Zych *Int J Concr Struct Mater* (2022) 16:71
https://doi.org/10.1186/s40069-022-00555-3

International Journal of Concrete Structures and Materials

REVIEW **Open Access**

History of the Development of Analytical Models of Cracking of Restrained Walls on a Given Edge Since 1968

Mariusz Zych*

Abstract

This research paper presents and comments on analytical models for calculating the widths of cracks formed as a result of imposed deformations generating tensile stresses in reinforced concrete base-restrained members. This issue regarding the mechanics of concrete structures has been presented on the basis of calculation models since 1968. In accordance with the current regulations of the European standard, the mechanics of the cracking of base-restrained members have been presented in a very simplified way, which was justified by a limited number of research studies performed on such members as well as in a few subject publications. The main purpose of this work was to present especially those models that had the greatest practical significance within a specific period of time or formed the basis for further studies of other authors. In addition, future trends in the development of computational tools are presented. The chronologically presented development of design ideas, which takes into account varying degrees of advancement of the mechanics of cracking due to the distinctly different design consequences, is a valuable source of information and an inspiration for subsequent researchers. In the second part of the paper, a few of the most important issues connected with the calculation of the crack width in base-restrained walls are presented. It is shown that currently, on the basis of the up-to-date knowledge, there are possibilities to create more complementary standard guidelines, which is already taking place in the case of European guidelines.

Keywords: Restrained wall, Imposed deformation, Crack mechanics, Analytical models, Codes

1 Introduction

Almost every RC (reinforced concrete) structure is accompanied by the occurrence of imposed deformations. The main question is whether they are large enough to significantly increase the stress imposed on the structure or its members. Depending on the type of the structure and the type of the load, the mechanism of cracking is different. A special case of such structural members are walls or their segments constructed on a sufficiently stiff foundation which, at the time of the occurrence of imposed deformations in the wall, does not enable the free shortening of the wall. This frequently results in excessive cracking (Jędrzejewska et al., 2020), which should always be controlled due to the serviceability limit state (SLS). This research paper analyses the basic assumptions of the mechanics of cracking under the influence of imposed deformations. However, it should be emphasised that the range of influences contributing to the cracking of RC structures is diverse. With regard to the current guidelines for the standards and the influences of external loads, this range has been described in detail by Knauff (2012). By contrast, in their research paper, Knauff et al. (2018) discuss the current provisions regarding calculation of crack widths, formed under the thermal–moisture influences, also taking into account German amendments to DIN EN

Journal information: ISSN 1976-0485/eISSN 2234-1315

*Correspondence: mzych@pk.edu.pl

Department of Prestressed Structures, IMiKB, Faculty of Civil Engineering, Cracow University of Technology, Warszawska St, 24, 31-155 Krakow, Poland

1992–1 (2010), contained in DIN EN 1992-1/NA (2011). In general, however, the issue of the mechanics of cracking from imposed deformations requires a much wider view, including material aspects and the technology of constructing massive elements (Kiernożycki, 2003). In addition, in unfavourable computational situations of imposed loads acting on the structure, thermal influences occurring after the erection of the entire structure and the influence of external loads, the risk of cracking and creating excessive crack widths are intensified. One such example was analysed by Buczkowski (1993). In the specific case of the thermal load of rectangular tanks buried in the ground, Buczkowski (1993) demonstrated that the effects of ground pressure are significantly increased by the thermal load.

One of the first studies of base-restrained members was conducted by Stoffers (1978) and analysed the influence of reinforcement, wall geometry and restraint conditions on the morphology of cracks, their spacing and their widths, which enabled the introduction of such factors in the mechanism of cracking which would enable the calculation of crack width depending on the diameter and spacing of the reinforcement. Subsequent studies of various authors focused on the formulation of the computational model, assuming such a relationship of height to length for which exceeding the tensile strength of concrete led to the formation of dilatation cracks. As a result of making significant progress in determining the development of the heat of hydration and its influence on the development of physical properties, as well as the formal description of these phenomena, further research studies enabled attempts to combine and expand the problem of the mechanics of cracking by taking the development of thermal stresses increased by restraints along the edges of the member into consideration: Van Breugel (1982, 1995), Emborg (1989), Rostásy and Onken (1994). As far as analyses using FEM (fine element method) are concerned, attention should be paid to the research studies of the team of Pettersson and Thelandesson (2001a, 2001b) and Pettersson et al. (2002). These studies present a wide parametric analysis of the influence of the properties of concrete, the amount of reinforcement and the boundary conditions on the maximum crack width. The issue was simplified to 2D, i.e., only average strains were considered on the wall thickness. The intensive development of numerical methods enabled further refinements to the models, as exemplified by research studies performed by the team of Flaga and Klemczak (2016), Flaga (2011), which contain the proposal of an advanced numerical model and an engineering model which, from the point of view of designers who do not have access to advanced computer programs, allowing both the size of the deformation and its effects to be determined. The problem of the mechanics of cracking under the influence of imposed deformations takes multiple forms. For example, Klemczak and Knoppik-Wróbel (2015) and Knoppik-Wróbel (2015) presented a significant influence of the support conditions on the degree of restraint. If wall rotation is considered, the degree of restraint in the structural joint increases, but it decreases in the upper part of the wall. This effect is more visible in the case of longer walls and it is almost imperceptible in the case of shorter walls.

This research paper attempts to comment on some of the most common models (since 1968) to calculate crack widths in base-restrained members. Their development followed the progress in the research performed on these members and the conducted parametric analyses. The activities performed in various scientific centres were finalised with the issuance of the first European standard EN 1992-3 (2006), which proposes, for example, an approach to determine crack widths in base-restrained members. Much of the information contained in EN 1992-3 (2006) is quoted from the British Standard BS 8007 (1987) regarding the design of tanks for liquids. Separate provisions in this regard also apply in the United States (ACI 207.2R-95, 1995) and in Japan (JCI, 2008).

First, this paper presents the development of the approach to design in the field of the fundamental issue of the calculation of the width of the cracks in maturing concrete and it thus provides the inspiration for the improvement of the current design guidelines and also for the creation of new computational models.

2 Chronological List of Selected Analytical Models

2.1 Evans & Hughes, 1968 Model

Evans and Hughes (1968) were among the first to perform studies on imposed strains in a real structure. The results of their research confirmed a larger scale of strains caused by a change of concrete hardening than by its shrinkage. They proposed the following equation for cracks spacing in a wall restrained along the bottom edge:

$$\frac{f_{ct} \cdot \phi}{f_b \cdot 2 \cdot \rho} \geq S \geq \frac{f_{ct} \cdot \phi}{f_b \cdot 4 \cdot \rho} \tag{1}$$

where f_{ct} is tensile strength of concrete and f_b is mean bond strength.

