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Linking Concrete Rheology to Strength: Sustainability Model Approach

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A requirement for achieving sustainable concrete structures is to develop a quantitative method for designing concrete mixtures that yields the target rheological properties and compressive strength. Toward this objective, this paper proposes a mathematical model approach to improve the sustainability of the concrete industry. A postulation that packing density, a function of the concrete mixture, provides the link between concrete mixture, rheological properties, and compressive strength was investigated. Rheological models for yield stress and plastic viscosity, and a compressive strength model were adopted with packing density as a central variable. The rheological models employ a cell description that is representative of fresh concrete. The compressive strength model is based on excess paste theory to account for the concrete mixture proportions, gradation of aggregate particles, and porosity. An experimental program was developed to calibrate and test these models. Results revealed that packing density provides a consistent and reliable link, and that the concrete mixture composition can be designed to achieve the target rheological properties and hardened properties and ensure quality control. Consequently, a new mixture proportioning methodology was developed and proposed as an improvement to the ACI 211.1 mixture design method. Furthermore, a case study was conducted to test for the applicability and adequacy of this proposed method. This research outcome, which provides a quantitative approach to design concrete mixtures to meet specific strength requirements and rheology, can also be used to ensure quality control before concrete is cast.

Keywords: packing density; rheology; strength; viscosity; yield stress.

INTRODUCTION

Improving the sustainability of structures has become a global objective over the past few decades (Moutassem and Miqdadi 2020). Sustainability of civil engineering structures encompasses effective selection of materials, which includes the use and recyclability of the material and building components, effective use of energy, and resiliency of structures. Traditionally, concrete mixture design aims to meet workability and compressive strength requirements (ACI Committee 318 2005). Research has shown that the workability of fresh concrete can be quantified using Bingham's rheological properties—namely, the yield stress and plastic viscosity (Ferraris and de Larrard 1998; Chidiac et al. 2000; de Larrard 1999). The main variables that affect these properties include the size, shape, and volume fraction of the solid particles, and method of compaction (de Larrard 1999). Different apparatuses such as the slump rate machine II (SLRM II) can be used to estimate the rheological properties of concrete (Chidiac and Habibbeigi 2005). Numerous fundamental and phenomenological rheology models have been proposed in literature. Chidiac and Mahmoodzadeh (2009, 2013) reviewed the predictive capabilities of the rheological models and concluded that the fundamental models developed based on excess paste theory and the cell method provide good and consistent predictions. Both the plastic viscosity and yield stress models employ packing density of fresh concrete as a central variable.

The strength of hardened concrete depends on its porosity and the bond strength of the hydrated cement which are function of the aggregate types, shapes and gradation, cement composition and degree of hydration, amount of entrained air, mixture proportions, and placement protocols (de Larrard 1999; Moutassem 2010). Moutassem and Chidiac (2016) reviewed the predictive capabilities of the compressive strength models proposed in literature and concluded that the comprehensive model developed by Chidiac et al. (2013) provides the highest degree of correlation to experimental data.

Research has shown that the optimum packing density of aggregates and concrete yields optimum rheology and strength by reducing concrete porosity (Johansen and Andersen 1996; Wong and Kwan 2008; Tasi et al. 2006; Shilstone and Shilstone 1993). Studies have revealed the following: rheology is a function of concrete packing density (de Larrard 1999; Chidiac and Mahmoodzadeh 2009, 2013), compressive strength is a function of aggregates packing density (de Larrard 1999; Moutassem 2010; Chidiac et al. 2013), and aggregate packing density and concrete packing density correlate (de Larrard 1999). Accordingly, packing density is a common variable that relates the mixture composition to these properties. Packing density, defined as the ratio of the volume of the solid particles to the bulk volume occupied by these particles, can be measured experimentally in accordance with ASTM C29 (1997). Moutassem and Chidiac (2008) evaluated the suitability of the packing density models proposed in the literature for concrete applications and concluded that the compressible packing model (CPM) provides the highest predictability.

Some researchers have attempted to establish a link between the rheological properties and compressive strength using experimental means (Chidiac et al. 2003; Laskar and Talukdar 2007, 2008). Chidiac et al. (2003) worked on

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correlating the rheological properties to the strength and durability of hardened concrete made with a mixture composed of a high water-cement ratio (*w*/*c*). It was observed that concrete compressive strength increases as the yield stress and plastic viscosity increase up to an optimum value. Laskar and Talukdar (2007, 2008) worked on correlating the rheological properties to the compressive strength of high-performance concrete. It was observed that the compressive strength increases with yield stress and that the increase is rapid at low values of yield stress. For the plastic viscosity, it was observed that there exists an optimum value corresponding to maximum compressive strength. These findings strongly suggest that a link exists between rheology and strength. However, they do not provide explanations, nor do they identify the common variables that permit linking these properties. Research conducted by Mehdipour and Khayat (2019) studied the effect of particle packing on rheology and strength development of dense cementitious suspensions. Particle packing was determined through altering the water content and determining the optimum water demand corresponding to maximum solid concentration and the minimum water demand required to initiate flow. The author suggests that the outcome of this study can be useful in optimizing the design of dense cementitious suspensions. However, no actual link between the mixture variables, rheology, and strength was produced to permit the design of optimum mixtures.

This paper presents an experimental program as well as an analytical methodology to demonstrate whether a link can be established between the concrete mixture, rheological properties, and compressive strength with packing density identified as the common variable.

RESEARCH SIGNIFICANCE

Current design practice is based on meeting workability and strength requirements (ACI Committee 211 1991). The slump test, which has been the standard test for workability, is not sufficient and the two rheological properties parameters—namely, yield stress and plastic viscosity—are needed to quantify workability (Tattersall and Banfill 1983; Ferraris and de Larrard 1998; Chidiac et al. 2003). For the compressive strength, current design targets the cement content and *w*/*c*. However, research has shown that other factors can be considered for a better prediction (de Larrard 1999). In addition, the current mixture design approach is not fundamental but is rather statistical and does not provide a link between the mixture proportions, slump, and compressive strength, which is needed to provide control over the design. Recognizing that packing density is a statistically significant variable for both strength and slump (rheology), and that it is a mixture property, it was postulated that packing density is a central variable providing a continuous link between concrete mixture, rheology, and compressive strength. On this basis, an improved version of the ACI 211.1 mixture proportioning methodology that incorporates the rheological properties and optimizes packing density of aggregates needs to be developed. This research outcome, which provides a quantitative approach to design concrete mixtures to meet specific strength requirements and rheology, can also be used to ensure quality control before concrete is cast for more sustainable structures.

EXPERIMENTAL PROGRAM

Materials

The concrete was prepared using a mixture of crushed limestone, siliceous sand, ordinary portland cement (OPC), air-entraining admixture (AEA), and water. Hydraulic Type 10 (Type GU) cement was used in this study. The chemical and physical properties are summarized in Table 1. Crushed limestone coarse aggregate (CA) with 20 and 14 mm nominal maximum aggregate sizes were used. The aggregates were obtained from a quarry located in Dundas, ON, Canada. The specific gravities, absorption values, and bulk density for the 20 mm CA are 2.75, 0.92%, and 1636 kg/m^3; and 2.74, 0.88%, and 1576 kg/m^3 for the 14 mm CA, respectively. Commercially distributed sand was obtained. The fineness modulus, specific gravities, absorption values, and bulk density for the sand are 2.72, 2.71, 1.58%, and 1812 kg/m^3, respectively. The bulk density, specific gravity, and absorption for CA and sand were obtained following ASTM C127 (2015) and ASTM C128 (2015), respectively. The particle size distribution test was carried out in accordance with ASTM C136 (2014) and was found to conform to the specification requirements. A commercial AEA meeting the requirements of ASTM C260 (2010) was used to entrain air.

