17, which also shows for comparison the results of using a rectangular variation over an arbitrarily chosen width of 0.5ℓ. (This value was chosen because it forms the basis for load distribution rules suggested by others.) It gives a moment in the most heavily loaded plank, which is 22.5 percent higher than that derived using the triangular variation.

The total required resisting moment is 22.4 P ft-kips. This is 12 percent higher than the total applied moment because each plank was designed for the maximum, rather than the average, moment per unit width tributary to it. However, in this case, if the average were used instead, the design moment for the most heavily loaded plank would not change, because the average and maximum moments per unit width are the same in the critical plank. Since all planks would probably be chosen to be identical, the design of the whole deck would be unaffected by the choice.

DISCUSSION

Exact solutions to the load-distribution problem are elusive and quite complex, depending on both the mathematical model and the numerical accuracy that can be obtained. Design rules can be distilled from them to any desired degree of accuracy and simplicity, but simplicity carries the penalty of conservatism. The rules presented here are an attempt to find a middle ground suitable for design office use. They are, in fact, more straightforward to use than they appear at first sight.

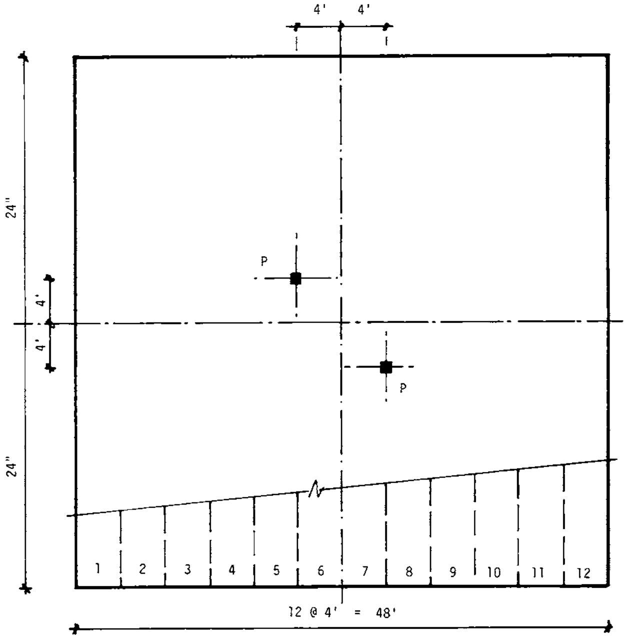

Fig. 16 — Deck for Example 4

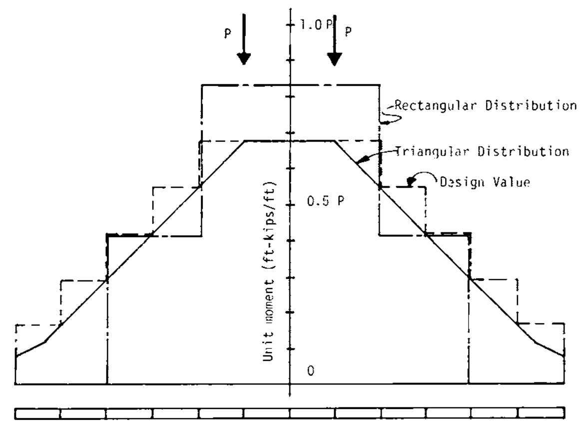

Fig. 17 — Transverse variation on unit moment in Example 4

Design for moment and deflection is for many cases less restrictive than under other proposed rules based on a rectangular variation of response. The equations for shear and torsion show higher values, especially for shear near midspan, that could control the design in some cases. This problem may be somewhat artificial, caused by the fact that the ACI strength equations separately address punching shear and beam shear. The distinction is not really behavioral, but rather an analytical convenience, and the result is that the shear strengths predicted for hollow-core sections reflect rather poorly the true strengths, generally underestimating them.

Transverse moment and shear are not addressed here, but are discussed in Reference 2. They are likely to cause difficulties primarily with short, wide planks.

Shear flexibility due to cell distortion causes concentration of the response under the load. Some uncertainty still exists[15] over the extent of its influence, but it is definitely most important in wide planks and in those that have roughly rectangular voids. Further research is needed to quantify its effects more accurately in those cases.

These rules were based on linear elastic analysis and thin plate theory with parameters (such as the size of the patch load) chosen to represent the real, rather complex, geometry as well as possible. The available experimental evidence in general corroborates the methods of calculation, but detailed experimental confirmation in the inelastic range is highly desirable.

CONCLUSIONS

Design rules are presented for analysis of precast decks subjected to concentrated loads. They are derived from the results of a wide parameter study and provide guidance for most situations that the designer is likely to encounter. They are more detailed than other extant rules but are still simple enough to apply in design.

ACKNOWLEDGMENTS

This research was supported by a grant from the Prestressed Concrete Institute and was conducted with the assistance of former graduate students Harald Oskarsson and Jung-Heum Yon.

NOTATION

A = area enclosed by a line drawn along the mid-thickness of the flanges and outer webs of a hollow-core section
B_d, B_m, B_t, B_v = distribution width, based on triangular transverse variation of response, for deflection, longitudinal bending moment, torsion, and shear
b = member width
C_1 = response modification factor for use with wide decks
C_e = response modification factor for use with loads near a free edge
e_1, e_2 = nearer and farther distances from the load to a free edge
h = overall thickness of member
i = flexural moment of inertia of member
J = torsion constant for member
ℓ = span length
M = total applied longitudinal bending moment
m_x = unit longitudinal bending moment
m_{xy} = unit torsional moment
Q_{eq}, Q_t, Q_v = total equivalent unit shear, unit shear caused by torsion, and unit shear caused by applied shear
S = distance between face of load and critical section for shear
T = applied torque
t = wall thickness
t_1 = thickness of outer web of hollow-core section
Σt = sum of web thicknesses in one hollow-core member
V = total applied shear force
V_1 = shear force tributary to one member
x = distance from one support
Δ = midspan deflection
ξ_L, ξ_R, ξ_{LR} = longitudinal load location, response location, and distance between response and center of load, each divided by ℓ
τ = shear stress

CONVERSION FACTORS

1 in. = 25.4 mm
1 ft = 0.3048 m
1 kip = 4.448 kN
1 kip/ft = 14.593 kN/m
1 ft-kip = 1.356 kN-m
1 ft-kip/ft = 4.448 kN-m/m

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