Evans and Hughes (1968) assumed that the initial crack spacing halved when further cracking formed. By contrast, stresses in concrete increase linearly from zero in the cracked cross section to the maximum value at the distance of s_{min} from cracked section. It follows that with a degree of reinforcement greater than ρ_{crit} (i.e., the degree of reinforcement at which the reinforcing steel does not become plastic), the maximum mean tensile strain in the uncracked cross section along the length s_{min} adjacent to the crack is ½ε_{ctu}. If the next crack is formed at a distance of s (usually greater than s_{min}), then the mean strain at the length $s/2$ is equal to ½$\varepsilon_{ctu}(s/2s_{min})$. Thus, Evans and Hughes (1968) proposed a formula for calculating the crack width in the following form:

$$w = s \cdot \left(\varepsilon_{sh} + \varepsilon_{th} - s\varepsilon_{ctu}/4s_{\min}\right) \tag{2}$$

where ε_{sh} is actual shrinkage strain including internal restraints from reinforcement and ε_{th} is strain from temperature changes.

2.2 Hughes & Miller, 1970 Model

Hughes and Miller (1970) performed their studies on three RC walls. Measurements were taken of strains, changes of moisture and temperature in the period of concrete hardening. They showed compatibility between the measurements and calculations done after Eqs. (1) and (2). Moreover, they stated that cracks develop first in the walls connections joints, next they can develop in the wall itself. They also stated that cracking in the period of concrete hardening is best restricted by the use of steel formwork and decreasing the temperature of the hardening concrete by watering it at the earliest possible moment, which is to result in earlier removal of heat from the structure.

2.3 Stoffers, 1978 Model

The research studies presented by Stoffers (1978) are still some of the most important contributions in the field of the assessment of cracking of members joined along the bottom edge. The studies included eighteen different research models, three of which were duplicates. Due to their small size, micro-concrete was used. Two basic types of members were executed, the bases of which were prestresed steel sections. Having concreted the members, the prestressing force was gradually reduced and thus the elongation at the lower edge of the member was introduced, which caused their cracking.

In the performed tests, the influence of reinforcement was clearly visible for the models of Series I and II, with a degree of reinforcement exceeding 0.5%. For lower degrees of reinforcement, this effect was less visible or not noticeable at all. In addition, in all cases, the number of cracks in the vicinity of the steel beam was the highest. For the members of Series I and II, the average crack width increased with the distance from the steel beam. In the case of Series III, the largest average crack width occurred 0.20 m above the steel beam.

The results of the experimental research formed the basis for the development of a model enabling the crack width and the minimum degree of reinforcement to be determined. Stoffers (1978) first describes the procedure of cracking based on the diagram of a bar joined at opposite ends. Later, this method was modified to include the influence of a joint along the bottom edge.

The following assumptions were adopted:

- a member fully restrained at opposite ends,
- linear distribution of strains in the concrete in the vicinity of cracks,
- full bond of concrete to reinforcement beyond regions of cracking,
- length of the section "z" along which the bond stress increases, takes into account: diameter of reinforcement, ratio of steel, concrete moduli of elasticity, degree of reinforcement, stresses in steel and ultimate value of bond stresses.

In the model of the base-restrained wall, it was assumed that:

- the number of cracks in the wall increases with decreasing distance y from the base,
- there is fixed joint and *L/H ratio* > *10*,
- stresses in the wall cross section remain constant at the entire height,
- cracking near the base has little effect on stresses on the opposite edge.

In addition, based on the experiments, it was assumed that the expected spacing of the dilatation cracks is equal to (1.0–1.5)H and that the average crack spacing is $\Delta l = y$ (y—distance from the base). This spacing is the result of restraining at the base, not of reinforcement.

In view of the above assumptions, the equations for calculating stresses in reinforcing steel and crack widths in the wall restrained along the lower edge take the following form:

$$\sigma_s = -\frac{-2f_b\alpha_e\rho y}{\phi} + \sqrt{\left(\frac{2f_b\alpha_e\rho y}{\phi}\right)^2 + \frac{4f_bE_sy\varepsilon_y}{\phi}} \tag{3}$$

$$w_y = \frac{\phi\sigma_s^2}{4f_bE_s} \cdot \frac{1-\rho}{1+(\alpha_e-1)\rho} \tag{4}$$

where f_b is bond stress, α_e is ratio of moduli of elasticity of steel and concrete, ϕ is diameter of reinforcement, y is distance of the considered wall level from joint with the foundation, ε_y is strain in the wall at y height, and ρ is degree of reinforcement.

2.4 Harrison, 1981 Model

Equation (17) formulated in 1990, presented later in the paper, is very similar to Harrison's proposal from 1981(Harrison, 1981):

$$w_{\max} = s_{\max} \cdot \left(0.5R_b \cdot (\varepsilon_{th} + \varepsilon_{sh}) - \varepsilon_{ult}/2\right) \quad (5)$$

which was a modification of the expression contained in BS 5337 (1976):

$$w_{\max} = s_{\max} \cdot \left(0.5\varepsilon_{th} + \varepsilon_{sh} - \varepsilon_{ult}/2\right) \quad (6)$$

In the course of changing expressions used to calculate the crack width, Harrison (1981) [Eq. (5)] introduced the main correction. It consisted of introducing the coefficient of the external degree of restraint, which enabled the prediction of the changes in crack widths at a given height of the wall, while in BS 5337 (1976), a constant crack width was defined. In addition, in BS 5337 (1976), as with BS 8007 (1987), concrete creep was included in the 50% reduction in the restrained part of thermal strains.

2.5 BS 8007 (1987) Standard

In the proposed standard BS 8007 (1987), the method of calculating crack width was, to a certain extent, very similar to the current provisions of EN 1992-3 (2006). The crack width was calculated from the following formula:

$$w_{\max} = s_{\max} \cdot \varepsilon, \quad (7)$$

where the spacing of the cracks was defined in the same manner as in the Evans and Hughes's model (Evans & Hughes, 1968):

$$s_{\max} = \left(f_{ct}/f_b\right) \cdot \phi/2\rho \quad (8)$$

The imposed strain was determined as

$$\varepsilon = \left[(\varepsilon_{cs} + \varepsilon_{te}) - 100 \cdot 10^{-6}\right] \ or\ \varepsilon = R \cdot \alpha_T \cdot \Delta T \quad (9)$$

where ε_{cs} is shrinkage strain, ε_{te} is thermal strain, R is coefficient of degree of external restraint, α_T is coefficient of thermal expansion of concrete, and ΔT is temperature change.

2.6 Rostásy & Henning, 1989 Model

Rostásy and Henning (1989) developed a procedure for designing reinforced concrete walls restrained along the bottom edge based on the results of Stoffers's research (Stoffers, 1978). The geometry and stiffness of the joined members on the values of the normal force N and the moment M were taken into account for calculating the stresses in the uncracked wall using the theory of elasticity of deep beams described by Schleeh (1962). Stresses in concrete were calculated for different slenderness of the wall L/H and also for two extreme cases of bending stiffness of the foundation: case *a)* $E_F \cdot I_F = 0$, case *b)* $E_F \cdot I_F = \infty$. In both cases, the tensile stiffness was assumed to be $E_F \cdot A_F = \infty$.