Concrete mixture design

The concrete mixture was proportioned following the statistical fractional factorial design method and the ranges recommended for designing and proportioning normal concrete mixtures (Kosmatka et al. 2002). The range selected

Table 1—Chemical and physical properties of Type 10 (Type GU) hydraulic cement

SiO_2, %	19.7
Al_2O_3, %	4.9
Fe_2O_3, %	2.6
CaO, %	62.1
MgO, %	2.8
SO_3, %	3.2
Na_2O, %	1.3
Loss on ignition, %	3.1
C4AF, %	8.0
C3A, %	9.0
C3S (CSA), %	58
C2S (CSA), %	13
C3S (ASTM), %	50
C2S (ASTM), %	19
Equivalent alkalies, %	0.78
Specific surface area (Blaine), cm^2/g	4205
% Passing 325 (45 um) mesh, %	92.1
Time of setting—initial, min	113
Compressive strength—28-day, MPa	41.9

Table 2—Concrete mixture design composition

Mixture No.	*w*/*c*	Size	Water, kg/m^3	Cement, kg/m^3	Coarse aggregate (bulk volume)	Coarse aggregate, kg/m^3	Sand, kg/m^3	Air, %
1	0.40	14	193	483	0.50	794	851	5.9
2	0.60	14	193	322	0.62	971	815	5.3
3	0.40	14	205	513	0.62	971	618	4.8
4	0.60	14	205	342	0.50	794	939	5.1
5	0.40	20	184	460	0.69	1134	563	4.6
6	0.60	20	184	307	0.57	928	898	5.6
7	0.40	20	197	493	0.57	928	703	4.8
8	0.60	20	197	328	0.69	1134	641	3.1
9	0.50	14	193	386	0.504	794	934	5.7
10	0.70	14	175	250	0.504	794	1100	8.8
11	0.50	14	205	410	0.504	794	881	5.1
12	0.70	14	193	276	0.504	794	1029	7.5
13	0.50	14	193	386	0.616	971	759	5.1
14	0.70	14	175	250	0.616	971	925	8.7
15	0.50	14	205	410	0.616	971	706	4.8
16	0.70	14	193	276	0.616	971	854	5.6
17	0.50	14	199	398	0.560	883	820	5.0
18	0.50	14	199	398	0.448	706	994	7.8
19	0.50	14	199	398	0.672	1059	645	6.7
20	0.50	14	199	398	0.560	883	820	5.8
21	0.40	14	216	540	0.50	794	807	2.1
22	0.60	14	216	360	0.62	971	787	1.0
23	0.40	14	228	570	0.62	971	574	1.8
24	0.60	14	228	380	0.50	794	912	1.2
25	0.40	20	205	513	0.69	1134	542	1.6
26	0.60	20	205	342	0.57	928	892	1.5
27	0.40	20	216	540	0.57	928	692	1.4
28	0.60	20	216	360	0.69	1134	643	1.5

for *w*/*c* was from 0.4 to 0.7. The water content values ranged from 175 to 228 kg/m^3, covering the full range of slump for non-air-entrained and air-entrained concrete. The cement content ranged from 250 to 570 kg/m^3. The bulk volume of CA per unit volume of concrete, V_{CA}, ranged from 0.45 to 0.69. The CA maximum sizes were 20 and 14 mm. For the air-entrained concrete mixtures, an AEA was used to achieve 5% air content. The total number of concrete mixtures was 28 and the corresponding proportions are given in Table 2. Of the 28 concrete mixtures, 20 were air-entrained and eight were non-air-entrained.

Packing density

The maximum packing density, ϕ^*, for each mixture was computed using the compressible packing model (de Larrard 1999), which requires the dry packing densities, volume fractions, and the characteristic diameters of the particles as an input. The dry packing density of sand and CA (oven dried) were measured in accordance with ASTM C29 (1997) with rodding as the method of compaction. The test was carried out using three specimens, and both the mean value and the standard deviation are reported. The mean packing densities of sand, 14 mm maximum CA size, and 20 mm maximum CA size were measured to be 0.669, 0.575, and 0.595, respectively, and the corresponding standard deviation is 0.016, 0.011, and 0.012. The mean characteristic diameters corresponding to 63.2% passing, as recommended by Goltermann et al. (1997), were measured for sand, 14 mm maximum aggregate size, and 20 mm maximum aggregate size and found to be 1.1, 10.4, and 14.3 mm, respectively. The corresponding standard deviations are 0.05, 1.48, and 1.22 mm, respectively. The mean values of the mean diameters, corresponding to 50% passing, needed for the proposed average paste thickness (*APT*) model, were determined to be 0.74, 9.1, and 12 mm for sand, 14 mm maximum CA size, and 20 mm maximum CA size, respectively. The corresponding standard deviations are 0.06, 0.48, and 1.55 mm, respectively.

Rheological properties

Bingham material properties, namely the yield stress and plastic viscosity, were estimated using the SLRM II (Chidiac and Habibbeigi 2005). Once the mixing procedure was completed, the slump was measured in accordance with ASTM C143 (2015) and the SLRM II was used to measure the concrete slump as a function of time, the slump flow S_f, the slump S_l, and the time of slump t_{slump}. Subsequently, the rheological properties were calculated as follows

$$\tau_o = \frac{4gV\rho}{\sqrt{3}\,\pi S_f^2} = 0.0397\left(\frac{\rho}{S_f^2}\right) \tag{1}$$

$$\eta = \frac{\rho g H V}{150\pi S_l S_f^2} t_{slump} \tag{2}$$

where ρ is the concrete density; H is the height of the slump cone; V is the volume of slump cone; and g is the gravitational acceleration.

Compressive strength

For every mixture, six standard cylinders, 100 mm in diameter and 200 mm high, were cast, consolidated by rodding, and finished in accordance with ASTM C192 (2016). The cylinders were sealed for 24 hours then placed in a moist curing room, where the relative humidity was more than 95% and the temperature was 24°C. The concrete compressive strength was evaluated in accordance with ASTM C39 (2016) at 28 days. Three specimens were tested for each mixture.

ANALYTICAL MODELS

Packing density

Moutassem and Chidiac (2008) reviewed the literature and evaluated the adequacy of many models proposed for predicting the maximum packing density of aggregates. Of the nine packing density models investigated, only the CPM, modified Toufar model (MTM), and theory of particle mixtures model (TPM) were found to correctly predict the maximum packing density of aggregate used in concrete (Moutassem and Chidiac 2008). In this study, the maximum packing density of aggregates, ϕ^*_{agg}, and concrete, ϕ^*_{conc}, are predicted using the CPM (de Larrard 1999). To account for the different sizes of fine aggregate, coarse aggregate, and cement particles, a characteristic diameter concept corresponding to 63.2% passing was introduced (Goltermann et al. 1997). Given the method of compaction and knowing the following: volume fractions of cement, fine aggregate, and coarse aggregate; their characteristic diameters; and their maximum packing densities—the CPM model can be employed to calculate ϕ^*_{agg} and ϕ^*_{conc}.

Rheology

Chidiac and Mahmoodzadeh (2009, 2013) carried out a review of the models reported in the literature for predicting the plastic viscosity and yield stress of fresh concrete. They revealed that there are different types of models reported in the literature and that there are few models that can predict the properties. However, closer examination of the results revealed that the model proposed by Chidiac and Mahmoodzadeh, which is a fundamental model, yielded both good and consistent predictions.

Plastic viscosity—Chidiac and Mahmoodzadeh (2013) developed the following model for determining the plastic viscosity of fresh concrete based on the composition of the mixture

$$\eta_r \cong \eta_i y^3 \frac{4(1-y^7)}{4(1+y^{10}) - 25y^3(1+y^4) + 42y^5} \tag{3}$$

where η_i is the dynamic intrinsic viscosity and is a function of the particle shape; and y is the ratio of the radius of the cell to the radius of the particle and is given by (Frankel and Acrivos 1967)

$$y = (\phi/\phi^*)^{1/3}(1-\psi) \tag{4}$$

where ϕ and ϕ^* are the packing density of concrete (volume fraction of solids) and the maximum packing density of the concrete mixture, respectively; and ψ is a function of the concrete mixture and is defined as follows

$$\begin{cases} \psi = C_1 \times \dfrac{M_c}{M_w} \\ \psi = C_1 \times \dfrac{M_{HRWRA}}{M_c} \times \dfrac{M_W}{M_c + M_{FineSand+Sand}} \end{cases} \tag{5}$$

where M_c, M_w, M_{HRWRA}, and $M_{FineSand+Sand}$ correspond to the mass of cement, water, high-range water-reducing admixture (HRWRA), and the total sand, respectively; and C_1 is a calibration constant whose value depends on the method of compaction and method of measuring the rheological properties.

Yield stress—Chidiac and Mahmoodzadeh (2013) followed the same analogy to develop a fundamental model for yield stress and is given by

$$\tau_o \cong \tau_i y^3 \frac{4(1-y^7)}{4(1+y^{10}) - 25y^3(1+y^4) + 42y^5} \tag{6}$$

where τ_i is referred to as intrinsic yield stress and is a function of the particle shape; and y is given in Eq. (8) with ψ for yield stress model defined as

$$\psi = C_1\left(\frac{M_G}{M_w} + \rho_w \times V_{air}\right) \tag{7}$$

where M_G and M_w are the mass of gravel—that is, CA—and mass of water in the concrete mixture, respectively.