The basic parameters defining the influence of the foundation on the limited imposed strains of the wall are as follows: S_D—ratio of tensile stiffness of the wall and of the foundation, S_B—ratio of bending stiffness of the wall and of the foundation, and p—parameter characterising the height. The criterion of ultimate strains was assumed as the criterion of thermal crack formation during the period of member cooling:

$$\varepsilon_0 \leq crit\varepsilon_r = f_{ctm}/E_W = \varepsilon_{ct} \quad (10)$$

The ultimate strain was assumed as: "critε_r = *0.005%—for early age concrete and* critε_r = *0.008% for hardened concrete*". When the wall self-heating temperature reaches the limit value, dilatation cracks is formed in the spacing of $s_D = 2H$, and after the limit value of the self-heating temperature is exceeded, the crack spacing is equal to $s_D = H$. This condition is described by the parameter:

$$m_{De} = \alpha_T \cdot \max \Delta T \cdot E_W/f_{ctm} \quad (11)$$

Assuming a mean length of relaxation l_{Em} according to the formula:

$$l_{Em} = c + 0.3s + 0.1\phi/\rho_r \quad (12)$$

Stresses in reinforcing steel and crack widths for their stabilised spacing are described by the following formulas:

$$\sigma_{sre} = \frac{f_{ctm}}{\rho} \cdot \frac{m_{De} \cdot \overline{v}_o}{1 + \frac{2l_{Em}}{3Hn\rho}} \quad (13)$$

$$w_{se} = \frac{2f_{ctm}}{3E_s\rho} \cdot \frac{m_{De} \cdot \overline{v}_o}{1 + \frac{2l_{Em}}{3Hn\rho}} l_{Em} \quad (14)$$

This model takes into account stiffness of the joined members without considering their length. The authors provide a geometrical criterion for the formation of cracks, according to which, the dilatation crack may be formed with the ratio of $L/H > 2$.

2.7 Al-Rawi, Kheder & Fadhil, 1990 Model

Al-Rawi and Kheder (1990), while modifying Eq. (8) for the crack spacing contained in BS 8007 (1987), assumed that in the base-restrained walls, this formula depends on both the strength of reinforcement and the joint along the lower edge. Therefore, they took into account the height of the wall in the expression for the crack spacing:

$$s_{\min} = \frac{k \cdot \phi \cdot H}{\rho \cdot H + k \cdot \phi} \text{ and } s_{\max} = 2s_{\min} \tag{15}$$

where $k = f_t/(4f_b) = 0.57$, 0.68 and 0.85 for ribbed, deformed and smooth rebars, respectively, ρ is degree of reinforcement, ϕ is diameter of reinforcement, and H is height of the wall.

Kheder and Fadhil (1990), continuing the approach of Al-Ravi and Kheder (1990), took into account the effect of the elastic shortening of the foundation with the coefficient K according to ACI 207 (1973):

$$K = \left(\frac{1 + A_w \cdot E_w}{A_f \cdot E_f}\right)^{-1} \tag{16}$$

where A_w is cross-sectional area of wall, E_w is modulus of elasticity of concrete in wall, A_f is cross-sectional area of foundation, and E_f is modulus of elasticity of concrete in foundation.They then modified the expression to the maximum crack width contained in BS 8007 (1987). Finally, they came up with an expression dependent upon, among other factors, the degree of restraint and elastic shortening of the foundation:

$$w_{\max} = 0.5s_{\max} \cdot (0.5KR \cdot (\varepsilon_{th} + \varepsilon_{sh}) - \varepsilon_{ctu}) \tag{17}$$

where R is coefficient of restraint determined on the basis of diagrams obtained from numerical calculations without taking creep into account, remaining denotations are as in Eq. (2).

Kheder and Fadhil (1990) decided that limiting the width of cracks in the walls restrained along the lower edge results from both reinforcement and restraint at the base, and therefore, less reinforcement can be used than in members restrained along the opposite edges. In addition, they stated that to use more economical solutions, the degree of reinforcement should depend on the variability of the degree of wall restraint.

2.8 Al- Kheder et al., 1994a, 1994b Model

In their next study, Kheder et al. (1994a) defined the formula for crack width in the following form:

$$w_{\max} = s_{\max} \cdot \left[C_1(R_b - C_2R_a) \cdot \varepsilon_{free} - \varepsilon_{ctu}/2\right] \tag{18}$$

where R_b is restraint coefficient before cracking in the middle of wall length, R_a is restraint coefficient after cracking on wall edge (defined using FEM for segment with L/H two times smaller and without reinforcement), C_1 is factor including influence of creep equal to 0.6, and C_2 is factor equal to 0.8 (value estimated based on crack width measurement at the level just above foundation).

In 1994, Kheder et al., (1994a, 1994b) also concluded that in walls with the ratio $L/H > 5$, the crack width increases from the base toward the upper edge. However, for walls meeting the condition $2 < L/H < 5$, the crack develops from the base upward, reaching its maximum width at a height of 0.2 to 0.4H. However, above this level, the crack width decreases.

2.9 Ivanyi (1995) Model

In the model developed by Ivanyi (1995), it was assumed that the wall was of infinite length, only dilatation cracks were considered, and in the place where the wall was joined to the foundation, a fixed joint was assumed. The consequence of these assumptions was the constant value of stresses along the wall height, as well as the fact that the largest crack widths occurred on the upper edge of the wall. With reference to the model of Rostásy and Henning (1989), Ivanyi (1995) makes the assumptions in his model that it "moves" within the L/H ratio from 10 to ∞, i.e., these are constant stresses in the cross section, hence in the model (Rostásy & Henning, 1989) $\eta_b = 0.0$, $n_b = 1.0$. The basis for the formulated relationships were the results of calculations made in the *NISA* program using a linear-elastic material. Another assumption (as in Stoffers (1978)) is that cracks can be formed in the spacing a equal to the wall height H or 1.5H. In the case of long, unreinforced walls, the width of cracks on their upper edge results from the free shortening of both segments of the wall:

$$w_2 = 2k_o H \varepsilon_o \tag{19}$$

where ε_o is strain which would occur in free member.

It should be noted that with this assumption, the crack width would increase with the height of the wall, and the assumption of $k_o = (1.0–1.5)$ provides the possibility of obtaining a large range of results. The influence of reinforcement was modelled with elastic members. To determine the stiffness c_s of reinforcement, model assumptions presented by Falkner (1969) and Leonhardt (1978) were used:

$$2l_E \varepsilon_{sm} = l_o \varepsilon_s \tag{20}$$

where l_E is length of reinforcement relaxation zone in crack zone, ε_s is strain of reinforcement in cross section through crack, and ε_{sm} is mean strain of reinforcement in stress relaxation zone.With the assumptions $2l_E = 50 + 0.2\phi/\rho_r$ and $\varepsilon_{sm}/\varepsilon_s = 0.4$, the value of c_s was determined:

$$c_s = \frac{E_s \cdot 2A_s}{l_c} = \frac{E_s 2\varphi^2\pi}{4(0.5l_o)} = \frac{E_s\varphi^2\pi}{0.4a_m} \tag{21}$$

The crack width in a reinforced wall is described by the formula:

$$w_{2s} = \frac{F_s}{c_s} = 2\left(\varepsilon_o k_o H - \frac{F_s}{E_b ds} k_o H\right), \tag{22}$$

in which the value of the force F_s is

$$F_s = \frac{\varepsilon_o k_o H}{1/c_s + k_o H / E_b d \cdot s} \tag{23}$$

2.10 Paas, 1998 Model

The Paas's model (Paas, 1998) is an extension of the approach proposed by Ivanyi (1995). The basic assumption of this model is the analysis of the dilatation cracks, supported by individual studies of the cracking of the walls joined along the bottom edge. The crack width in the unreinforced wall is described by the product of horizontal strain $\varepsilon_o(x, t_R)$ from volume changes and the theoretical length of the wall strip $l_e(x, t_R)$, where t_R represents the time at the moment of cracking:

$$w_e(x, t_R) = \varepsilon_o(x, t_R) \cdot l_e(x, t_R) \tag{24}$$

Similar, to the Ivanyi's model Ivanyi (1995), the length l_e depends on the factor k and the wall height H:

$$l_e(x, t_R) = k_e(x, t_R) \cdot H \tag{25}$$

where the factor $k_e(x, t_R)$, called here the geometric factor, is determined on the basis of the calculated strains for appropriate cross sections x. For this purpose, diagrams are used to determine the factor k_e on particular ordinates from $0.125H$ to H for the scheme of the wall joined along the lower edge. For the ratio $L_e/H \geq 2.5$, all the values of $k_e = const$:

$$k_e(x, t_R) = \frac{w_e(x, t_R)}{\varepsilon_o(x, t_R) \cdot H} \tag{26}$$

The next step is to determine the crack width in the reinforced wall. Its one-sided width is defined as

$$w_{s,e}(x, t_R) = \frac{w_e(x, t_R)}{1 + \frac{c_{s,e}(x)}{c_{b,e}(x, t_R)}} \tag{27}$$

where $c_{b,e}(x, t_R)$ is elastic tensile stiffness of concrete and $c_{s,e}(x, t_R)$ is elastic tensile stiffness of reinforcement are described by the following equations:

$$c_{b,e}(x, t_R) = \frac{E_b(x, t_R) \cdot A_b(x)}{l_e(x, t_R)} \tag{28}$$

$$c_{s,e}(x, t_R) = 2E_s A_s(x) / l_o(x) \tag{29}$$

where $A_b(x)$, $A_s(x)$ are surface areas of concrete and steel, respectively.

2.11 Flaga, 2004 Model

Flaga (2004) presented some issues regarding the influence of moisture and thermal fields on the additional stress of concrete structures. The provisions and regulations of EN 1992-1-1 (2003) were demonstrated with reference to determining shrinkage strains, as well as the method of determining shrinkage stresses, taking into account the ultimate tensile strain of concrete. Guidelines were provided to determine the calculation width of the surface zone and the method of "filling" the field of shrinkage tensile stresses. As far as the base-restrained walls are concerned, the following were presented: the method of determining the distribution of thermal and shrinkage stresses; the method of determining the minimum reinforcement; the criterion for limiting the crack width, which results from the EN 1992-1-1 guidelines EN 1992-1-1 (2003), and in particular from the simplified method by determining the permissible diameter of reinforcement.

2.12 Beeby & Forth, 2005 Model

Beeby and Forth (2005) analysed a simplified case of the wall joined along the lower edge (Fig. 1). They assumed a lack of reinforcement and the fact that with the increase of distance to the crack, the stresses are increasingly transferred to the wall by shearing at the contact with the base, until at a certain distance L_o from the crack, stress distribution is constant. Such an assumption is fundamentally different if compared to a member restrained along the opposite edges, where the effect of cracking reduces stiffness globally. In this case, the formation of cracks causes the stiffness to change locally. Outside the L_o area, it is assumed that the stress state is undisturbed and that cracks do not affect the widths of other cracks. Similar assumptions were adopted by Bamforth et al. (2015). They found that a greater degree of restraint

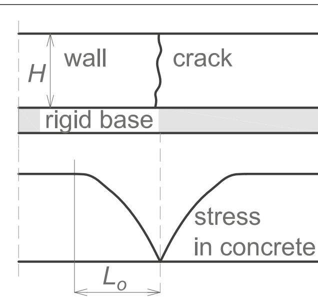

Fig. 1 Distribution of stresses after cracking in a member restrained along lower edge

would result in wider cracks and that the formation of a new crack did not affect the width of the existing cracks.

2.13 EN 1992-1-1 (2004) Standard

According to Appendix M (EN1992-3, 2006), the general expression (7.8) contained in EN 1992-1-1 (2004) and designed to describe crack widths in bending or tensile members should be used to calculate the width of cracks in tank walls.

According to EN 1992-1-1 (2004), in walls subjected to changes in strain from the self-heating of early age concrete, in which the area of horizontal reinforcement A_s does not meet the condition of the minimum degree of reinforcement, and the wall is restrained by a previously executed foundation, the maximum crack spacing can be adopted as 1.3*H*, (*H*—wall height). However, according to EN 1992-3 (2006), for the case of base-restrained members that meet the condition of the minimum degree of reinforcement, and if the spacing of rebars is not greater than $5(c+\phi/2)$, the crack spacing is determined from Eq. (7.11) is in (EN1992-1-1, 2004). In this equation, the effective area of concrete in tension surrounding the reinforcement is taken into account, which results from the introduced concept of primary cracks. Primary cracks in a member section refer to the longest cracks, i.e., cracks which are located only or usually in the area under tension and secondary cracks which result from effective height of the area in tension $h_{c,eff}$ (Fig. 2).

Thus, an adequate amount of reinforcement should provide an adequate amount of secondary cracks to absorb the imposed deformation. Thus, the influence of the strain-restraining member on crack spacing is not taken into account, assuming that it depends only on reinforcement, i.e., the model of the bar restrained along opposite edges. This assumption is completely different with reference to some of the above-mentioned models. The foundation, through a rigid joint to the slab in which shear stresses occur, contributes to the increase of stresses in the wall even without the presence of reinforcement (Fig. 4).

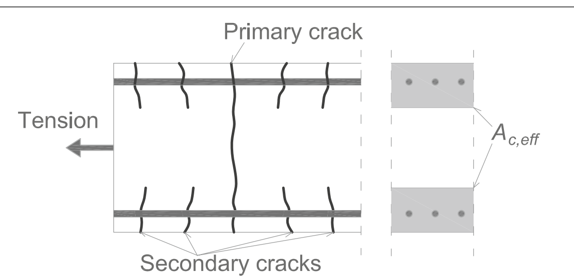

Fig. 2 Crack layout in the tie model

According to EN 1992-3 (2006), the difference in mean strains between steel and concrete is calculated according to the following expression:

$$\varepsilon_{sm} - \varepsilon_{cm} = R_{ax} \cdot \varepsilon_{free} = \varepsilon_r \tag{30}$$

where R_{ax} is index determining degree of restraint of imposed strains resulting from axial restraint induced by members joined with the analysed member, ε_{free} is strain that could occur in a completely free element, and ε_r is restrained part of strains.