Compressive strength

Moutassem and Chidiac (2016) reviewed the literature and evaluated the adequacy of many concrete compressive strength models. The model proposed by Chidiac et al. (2013) is adopted because it provides a high degree of correlation to the experimental data and mathematically accounts for packing density. The model is given by

$$f_c'(t) = KR_{c28}\left(\frac{APT}{D}\right)^{A} B^{\frac{W+EA}{C}}(\alpha(t) - \alpha_{cr}) \tag{8}$$

where f_c' is the compressive strength of concrete; α is the degree of cement hydration at age t; α_{cr} is the critical degree of hydration; K is the aggregate to paste bond constant; R_{c28} is the standard cement strength at 28 days; APT is the average paste thickness; D is the mean diameter of all aggregate particles in the mixture; $(W+EA)/C$ is the ratio of the volume fractions of water + entrapped and entrained air to cement; and A and B are calibration constants. Constant A depends on the shape of the fine and coarse aggregate particles, while constant B depends on specimen shape and test conditions. APT, which is a function of aggregate gradations, mean sizes, volume fractions, and method of compaction, can be obtained as follows

$$APT \approx -\frac{1}{2}\left(D_s + \frac{\phi_{ca}D_s^2}{\phi_s D_{ca}} + \frac{\phi D_s^2(1-\phi^*)}{\phi_s\phi^* D}\right) + \frac{1}{2}\sqrt{\left(D_s + \frac{\phi_{ca}D_s^2}{\phi_s D_{ca}} + \frac{\phi D_s^2(1-\phi^*)}{\phi_s\phi^* D}\right)^2 + \frac{4}{3}\frac{(\phi^* - \phi)}{\phi^*}\frac{D_s^2}{\phi_s}} \tag{9}$$

where D_s and D_{ca} are the mean diameter of sand and CA particles, respectively; and ϕ, ϕ_s, and ϕ_{ca} are the volume fractions of the aggregates, sand particles, and coarse particles, respectively.

The degree of cement hydration, α, can be obtained using a cement hydration model such as the one proposed by Schindler and Folliard (2005) and adopted in this study. The critical degree of hydration, α_{cr}, was predicted according to Rasmussen et al. (2002). The adopted models are as follows

$$\alpha_{cr} = k \times w/c \tag{10}$$

$$\alpha = \alpha_u \cdot \exp\left(-\left[\frac{\tau}{t_e}\right]^{\beta}\right) \tag{11}$$

$$\tau = 66.78p_{C3A}^{-0.154}p_{C3S}^{-0.401} \cdot \text{Blaine}^{-0.804} \cdot p_{SO3}^{-0.758} \cdot \exp(2.187p_{SLAG} + 9.5p_{FA}p_{FA\text{-CaO}}) \tag{12}$$

$$\beta = 181.4p_{C3A}^{0.146}p_{C3S}^{0.227} \cdot \text{Blaine}^{-0.535} \cdot p_{SO3}^{0.558} \cdot \exp(-0.647p_{SLAG}) \tag{13}$$

$$\alpha_u = \frac{1.031\,w/c}{0.194 + w/c} + 0.50p_{FA} + 0.30p_{SLAG} \leq 1.0 \tag{14}$$

$$t_e = \sum_0^t \exp\left(\frac{E}{R}\left(\frac{1}{293} - \frac{1}{T+273}\right)\right)\Delta t \tag{15}$$

where t_e is the equivalent age (hours); α_u is the ultimate degree of cement hydration; T is the concrete temperature (°C); E is the activation energy (33,500 J/mol for $T > 20°C$); R is the universal gas constant (8.3144 J/mol/K); τ and β are the hydration time parameter and hydration shape parameter, respectively; p_{C3A}, p_{C3S}, p_{SO3}, p_{FA}, p_{SLAG}, $p_{FA\text{-CaO}}$ are the weight ratios in terms of total cement content; and k is a calibration constant (0.43 for OPC concrete).

MODEL CALIBRATION AND PREDICTION

Calibration procedure

The experimental test results are used to calibrate the parameters η_i, τ_i, and C_1 in the plastic viscosity and yield stress models and the parameters K, A, and B in the compressive strength model. Model calibration was achieved using least-squares analysis. The values of the constants were selected by minimizing the model standard error (σ), which provides a global assessment of the model prediction (Montgomery and Runger 2003)

$$\sigma = \sqrt{\frac{\sum_i(Model_i - Experiment_i)^2}{n - p}} \tag{16}$$

where $Model_i$ and $Experiment_i$ are the model and experiment values corresponding to mixture i, respectively; n is the number of test points; and p is the number of model constants. In addition, the correlation coefficient (R^2), which measures the degree of correlation between the model and experiment, was also calculated.

Model prediction

The properties of the concrete mixture listed in Table 2 were evaluated. Table 3 shows the experimental and predicted values for yield stress, plastic viscosity, and compressive strength. Errors as a percent difference between experiment and model predicted values are also presented. Table 4 presents the goodness of fit—that is, σ and R^2—for non-air-entrained and air-entrained concretes. These results provide a reasonable degree of correlation to the experimental data. Figure 1 illustrates graphically the goodness of fit of the rheological properties and compressive strength prediction models for air-entrained and non-air-entrained mixtures. Because the compressive strength model accounts for air, the corresponding graph consists of both.

CORRELATION BETWEEN MIXTURE DESIGN, RHEOLOGY, AND COMPRESSIVE STRENGTH

Production and potential use of correlation nomographs

The identified models provided in this paper were used to produce correlation nomographs to illustrate graphically the postulation that packing density provides a continuous link between concrete mixture, rheology, and strength (Moutassem 2010). The input parameters include w/c, water content, air content, V_{CA}, and maximum CA size. The nomographs shown in Fig. 2 to 5 correspond to air-entrained concrete mixtures. The nomographs shown in Fig. 6 to 9 correspond to non-air-entrained concrete mixtures. Each figure contains three charts. The dashed lines in chart 1 correspond to the yield stress versus ϕ/ϕ^*_{conc}, whereas the solid lines correspond to the water content versus ϕ/ϕ^*_{conc}. Chart 2 is similar to chart 1 except that the rheological property is the plastic viscosity instead of the yield stress.

Table 3—Experimental and predicted rheological properties and compressive strength results

Mixture No.	$(\phi/\phi^*)_{agg}$	$(\phi/\phi^*)_{conc}$	τ_{oexp}, Pa	τ_{omodel}, Pa	Error, %	μ_{exp}, Pa-s	μ_{model}, Pa-s	Error, %	$f_c'{}_{exp}$	$f_c'{}_{model}$	Error, %
1	0.741	0.880	1763	1398	21	49	34	30	37.8 ± 2.3	36.5	3
2	0.803	0.863	1046	918	12	12	9	25	23.1 ± 1.2	24.3	5
3	0.716	0.883	1530	1525	0	31	40	31	36.4 ± 1.0	38.1	5
4	0.790	0.855	645	761	18	6	7	17	27.2 ± 1.1	24.9	9
5	0.752	0.895	2228	2200	1	80	84	5	35.7 ± 1.8	37.9	6
6	0.810	0.865	887	962	8	11	10	13	23.9 ± 2.0	23.7	1
7	0.721	0.885	2105	1610	23	56	45	20	38.4 ± 1.4	38.0	1
8	0.796	0.873	600	1182	97	9	13	42	23.8 ± 2.3	27.3	15
9	0.783	0.867	738	1011	37	8	13	63	30.4 ± 0.8	30.0	1
10	0.839	0.864	1391	946	32	—	—	—	14.9 ± 0.8	14.7	2
11	0.762	0.862	565	902	60	5	11	121	32.9 ± 1.0	31.1	5
12	0.815	0.850	720	680	6	8	5	34	18.4 ± 0.7	16.7	9
13	0.778	0.871	876	1103	26	9	15	66	30.7 ± 2.6	30.6	0
14	0.827	0.856	841	779	7	9	6	30	13.6 ± 0.6	14.8	9
15	0.756	0.865	864	950	10	10	12	19	32.5 ± 0.2	31.4	3
16	0.818	0.861	604	863	43	8	7	11	18.6 ± 2.0	18.7	0
17	0.770	0.867	922	1005	9	9	13	44	31.9 ± 0.6	31.0	3
18	0.761	0.844	759	592	22	7	6	13	26.3 ± 1.0	27.7	5
19	0.755	0.853	742	718	3	9	8	12	25.5 ± 1.3	28.9	13
20	0.763	0.859	794	840	6	—	—	—	33.1 ± 2.2	30.0	9
21	0.729	0.904	1208	1210	0	20	25	29	42.6 ± 1.5	41.7	2
22	0.803	0.884	607	649	7	—	—	—	31.0 ± 0.3	30.3	2
23	0.702	0.902	1464	1138	22	26	22	14	43.8 ± 3.5	42.1	4
24	0.788	0.873	728	463	36	6	5	16	29.7 ± 0.3	30.2	2
25	0.744	0.915	2131	2027	5	48	47	2	40.5 ± 2.4	42.4	5
26	0.816	0.888	679	732	8	7	8	22	31.9 ± 2.3	29.8	7
27	0.719	0.910	1134	1560	38	—	—	—	43.3 ± 1.4	42.9	1
28	0.786	0.877	597	546	8	9	5	40	27.3 ± 0.7	29.8	9

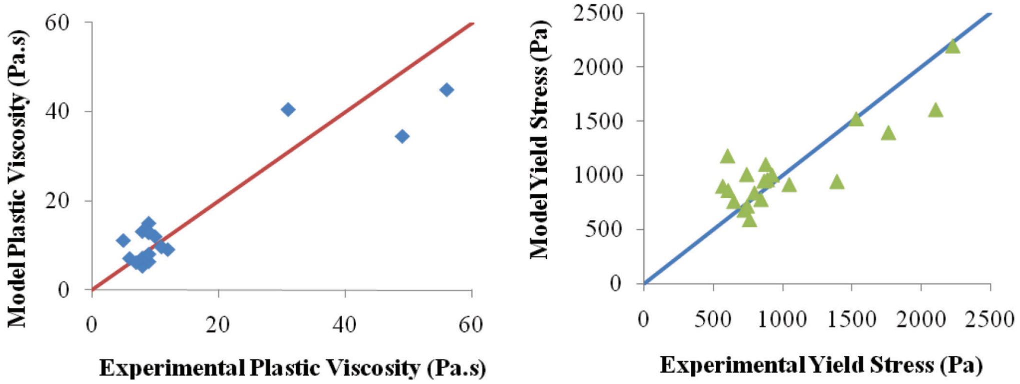
Table 4—Goodness of fit

	Plastic viscosity, Pa-s		Yield stress, Pa		Compressive strength, MPa	
Air consideration	σ	R^2	σ	R^2	σ	R^2
Air-entrained	6.1	0.92	269	0.74	1.8	0.96
Non-air-entrained	3.9	0.95	250	0.83		

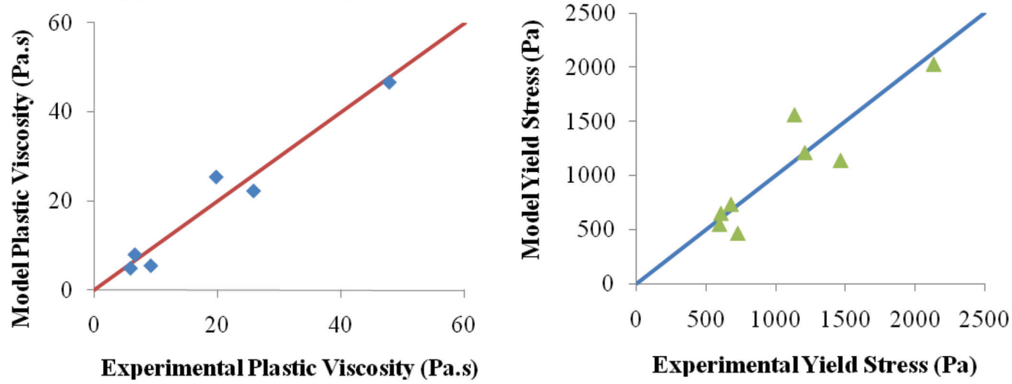
The dashed lines in chart 3 correspond to ϕ/ϕ^*_{conc} versus ϕ/ϕ^*_{agg}, whereas the solid lines correspond to ϕ/ϕ^*_{agg} versus compressive strength. Following the same procedure, additional nomographs can be produced to cover a wider range of input parameters.

Applicability and adequacy of correlation nomographs

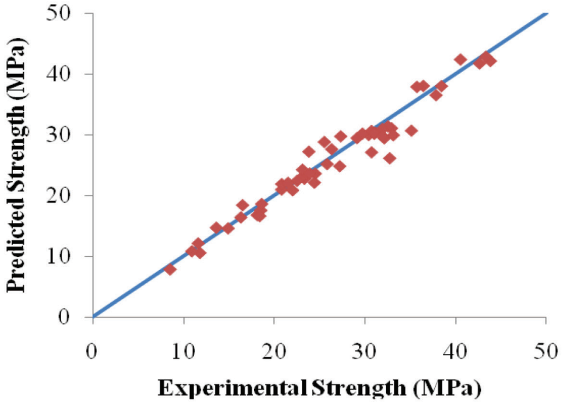
Once the mixture design is known, charts 1 and 2 can be used to determine ϕ/ϕ^*_{conc} and the corresponding yield stress and plastic viscosity values. Knowing ϕ/ϕ^*_{conc} from charts 1 or 2, the ϕ/ϕ^*_{agg} can be determined using the dashed lines shown in chart 3. With ϕ/ϕ^*_{agg} determined, the solid lines in chart 3 can be used to determine the corresponding compressive strength of concrete. To illustrate the applicability and test the adequacy of the correlation nomographs, a concrete mixture was randomly selected from Table 2 (No. 2) and the relevant nomograph was manually used to ensure that it yields rheological and strength results similar to those shown in Table 3. Entering the first chart of Fig. 4 with a water content of 193 kg/m^3 and intersecting the bold line corresponding to V_{CA} of 0.62 yields a ϕ/ϕ^*_{conc} of approximately 0.863 on the x-axis. Moving vertically upwards from this point and intersecting the dashed line corresponding to



(a) Plastic Viscosity, air entrained (b) Yield Stress, air entrained



(c) Plastic Viscosity, non-air entrained (d) Yield Stress, non-air entrained



(e) Compressive Strength

Fig. 1—Prediction capability of yield stress, plastic viscosity, and strength models.

V_{CA} of 0.62 yields a yield stress of approximately 950 Pa. Entering the second chart with a water content of 193 kg/m^3 and intersecting the bold line corresponding to V_{CA} of 0.62 yields a ϕ/ϕ^*_{conc} of approximately 0.863 on the x-axis. Moving vertically upwards from this point and intersecting the dashed line corresponding to V_{CA} of 0.62 yields a plastic viscosity of approximately 9.5 Pa-s. Entering the third chart with ϕ/ϕ^*_{conc} of approximately 0.863 and intersecting the dashed line corresponding to V_{CA} of 0.62 yields a ϕ/ϕ^*_{agg} of approximately 0.805 on the x-axis. Moving vertically upwards from this point and intersecting the bold line corresponding to V_{CA} of 0.62 yields a compressive strength of approximately 24.6 MPa. Comparison reveals that these results are similar to those shown in Table 3. Hence, these nomographs demonstrate that there exists a continuous link between concrete mixture, rheology, and compressive strength through packing density. The small variability in the results is due to human error when manually using the charts, and because these nomographs for air-entrained concrete were constructed from the models by assuming a target air content of 5%, whereas the actual air content for this mixture is 5.3%.

Correlations trend

The effect of *w/c*, water content, V_{CA}, maximum aggregate size, and air content on the properties were investigated to verify their consistence with what is reported in literature. These figures reveal that an increase in *w/c* due to an increase in water content or a reduction in the cement content results in a decrease in yield stress, plastic viscosity,

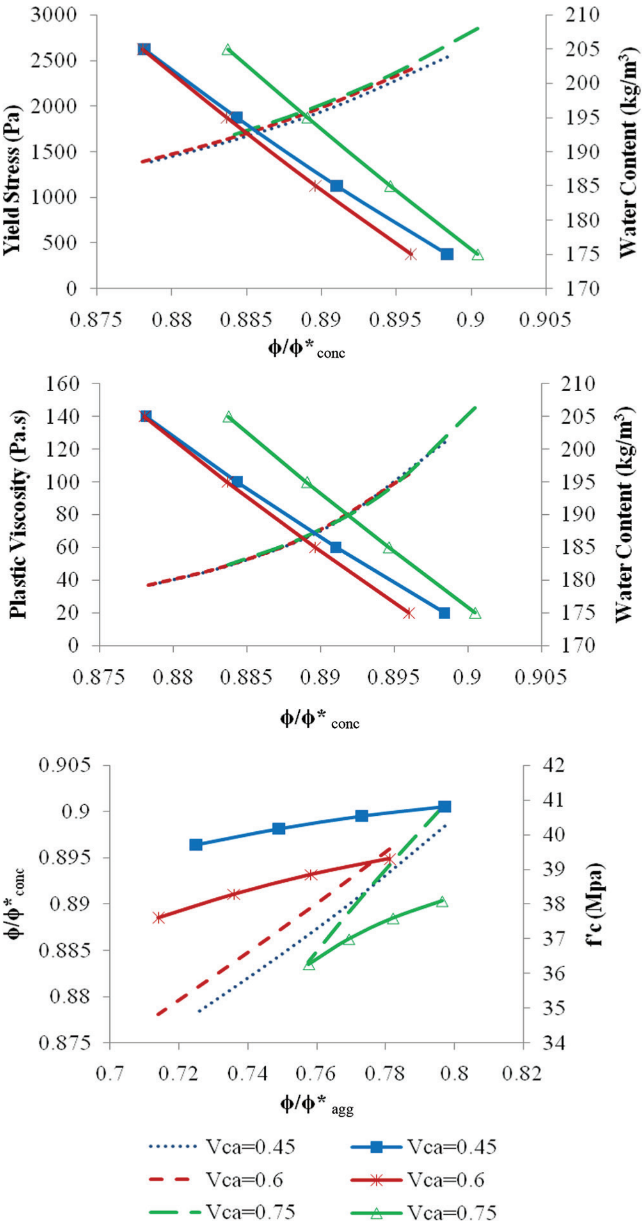


*Fig. 2—Linking rheological properties to strength: Nomograph—*w/c *= 0.40, 14 mm, air-entrained (5% air).*