Considering the alternative form of Eq. (9), it is identical to Eq. (30). According to the assumptions of EN 1992-3 (2006), it can be stated that in the case of the wall restrained along the lower edge, the crack width is proportional to the restrained part of the strain, i.e., the difference between the actual strain of the member and the strain that would have been formed if the member remained unrestrained.

2.14 ACI 207.2R-07, 2007 Standard

According to ACI 207.2R-07 (2007), the first crack (1) is formed roughly at half the length of the member and develops upward. If $L/H > 2.0$ and the crack develops to a height of 0.2*H*–0.3*H*, then its further development may become unstable and the crack will quickly develop to the very top of the member. After the first crack is formed, there is a redistribution of restraint at the bottom base. A new pair of cracks (2) are formed at roughly half the length of the uncracked area. If $L`/H > 1.0$, ($L` = L/2$) the cracks then develop upward according to the above-mentioned rule. The subsequent cracks develop in the same way until the sum of the crack widths compensates for the changes in volume.

The current standard ACI 207.2R-07 (2007) does not provide guidelines for determining crack widths in members subjected to imposed deformations but only refers to ACI 224R-01 (2001) dedicated to crack control in concrete structures. The standard provides a number of formulas to determine the crack width for typical RC and prestressed members. One of these formulas, which was not designed for cracking from imposed deformations but according to the authors of the standard is the most appropriate, is the formula defining tensile cracking:

$$w = 0.10 \cdot \sqrt[3]{d_c \cdot A} \cdot f_s \cdot 10^{-3} \tag{31}$$

Equation (31) is also included as a reference in ACI 207.2R-95 (1995) which, among many approaches describing crack width, adopted the equation developed by Gergely and Lutz (1968). The formula defining the crack width is based on statistical research, and in the case of massive structures, takes the following form:

$$w = 0.076 \cdot \sqrt[3]{d_c \cdot A} \cdot f_s \cdot 10^{-3} \tag{32}$$

where w is maximum crack width on surface [in.] (note: 1 in = 254 mm), d_c is concrete cover to bar axis [in.], A is mean effective tension area of concrete around reinforcing rebars ($2d_c$ spacing), [in.^{2}], and f_S is calculated stress in steel [ksi] (note: 1 ksi = 6,89 MPa).

Therefore, stresses in reinforcing steel can be calculated by converting the Gergely–Lutz equation Gergely and Lutz (1968) for the crack width, assuming concrete cover of the rebars and their spacing as follows:

$$f_s = \frac{w \cdot 10^3}{0.076 \cdot \sqrt[3]{d_c \cdot A}} \tag{33}$$

The next assumption is to assume stresses at the top of the crack equal to f_t\`. In addition, the sum of the crack widths at each level must be approximately equal to the total change in volume ($K_R L C_T T_E$) minus the $L\,f_t$\`$/E_c$ concrete extension. Hognestad (1962) stated that the mean value of the ratio of the maximum crack width to the mean width is 1.5. If N is assumed to be the number of cracks, and w the maximum crack width, $Nw/1.5$ is the sum of the crack widths on a given length, hence:

$$N \cdot w/1.5 = 12 \cdot L\left(K_R \cdot \alpha_T \cdot T_E - f_t^{'}/E_c\right) \tag{34}$$

where T_E is temperature change of concrete.

For the mean crack spacing L\`:

$$L^{'} = w\Big/\left[18 \cdot \left(K_R \cdot C_T \cdot T_E - f_t^{'}/E_c\right)\right] \tag{35}$$

Calculating the mean crack spacing is necessary to determine the moment transferred by reinforcement. The crack may or may not develop over the entire height of the wall. It depends mainly on the L/H ratio. If the crack develops along a part of the height, only the reinforcement in this part is involved in the moment transfer ($T_C x + A_s^{'} f_s h_c/2$). Reinforcing bars at height h are needed to impose crack spacing equal to L'. For this case, the value of this moment is defined as

$$M_{Rh} = 0.20 \cdot f_t^{'} \cdot B \cdot h^2\left(1 - L^{'}\Big/2h\right) \tag{36}$$

where x is distance from base to resultant tensile stress in concrete, T_C is resultant tensile stress in concrete, h_c is crack height, and $A_s^{'}$ is total area of reinforcement in cross section through crack.

Therefore, the required reinforcement in each strip above the base is defined as

$$A_b = 0.4 \cdot \frac{f_t^{'} \cdot B \cdot h}{f_s N_H}\left(1 - \frac{L^{'}}{2h}\right) \tag{37}$$

where h is analysed distance range above base, N_H is total number of rebars at height h above base, A_b is area of reinforcement required on each wall surface, and A_s\`h/N_H is A_b.

2.15 CIRIA C660 (2007)

In CIRIA C660 (2007), the method of calculation of crack width in imposed deformations is based on the relationship between standards EN 1992-1-1 (2004) and EN 1992-3 (2006). However, the difference is that in the case, where external restraint have their effect, the imposed deformation which determines the crack width is defined as

$$\varepsilon_{cr} = \varepsilon_r - 0.5\varepsilon_{ctu}, \tag{38}$$

In the equation above, it is assumed that after the occurrence of cracks, the average residual strain in concrete amounts to half of the ultimate tensile strain capacity of concrete. This prevents overestimation of the computational width of cracks as stated in EN 1992-3 (2006). Moreover, the restrained area of the imposed deformation ε_r is defined as

$$\varepsilon_r = K_1[(\alpha_c \cdot T_1 + \varepsilon_{ca}) \cdot R_1 + \alpha_c \cdot T_2 \cdot R_2 + \varepsilon_{cd} \cdot R_3], \tag{39}$$

where T_1 is the difference between peak temperature and mean ambient temperature, T_2 is the long-term fall of concrete temperature, α_c is the coefficient of thermal expansion, ε_{ca} is the autogenous shrinkage, ε_{cd} is the drying shrinkage, k_1 is coefficient which takes into consideration stress relaxation caused by concrete creep in the long term ($k_1 = 0.65$ see CIRIA C660 (2007)), R_1 is restraint coefficient in the period when concrete becomes mature, and R_2, R_3 are restraint coefficient for long-term thermal strains and drying shrinkage, respectively.

Strain ε_r used in the expression (38), unlike expression (30) in EN 1992-3 (2006) takes into consideration also strains occurring later, e.g., long-term thermal and shrinkage strains apart from imposed strains occurring during maturing of concrete (i.e., $\alpha_c T_1$ and ε_{ca}).

In addition, in CIRIA C660 (2007) the cracks inducing strain for the case of internal restraint are defined as

$$\varepsilon_{cr} = K_1 \cdot \Delta T \cdot \alpha_c \cdot R - 0.5\varepsilon_{ctu}, \tag{40}$$

where ΔT is the difference in temperature between the centre and the surface of the element and R is the coefficient of internal restraint (acc. to CIRIA C660 (2007) recommended value is 0.42).