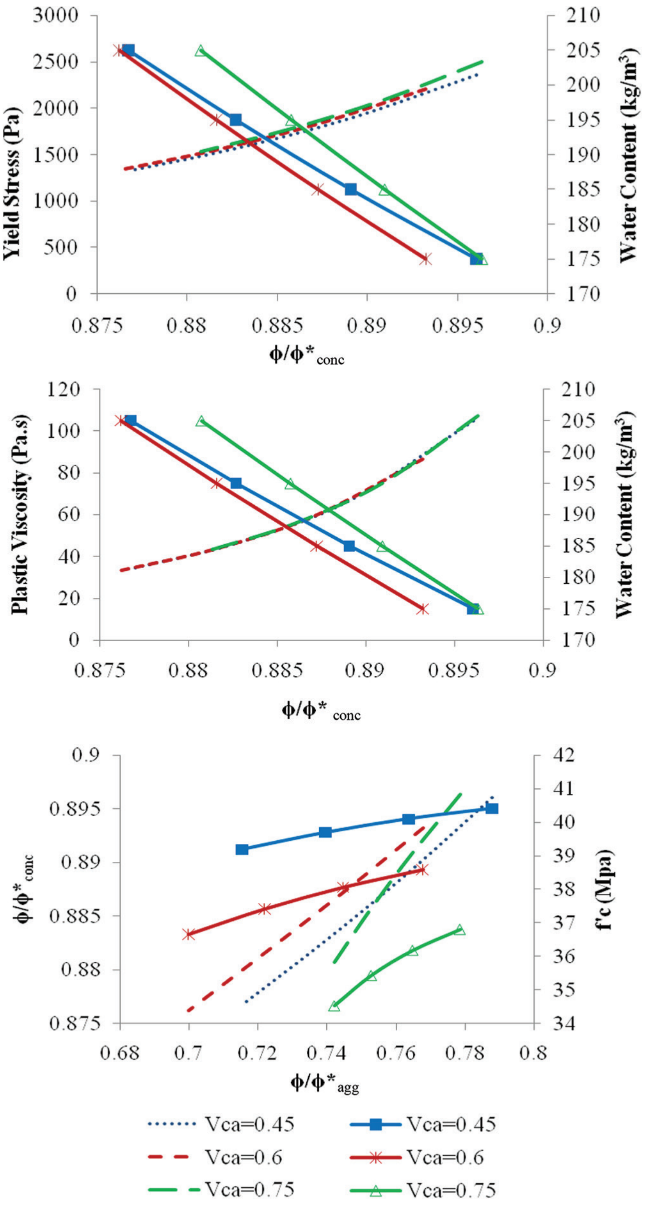


*Fig. 3—Linking rheological properties to strength: Nomograph—*w/c *= 0.40, 20 mm, air-entrained (5% air).*

and compressive strength, which is consistent with literature (Quiroga 2003). The volume fraction of solids, ϕ_{conc}, can be determined from $1–V_{air}–V_{water}$. An increase in water content or a reduction in cement content reduces ϕ_{conc} significantly relative to its effect on ϕ^*_{conc}, which depends on the mixture gradation. This results in a reduction in ϕ/ϕ^*_{conc} and a reduction in the yield stress and plastic viscosity, according to Eq. (3) and (6). In addition, an increase in *w/c* results in a reduction in strength due to an increase in the water-filled capillary porosity and in accordance with Eq. (8).

These figures also reveal that an increase in the water content for fixed *w/c* results in a reduction in yield stress, plastic viscosity, and strength, which is consistent with literature (Quiroga 2003). The reduction in yield stress and plastic viscosity is due to the presence of a larger volume of excess paste beyond what is required to fill the voids between the particles, which results in further lubrication. An increase in V_{CA} for fixed *w/c* and water content (that is, fixed ϕ_{agg}) influences the sand-aggregate ratio (S/A), which can result in either a decrease or increase in ϕ^*_{agg}, depending on the total aggregates gradation. Research has shown that there is an optimum S/A that equals the maximum binary packing of these elements. This optimum S/A corresponds to minimum porosity and thus maximum workability and strength (Johansen and Andersen 1996; Quiroga 2003). Comparing the figures with 14 mm maximum aggregate size to the figures with 20 mm maximum aggregate size, it is revealed that an increase in the maximum aggregate size results in an increase in workability and a slight reduction in compressive strength, which is consistent with literature (ACI Committee 318 2005; de Larrard 1999; Quiroga 2003). The figures also reveal that an increase in the air content through air entrainment for fixed mixture constituents results in a reduction in yield stress, plastic viscosity, and compressive strength, which is consistent with literature (Tattersall and Banfill

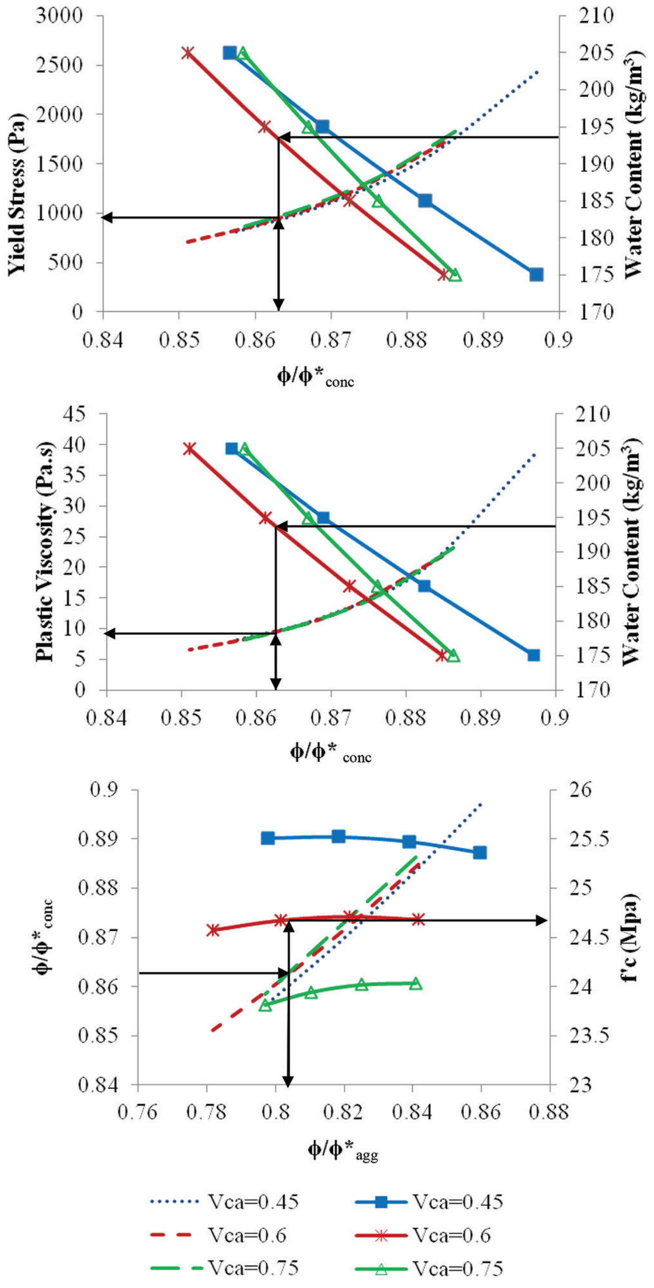


*Fig. 4—Linking rheological properties to strength: Nomograph—*w/c *= 0.60, 14 mm, air-entrained (5% air).*

1983). In the context of packing density, an increase in the amount of air results in smaller ϕ_{conc} but has no influence on ϕ^*_{conc}. Therefore, ϕ/ϕ^*_{conc} would decrease and lead to a reduction in the yield stress and plastic viscosity, according to Eq. (3) and (6). An increase in air content would increase the porosity and result in a reduction in the compressive strength of concrete (ACI Committee 318 2005).

NEW METHODOLOGY FOR CONCRETE MIXTURE DESIGN AND CASE STUDY

New concrete mixture design methodology

Recognizing that packing density is a statistically significant variable for both strength and slump (rheology), and that it is a mixture property, it was postulated and then confirmed that packing density is a central variable providing a continuous link between concrete mixture, rheology, and compressive strength. On this basis, the following methodology is proposed to integrate packing density and rheology in the design of concrete mixture:

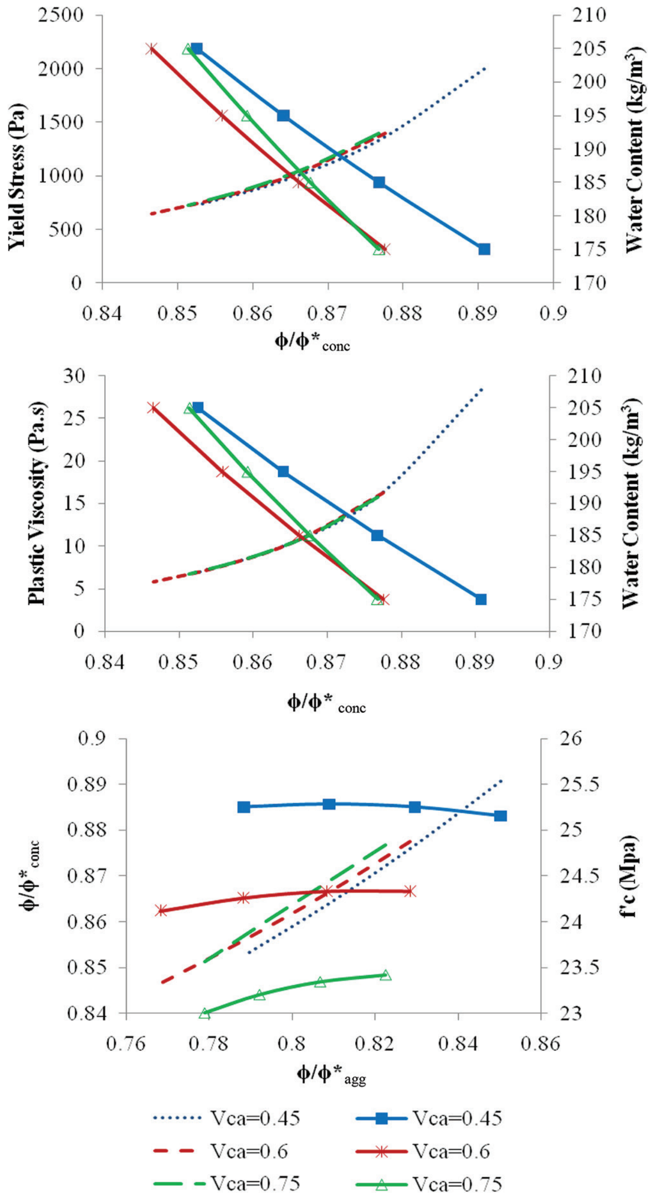


*Fig. 5—Linking rheological properties to strength: Nomograph—*w/c *= 0.60, 20 mm, air-entrained (5% air).*

Proposed methodology

- Design requirements: compressive strength, yield stress, plastic viscosity (f_c', τ_o, μ).
- From $f_c'_{required}$ and according to ACI 211.1 guidelines → Estimate *w/c*.
- From $\tau_{o\ required}$, compute slump using model (Chidiac et al. 2000; Chidiac and Habibbeigi 2005) and from ACI 211.1 guidelines → Estimate water content.
- From *w/c* and water content → Determine cement content.
- Determine V_{CA}/V_{agg} through optimization of ϕ^*_{agg} using a suitable packing density model such as MTM, CPM, or TPM.
- From $V_{agg} = 1 - V_w - V_c - V_{air}$ and from known V_{CA}/V_{agg} → Calculate V_{CA}.

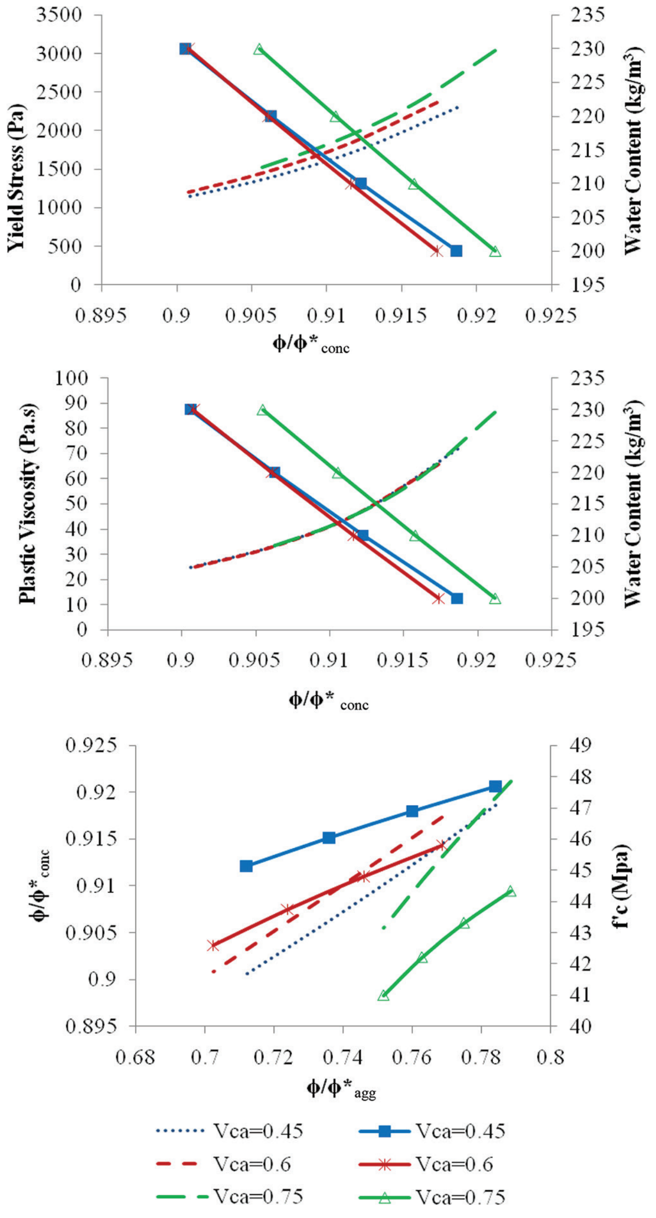


Fig. 6—Linking rheological properties to strength: Nomograph—w/c = 0.40, 14 mm, non-air-entrained.

- Determine V_s remaining and mixture proportions in kg/m^3.
- Predict f_c' using Eq. (8) or any using any other suitable f_c' model or using the nomographs, and then check that $f_c' >$ required f_c'.
- Determine $\phi_{concrete}$ from mixture and compute $\phi^*_{concrete}$ using MTM, CPM, or TPM.
- Using rheological models such as Eq. (3) and (6) or by using the nomographs → Predict τ_o and μ and check if OK.
- If $f_c'{}_{predicted} < f_c'{}_{required}$ → adjust w/c and iterate.
- If predicted τ_o or μ are not as desired → adjust water content and iterate.

Case study

The following design case study demonstrates the applicability and adequacy of the proposed methodology:

Given: $f_c'{}_{required} > 35$ MPa, $\tau_{o,required} < 1000$ Pa, $\mu_{required} < 30$ Pa-s, 14 mm maximum aggregate size, air-entrained concrete (~5%), materials properties, and gradations.

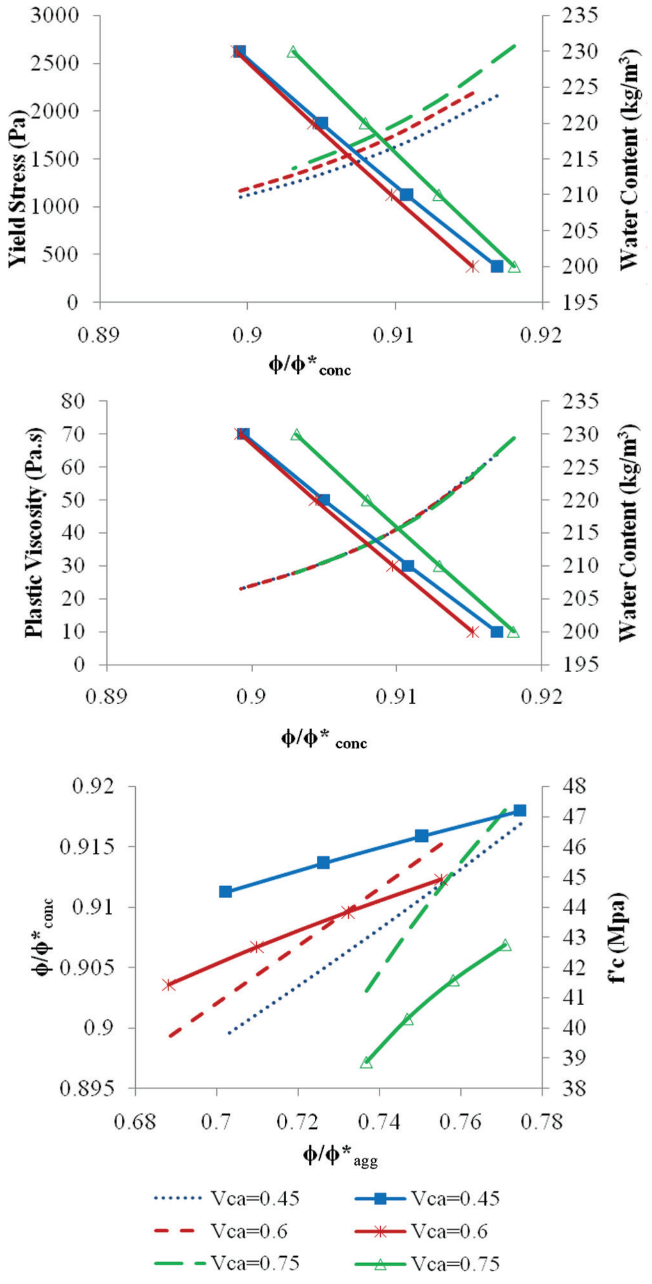


Fig. 7—Linking rheological properties to strength: Nomograph—w/c = 0.40, 20 mm, non-air-entrained.