In Eq. (40), only the temperature gradient during maturing of concrete was taken into account due to the fact that autogenic shrinkage is almost uniform in the member cross section. According to CIRIA C660 (2007), the general equation used to determine the crack width in base-restrained wall and in the member under the internal restraint is as follows:

$$w_k = s_{r,\max} \cdot \varepsilon_{cr}, \tag{41}$$

where $s_{r,max}$ is maximum crack spacing as it is defined in EN 1992-1-1 (2004), with the difference that the recommended value of coefficient k_1 of reinforcement steel is 1.14, which results from taking into account the reduction of bond strength by a factor of 0.7, when the sufficient bond conditions cannot be guaranteed (e.g., 0.8/0.7 = 1.14).

Thus, according to Eq. (41), the crack width depends mainly on the size of the imposed deformation and the degree of restraint.

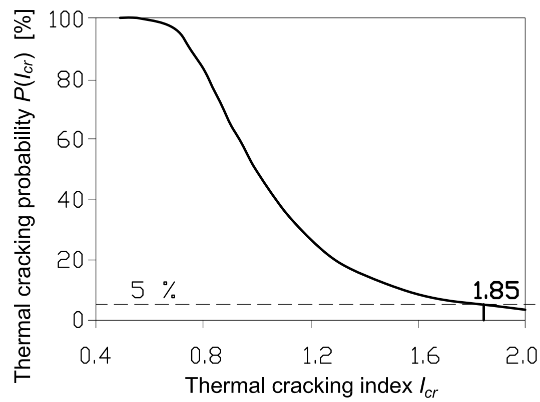

Fig. 3 Dependence between crack index I_{cr} and crack probability

2.16 JCI, 2008 Standard

A completely different approach to the cracking of hardening concrete is presented in JCI (2008) and it concerns the period from casting a member to the structure reaching a temperature equal to the ambient temperature. During this period, the influence of drying shrinkage and non-linear temperature distributions on the thickness of the member that may cause surface cracks are neglected. It is assumed that these phenomena can be effectively eliminated through technology and water curing. The crack control model is based on the probability of crack formation:

$$P(I_{cr}) = 1 - \exp\left[-\left(I_{cr}/0.92\right)^{-4.29}\right] \cdot 100 \tag{42}$$

where I_{cr} = tensile strength/tensile stress strength of concrete.

Based on the calculations of the parameter I_{cr} performed in 3D-FM, and comparisons with the state of cracking of 728 structural members, histograms of the crack frequency or its absence were prepared. These formed the basis for defining Eq. (42). The probability of cracking is defined as the ratio of crack frequency to the total number of cases in each crack index interval I_{cr} (Fig. 3).

To avoid cracking, a probability of 5% should normally be assumed to be the limit value. The limit value of probability should result from the quality and conditions of the structure. According to JCI (2008), cracks will not occur when the following condition is met:

$$P_c/P_t \leq 1.0, \tag{43}$$

where P_t is cracking probability limit value and P_c is calculated cracking probability.

In practice, this condition can be verified, as illustrated in Fig. 3 based on the limit value of I_{cr} = 1.85. The permissible crack width is determined by verifying the following condition:

$$\gamma_i w_c/w_a \leq 1.0, \tag{44}$$

where γ_i is safety factor, in general assumed as 1.0, w_a is limit crack width, and w_c is calculated crack width according to equation:

$$w_c = \gamma_a\left(-0.071/\rho\right) \cdot (I_{cr} - 2.04), \tag{45}$$

where ρ is degree of reinforcement (method calibrated for $\rho \epsilon$ < 0.25–0.93% >), I_{cr} is crack index greater than 1.85, γ_a is safety factor to determine crack width, which should fall within the range of 1.0–1.7.

Equation (44) results from the implementation of linear correlations between the measured maximum crack widths in the experiments and the crack index. The tangent of each angle of the regression line is presented as a function of reinforcement $y = 0.071/\rho$, which provides the basis for writing the equation for w_c in the form of Eq. (45). Equation (45) clearly demonstrates that the higher the degree of reinforcement, the smaller the efficiency of its increase. For example, for $\rho = 0.75$ and increasing up to 1.0%, as well as 1.5 and increasing up to 1.75%, the crack width is narrowed down to 33 and 17%, respectively.

2.17 Bamforth et al., 2009 Model

Bamforth et al. (2009) proposed a new method to control cracking for elements subject to continuous edge restraint. The basis of this concept was the fact that the edge is restrained in a similar way to the reinforcement by attracting some of the load and thus distributing the cracks. Thus, on the basis of the above mentioned assumptions, the expression $A_{s,min}$ according to EN 1992-1-1 (2004) is in Eq. (7.1) was modified as follows:

$$A_{s,\min} = \left(1 - 0.5 \cdot R_{edge}\right) \frac{k \cdot k_c \cdot f_{ct,eff}}{f_{yk}} \cdot A_{ct}, \tag{46}$$

The crack width is calculated as the sum of w_{k1} and w_{k2}, i.e., the width of the crack immediately after its occurrence (w_{k1}—concerns the transferred strain from concrete to steel) and the crack width w_{k2} as a further result of imposed strain. At the first stage, the mean residual strain of steel is defined according to Eq. (47), which is a modification of the expression (M.1) in EN 1992-3 (2006) for the restrained member on opposite edges:

$$\varepsilon_{smr} = \frac{0.5 \cdot \alpha_e \cdot f_{ct,eff}\left[\left(1 - R_{edge}\right) \cdot B + 1\right]}{E_s\left[1 - \frac{s_{r,\max}}{L_{eff}}\left[1 - 0.5 \cdot \left(B + \frac{1}{(1 - R_{edge})}\right)\right]\right]}, \tag{47}$$

where $B = k \cdot k_c / \alpha_e \cdot \rho + 1$, E_s, α_e and ρ acc. to EN 1992-1-1 (2004), $s_{r,max}$ is maximum crack spacing according to EN 1992-1-1 (2004) Eq. (7.11) taking into account the correction of coefficient k_1 according to CIRIA C660 (2007), L_{eff} is the effective length over which strain relief occurs $= k_L H/R_{edge}$, and k_L is the length coefficient $= 1.5$.

In the first stage, the crack width is described by the following equation:

$$w_{k1} = s_{r,\max}(\varepsilon_{smr} - 0.5\varepsilon_{ctu}), \tag{48}$$

In the second stage, the crack widens when $\varepsilon_r > \varepsilon_{ctu}$. By defining the residual free contraction ε_{res} taking into consideration the creep before the occurrence of a crack in the form:

$$\varepsilon_{res} = \varepsilon_{free} - \frac{\varepsilon_{ctu}}{R_{edge} \cdot k_1}, \tag{49}$$

and assuming that the average restraint within the zone of cracking is $0.5R_{edge}$ and the crack width increase is proportional to ($1\text{–}0.5R_{edge}$), the expression for w_{k2} is as follows:

$$w_{k2} = s_{r,\max}\left(1 - 0.5R_{edge}\right) \cdot k_1 \cdot \left(\varepsilon_{free} - \frac{\varepsilon_{ctu}}{R_{edge} \cdot k_1}\right). \tag{50}$$

2.18 DIN EN 1992-1-1/NA (2011) Standard

The necessity of taking into account the total effect of external loads and imposed strains on the width of cracks was first discussed in DIN EN 1992-1-1/NA (2011). Research performed by Turner et al. (2017) proves that summing up the effects of imposed strains and external loads is the correct approach. Moreover, it was demonstrated that loads occurring later can double the width of the crack in relation to its width during the maturing of concrete.