- From ACI 211.1 guidelines → $w/c = 0.39$
- For $\tau_o = 1000$ Pa → Slump = 200 mm. From ACI 211.1 guide → W = 205 kg/m → C = 526 kg/m^3.
- Using CPM → $V_{CA} = 0.58$ and $\phi^*_{agg} = 0.816$ → CA = 914 kg/m^3 and FA = 663 kg/m^3
- Using *APT* model: *APT* = 0.208 mm
- Using f_c' model: $f_c' = 37.8$ MPa > 35 → OK
- Using rheology models: $\mu = 29.8$ Pa-s < 30 Pa-s → OK; $\tau_o = 1201$ Pa > 1000 Pa → NO → Increase water content
- Assume W = 220 kg/m^3 → C = 564 kg/m^3
- From the CPM → $V_{CA} = 0.56$ and $\phi^*_{agg} = 0.816$ → CA = 883 kg/m^3 and FA = 620 kg/m^3
- Using *APT* model: *APT* = 0.240 mm
- Using f_c' model: $f_c' = 36.3$ MPa > 35 MPa → OK
- And: $\mu = 20.9$ Pa-s < 30 Pa-s → OK; $\tau_o = 975$ Pa < 1000 Pa → OK

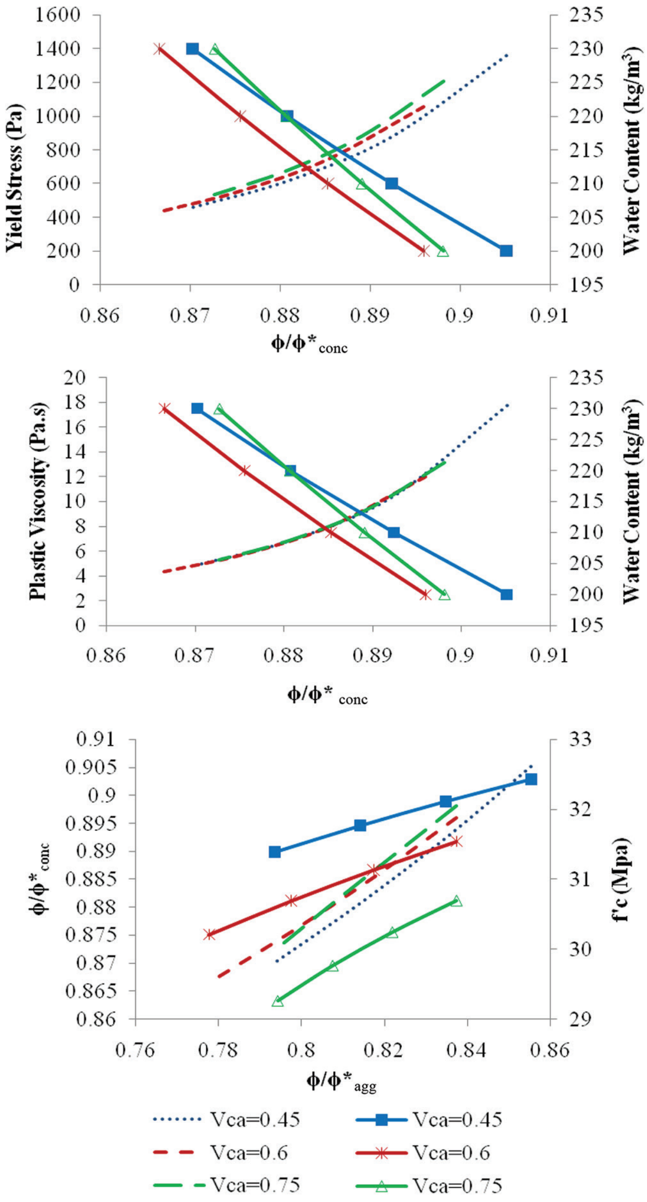


*Fig. 8—Linking rheological properties to strength: Nomograph—*w/c *= 0.60, 14 mm, non-air-entrained.*

CONCLUSIONS

The experimental and analytical study undertaken to investigate the postulation that packing density provides a link between the concrete mixture, the rheological properties, and the compressive strength has revealed the following:

1. The model analogy developed to investigate the desired link was evaluated and has shown to provide good degree of correlation to the experimental data. The correlation factors for yield stress, plastic viscosity, and compressive strength models were 0.74, 0.92, and 0.96 for air-entrained concrete and 0.83, 0.95, and 0.96 for non-air-entrained concrete, respectively.

2. The correlation nomographs demonstrate a continuous link between concrete mixture, rheology, and compressive strength through packing density.

3. The model correlation trends show that an increase in water-cement ratio (*w*/*c*), water content, maximum aggregate size, or air content through entrainment results in a reduction in the yield stress, plastic viscosity, and compressive strength whereas an increase in coarse aggregate volume can result in either a decrease or increase in the properties, depending on the corresponding maximum packing density of concrete. These trends were found consistent with what has been reported in literature.

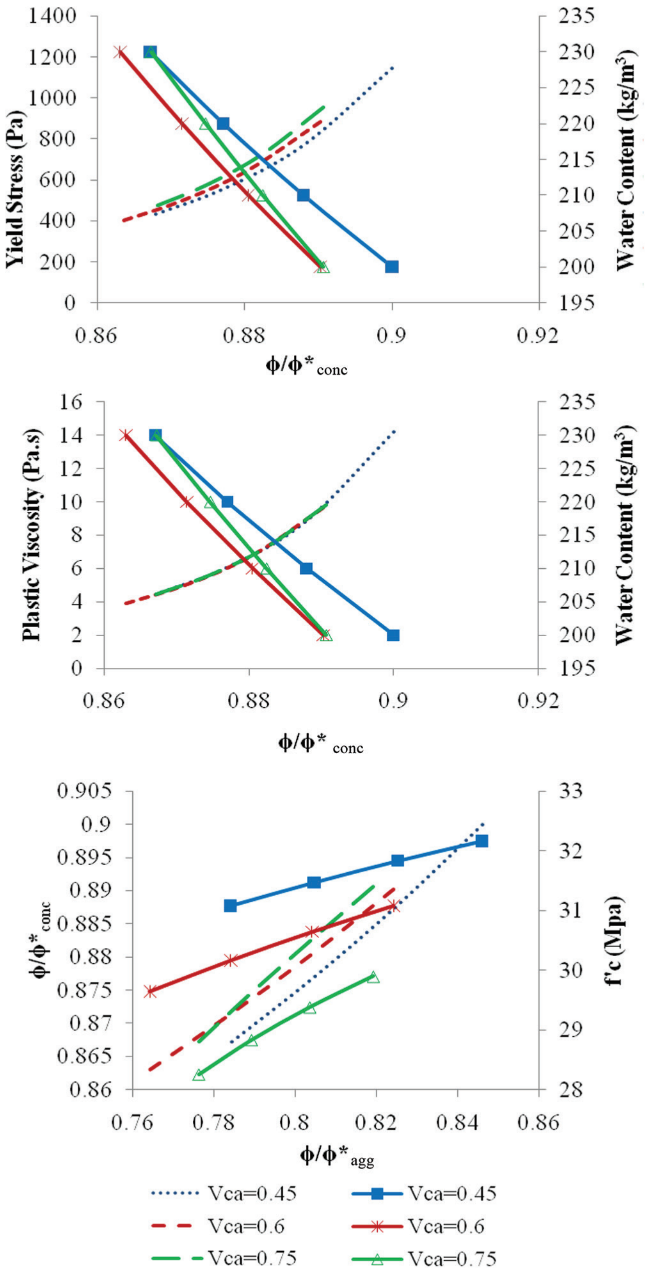


*Fig. 9—Linking rheological properties to strength: Nomograph—*w/c *= 0.60, 20 mm, non-air-entrained.*

Consequently, an improved version of the ACI 211.1 mixture proportioning methodology that incorporates the rheological properties and optimizes packing density of aggregates was proposed, and a case study was conducted to test for its applicability and adequacy. This research outcome, which provides a quantitative approach to design concrete mixtures to meet specific strength requirements and rheology, can also be used to ensure quality control before concrete is cast for more sustainable structures.