The basic principle adopted by DIN EN 1992-1-1/NA (2011) (after DBV (2006)) is that the maximum tensile force from the imposed strain resulting from the member crack propagation is restricted, up to a value of imposed strains of 0.8%, which is associated with, as shown in the research by Knauff et al. (2018), additional stresses in the reinforcement from imposed strains at a level of at least 160 MPa. Above this value, is the stage of stabilised crack spacing and the acting force will increase the cracks width (Fig. 4).

It must be pointed out that the approach according to DIN EN 1992-1-1/NA (2011) refers to the tie member and it is often adopted to the calculation of the width of cracks in base-restrained walls, i.e., as a tie model separated from a given section of a wall.

In DIN EN 1992-1-1/NA (2011), as with EN 1992-1-1 (2004), crack spacing is defined in detail only for a tie model. Such an approach to base-restrained walls may be the correct approach only when the wall is strongly reinforced (Zych & Seruga, 2019). However, a significant change in DIN EN 1992-1-1/NA (2011) in relation to EN 1992-1-1 (2004) is abandoning the concept of coefficients k_1, k_2, k_3, k_4. In addition, only steel reinforcement is taken into account. Additive element k_3c_{nom}, which takes into account the influence of the thickness of insulation on the bigger spacing of cracks, is neglected. Element k_3c_{nom}, in a paper by Zych and Jaromska (2019), is graphically interpreted as a section in which there is no bond between steel and concrete. Such an assumption was correct for smooth rebars which are not applied nowadays. Thus, the approach in DIN EN 1992-1-1/NA (2011) in this matter is also up to date. According to the author of DIN EN 1992-1-1/NA (2011) similar to EN 1992–1-1 (2004), the problem of crack spacing in base-restrained members remains unsolved. It is caused by the fact that the current guidelines used to calculate crack spacing refer only to structures which do not meet the requirements of minimum reinforcement. In addition, in DIN EN 1992-1-1/NA (2011) there are no definitions of mean

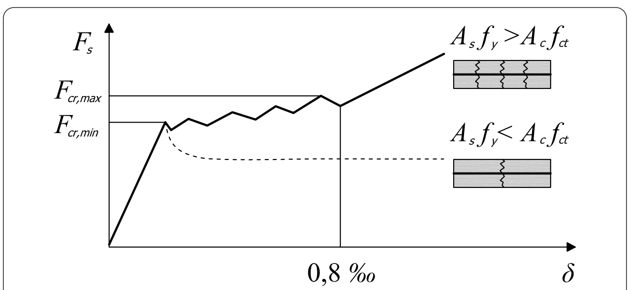

Fig. 4 Relation between imposed strain and tensile force (Krause & Horstmann, 2018)

strains between steel and concrete for base-restrained walls, which proves the necessity to apply Eq. (30) in EN 1992-11 (2004).

2.19 Model Code (2013)

Model Code (2013) does not consider the case of a base-restrained member. However, in the case of the calculation of crack width with the use of detailed method, the model considers the influence of shrinkage during drying on the higher value which is the difference of the mean strains between the reinforcement and concrete which is as follows:

$$\varepsilon_{sm} - \varepsilon_{cm} - \varepsilon_{cs} = \frac{\sigma_s - \beta \cdot \sigma_{sr}}{E_s} + \eta_r \cdot \varepsilon_{sh}, \tag{51}$$

where σ_s is the steel stress in a crack, σ_{sr} is the maximum steel stress in a crack in the crack formation stage, β is an empirical coefficient to assess the mean strain over $l_{s,max}$ depending on the type of loading, η_r is coefficient taking into account shrinkage occurrence, and ε_{sh} is the shrinkage strain.

Another important guideline concerns the differentiation between "crack formation stage" and "stabilised cracking stage" in which a different relation of f_{ctm}/τ_{bms} is considered for long-term load, which significantly affects crack spacing, and also different values of coefficients β and η_r dependent on the type of load and the size of the occurring shrinkage, respectively. According to Model Code (2013), the impact of imposed deformations should be considered together with the load impact. When the crack width is calculated, the superposition of these impacts should be performed at the stage of the determination of stresses in the reinforcement. However, Model Code (2013) does not provide any computational procedures in this matter. In general, the equations contained in Model Code (2013) and EN1992-1-1 (2003), are calibrated only in case of a tie element which is axially stretched by external force.

2.20 Schlicke & Tue, 2015 Model

Schlicke and Tue (2015, 2016) proposed a method for the determination of the minimum reinforcement to limit the crack width in base-restrained members while taking into account the deformation compatibility. This approach is supported by the physical basis of deformation changes in maturing concrete and their redistribution after crack occurrence as opposed to the guidelines contained in some other models, e.g., EN 1992-1-1 (2003).

According to Schlicke and Tue (2015), the concept of the effective concrete area "does not consider the fact that every new secondary crack will increase the steel strain in the primary cracks" (see Bödefeld, 2010). The number of secondary cracks n with considerable imposed deformation was described by equation:

$$n = \left(\frac{\sigma_{rest,\max}}{E_{eff}} \cdot l_{cr} \cdot \frac{1}{w_{k,\lim}} - 1 \right) \cdot 1.1, \tag{52}$$

where $\sigma_{rest,max}$ is imposed deformations stress, E_{eff} is effective modulus of concrete elasticity, $w_{ck,lim}$ is crack width limit, 1.1 is factor denoting a decrease in the width of the subsequent secondary cracks referred to the width of the primary crack, and n is value rounded up to the next integer.

The final minimum reinforcement is calculated in relation to the cracking force of the effective concrete area as well as from the number of secondary cracks ($n > 0$) (Bödefeld, 2010):

$$A_{s,\min} = \sqrt{\frac{d_s \cdot b^2 \cdot d_1 \cdot f_{ct,eff} \cdot (0.69 + 0.34 \cdot n)}{w_{k,\lim} \cdot E_s}} \tag{53}$$

where d_s is diameter of reinforcement, b is width in direction viewed, $f_{ct,eff}$ is effective tensile strength of concrete, and E_s is reinforcement elastic modulus.