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REFERENCES

ACI Committee 211, 1991, "Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete (ACI 211.1-91)," American Concrete Institute, Farmington Hills, MI, 38 pp.

ACI Committee 318, 2005, "Building Code Requirements for Structural Concrete (ACI 318-05) and Commentary (ACI 318R-05)," American Concrete Institute, Farmington Hills, MI, 430 pp.

ASTM C29-97, 1997, "Standard Test Method for Bulk Density and Voids in Aggregate," ASTM International, West Conshohocken, PA, 4 pp.

ASTM C39-16, 2016, Standard Test Method for Compressive Strength of Cylindrical Specimens," ASTM International, West Conshohocken, PA, 7 pp.

ASTM C127-15, 2015, "Standard Test Method for Specific Gravity and Absorption of Coarse Aggregate," ASTM International, West Conshohocken, PA, 5 pp.

ASTM C128-15, 2015, "Standard Test Method for Specific Gravity and Absorption of Fine Aggregate," ASTM International, West Conshohocken, PA, 6 pp.

ASTM C136-14, 2014, "Standard Test Method for Sieve Analysis of Fine and Coarse Aggregates," ASTM International, West Conshohocken, PA, 5 pp.

ASTM C143-15, 2015, "Standard Test Method for Slump of Hydraulic-Cement Concrete," ASTM International, West Conshohocken, PA, 4 pp.

ASTM C192-16, 2016, "Making and Curing Concrete Specimens in the Laboratory," ASTM International, West Conshohocken, PA, 8 pp.

ASTM C260-10, 2010, "Air-Entraining Admixtures for Concrete," ASTM International, West Conshohocken, PA, 3 pp.

Chidiac, S. E., and Habibbeigi, F., 2005, "Modelling the Rheological Behaviour of Fresh Concrete: An Elasto-Viscoplastic Finite Element Approach," *Computers and Concrete*, V. 2, No. 2, pp. 97-110. doi: 10.12989/cac.2005.2.2.097

Chidiac, S. E.; Maadani, O.; Razaqpur, A. G.; and Mailvaganam, N. P., 2000, "Controlling the Quality of Fresh Concrete – A New Approach," *Magazine of Concrete Research*, V. 52, No. 5, pp. 353-363. doi: 10.1680/macr.2000.52.5.353

Chidiac, S. E.; Maadani, O.; Razaqpur, A. G.; and Mailvaganam, N. P., 2003, "Correlation of Rheological Properties to Durability and Strength of Hardened Concrete," *Journal of Materials in Civil Engineering*, ASCE, V. 15, No. 4, pp. 391-399. doi: 10.1061/(ASCE)0899-1561(2003)15:4(391)

Chidiac, S. E., and Mahmoodzadeh, F., 2009, "Plastic Viscosity of Fresh Concrete – A Critical Review of Predictions Methods," *Cement and Concrete Composites*, V. 31, No. 8, pp. 535-544. doi: 10.1016/j.cemconcomp.2009.02.004

Chidiac, S. E., and Mahmoodzadeh, F., 2013, "Constitutive Flow Models for Characterizing the Rheology of Fresh Mortar and Concrete," *Canadian Journal of Civil Engineering*, V. 40, No. 5, pp. 475-482. doi: 10.1139/L2012-025

Chidiac, S. E.; Moutassem, F.; and Mahmoodzadeh, F., 2013, "Compressive Strength Model for Concrete," *Magazine of Concrete Research*, V. 65, No. 9, pp. 557-572. doi: 10.1680/macr.12.00167

de Larrard, F., 1999, *Concrete Mixture Proportioning: A Scientific Approach*, Spon Press, London, UK.

Ferraris, C. F., and de Larrard, F., 1998, "Testing and Modelling Fresh Concrete Rheology," *NISTIR 6094*, National Institute of Standards and Technology, Gaithersburg, MD.

Frankel, N. A., and Acrivos, A., 1967, "On the Viscosity of Concentrated Suspension of Solid Spheres," *Journal of Chemical Engineering Science*, V. 22, No. 6, pp. 847-853. doi: 10.1016/0009-2509(67)80149-0

Goltermann, P.; Johansen, V.; and Palbol, L., 1997, "Packing of Aggregates: An Alternative Tool to Determine the Optimal Aggregate Mix," *ACI Materials Journal*, V. 94, No. 5, Sept.-Oct., pp. 435-443.

Johansen, V., and Andersen, P. J., 1996, "Particle Packing and Concrete Properties," *Materials Science of Concrete II*, American Ceramic Society, Westerville, OH, pp. 111-147.

Kosmatka, S. H.; Kerkhoff, B.; and Panarese, W. C., 2002, *Design and Control of Concrete Mixtures*, seventh Canadian edition, Cement Association of Canada, Ottawa, ON, Canada.

Laskar, A. I., and Talukdar, S., 2007, "Correlation Between Compressive Strength and Rheological Parameters of High-Performance Concrete," *Research Letters in Materials Science*, V. 2007, pp. 1-4. doi: 10.1155/2007/45869

Laskar, A. I., and Talukdar, S., 2008, "A New Mix Design Method for High Performance Concrete," *Asian Journal of Civil Engineering*, V. 9, No. 1, pp. 15-23.

Mehdipour, I., and Khayat, K. H., 2019, "Elucidating How Particle Packing Controls Rheology and Strength Development of Dense Cementitious Suspensions," *Cement and Concrete Composites*, V. 104, p. 103413. doi: 10.1016/j.cemconcomp.2019.103413

Montgomery, D. C., and Runger, G. C., 2003, *Applied Statistics and Probability for Engineers*, third edition, John Wiley and Sons Inc., New York.

Moutassem, F., 2010, "Packing Density Links Concrete Mixture, Rheology, and Compressive Strength," PhD dissertation, McMaster University, Hamilton, ON, Canada.

Moutassem, F., and Chidiac, S. E., 2008, "Evaluation of Packing Density Models for Concrete Applications," *2nd Canadian Conference on Effective Design of Structures*, McMaster University, Hamilton, ON, Canada.

Moutassem, F., and Chidiac, S. E., 2016, "Assessment of Concrete Compressive Strength Prediction Models," *KSCE Journal of Civil Engineering*, V. 20, No. 1, pp. 343-358. doi: 10.1007/s12205-015-0722-4

Moutassem, F., and Miqdadi, I., 2020, "Sustainability-Model Approach for Chloride Permeability Based on Concrete Mixture," *Structures*, V. 28, pp. 983-990. doi: 10.1016/j.istruc.2020.09.041

Quiroga, P., 2003, "The Effect of the Aggregate Characteristics on the Performance of Portland Cement Concrete," PhD dissertation, The University of Texas at Austin, Austin, TX, 382 pp.

Rasmussen, R. O.; Ruiz, J. M.; Rozycki, D. K.; and McCullough, B. F., 2002, "Constructing High-Performance Concrete Pavements With FHWA HIPERPAV Systems Analysis Software," *Transportation Research Record: Journal of the Transportation Research Board*, V. 1813, No. 1, pp. 11-20. doi: 10.3141/1813-02

Schindler, A. K., and Folliard, K. J., 2005, "Heat of Hydration Models for Cementitious Materials," *ACI Materials Journal*, V. 102, No. 1, Jan.-Feb., pp. 24-33.

Shilstone, J. M. Sr., and Shilstone, J. M. Jr., 1993, "High Performance Concrete Mixtures for Durability," *Proceedings of the Symposium on High Performance Concrete in Severe Environments*, SP-140, P. Zia, ed., American Concrete Institute, Farmington Hills, MI, pp. 281-306.

Tasi, C. T.; Li, S.; and Hwang, C. L., 2006, "The Effect of Aggregate Gradation on Engineering Properties of High-Performance Concrete," *Journal of ASTM International*, V. 3, No. 3, pp. 1-12.

Tattersall, G. H., and Banfill, P. F. G., 1983, *The Rheology of Fresh Concrete*, Pitman Publishing, 356 pp.

Wong, H. H. C., and Kwan, A. K. H., 2008, "Packing Density of Cementitious Materials: Measurement and Modelling," *Magazine of Concrete Research*, V. 60, No. 3, pp. 165-175. doi: 10.1680/macr.2007.00004