When $n \leq 0$, the condition of deformation compatibility is met but a skin reinforcement is required to ensure a robust concrete surface (Schlicke, 2014). The fundamental assumption of the analytical model is taken into account, while determining the stress in the base-restrained wall along the lower edge, the influence of the dead load of the wall in the form of additional bending moment M_G apart from force N_W and moment M_W resulting from the analysis of the wall and the foundation cross section:

$$M_G = 0.5 \cdot \left(\gamma_c \cdot A_{ges} \cdot L_{eff,\max}^2 \right) \tag{54}$$

where γ_c is weight of concrete, A_{ges} is overall area of cross section, and $L_{eff,max}$ is distance from the wall edge to where the moment from the self-weight results in a constant value of stresses in the cross section of the wall and foundation (Schlicke & Tue, 2016):

$$L_{eff,\max} = \sqrt{\frac{2M_w}{\gamma_c A_{ges}} \cdot \frac{I_i}{I_w}} \leq \frac{L}{2} \tag{55}$$

where I_i is total moment of inertia, I_w is moment of inertia of wall cross section, and L is length of wall (accuracy of Eq. (55) in the context of wall length (L) was discussed by Schlicke (2014).

The crack height was analysed in detail by Rostásy and Henning (1990), however, disregarding the self-weight of wall. In the discussed model the crack height depends on stresses σ_R in concrete along the top edge of the crack. If σ_R is below the tensile strength, the crack height will be stopped. The value of stresses ($\sigma_R(h_R)$) depends on the remaining area of concrete without cracks $h_{R,}$ because the tensile force is transferred by concrete until the moment when the crack appears along the entire length of the wall height:

$$\sigma_R(h_R) = \frac{\kappa_R \cdot E_W \cdot b_w \cdot \left(h_R^3 - h_W^3\right) + 6 \cdot N_W \cdot (h_W + h_F)}{6 \cdot b_W \cdot h_R \cdot (2 \cdot h_W - h_R + h_F)} + \frac{\kappa_R \cdot E_W \cdot h_R}{2}, \tag{56}$$

For a cracked cross section, i.e., a smaller cross section of concrete, the curve is calculated after the formula:

$$\kappa_R = \frac{\sigma_{W,u} - \sigma_{W,o}}{E_W \cdot h_W} \tag{57}$$

where $\sigma_{W,u}$ is stresses along the bottom edge of wall and $\sigma_{W,o}$ is stresses along the upper edge of wall.Whereas the crack spacing l_{cr} depends on crack height h_{cr}:

$$l_{cr} = 1.2 \cdot h_{cr}. \tag{58}$$

To sum up, the approach, which is based on the deformation compatibility, is both safer and a more economical estimation of the minimum area of reinforcement in comparison with the guidelines described in EN 1992-1-1 (2003). The presented model defines the way of determining the reinforcement to reduce the width of cracks in maturing concrete. In addition, the model gives the basis for super positioning of additional deformations during the lifetime of constructions. The authors are working on the indispensable parameters necessary to develop the model further.

2.21 Flaga & Klemczak, 2016 Model

Flaga and Klemczak (2016) pointed to the occurrence of concrete decompression after the crack appearance and decrease of tensile stress in reinforcement steel. In the case of the near-surface reinforcement and the self-stresses occurring in it, authors rely on Eq. (7.1) from EN 1992-1-1 (2004). However, on the basis of Flaga (2011) the authors proposed the crack width correction w_{lim} which results in reality from Eq. (7.1) while taking into account the stress decrease $\sigma_s < \sigma_{s,lim}$ after crack occurrence and the tensile stress in concrete between cracks:

$$w_k^{RC} = \left(\frac{1}{3} \div \frac{2}{3}\right) w_k \tag{59}$$

In Eq. (7.1) in (EN1992-1-1, 2004), σ_s is determined for the reinforcement diameter ϕ_s according to guidelines in Rüch and Jungwith (1976) Eq. (60) which reflect the values of ϕ_s shown in Table 5.1 in EN1992-1-1 (2004):

$$\phi_s \leq \frac{3\tau_1 \cdot w_{\lim} \cdot E_s}{f_{yk}^2} \tag{60}$$

Taking into account the different reinforcement diameter, the stress correction is determined according to the following formula:

$$\sigma_{s.\lim} = f_{yk}\sqrt{\phi_s/\varphi} \tag{61}$$

For diagonal reinforcement and restraint stress occurring in it, it is recommended to apply Eq. (7.1) (EN1992-1-1, 2004). However, the essence of the proposed model is the determination of the crack width, i.e., the area in which the reinforcement should be placed. The crack width in base-restrained walls depends on the relations of the wall length to its height and the mean values of bond stress τ_{pm}. In the general case, it is defined as follows:

$$h_{crack} = \frac{\sigma_{wym}^{H=0} - f_{ctm}(t)}{\sigma_{wym}^{H=0} + \sigma_{wym}^{H=h}} \cdot h \tag{62}$$

where $\sigma_{wym}^{H=0}$and $\sigma_{wym}^{H=h}$ are stress in concrete caused by imposed deformations at the wall higher and lower edge, respectively.

2.22 Barre et al., 2016 Model

Barre et al. (2016) in Research Project CEOS.fr presented a multi-layered and detailed description of cracks in reinforced members. For walls restrained along the lower edge, Barre et al. (2016) use the equations from Model Code (2013) and EN1992-1-1 (2004). However, they propose a different equation for shrinkage strain ε_{cs} used in Eqs. (7.6 - 3) in Model Code (2013):

$$\varepsilon_{cs} = 0.5\varepsilon_{ca}(t) + \alpha_T[0.6(T_{\max} - T_{ini}) + T_{ini} - T_{\min}(t)], \tag{63}$$

where T_{ini} is the initial temperature of the concrete at the time of pouring, and T_{min} is the minimum temperature of the concrete up to time t.

In Eq. (63), coefficient 0.5 takes into account stress relaxation which is caused by autogenic shrinkage. While coefficient 0.6 also takes into account the increase of temperature thus causing compressive stresses. In the current guidelines EN1992-1-1 (2004) and in Model Code (2013), the favourable strains during the period of temperature increase are not taken into account in calculations.

2.23 Gilbert, 2017 Model

Gilbert (2017) pays attention to the fact that the influence of concrete shrinkage on the crack width is often not taken into consideration. Concrete shrinkage causes the excessive crack width. Gilbert emphasises that the difference in deformations, which is the stress-related strain resulting from restraint and consists of elastic and creep strains, is as follows:

$$\varepsilon_r = \varepsilon_{actual} - \varepsilon_{T+cs} = \varepsilon_{el} + \varepsilon_{creep} = \frac{\sigma_r}{E_c} + \chi\varphi\frac{\sigma_r}{E_c}, \tag{64}$$

where

$$\varepsilon_{T+cs} = \alpha_c \Delta T + \varepsilon_{cs}, \tag{65}$$

For engineering purposes, Eq. (64) is expressed in stress form:

$$\sigma_r = \varepsilon_r \cdot E_{aaef}, \tag{66}$$

where E_{aaef} is the age-adjusted effective modulus of the concrete:

$$E_{aaef} = \frac{E_c}{(1+\chi\varphi)}, \tag{67}$$

where ϕ is coefficient of concrete creep dependent on hydration time and χ is aging coefficient taking into account the fact that σ_r increases in concrete gradually.

During crack appearance, some of the mean restrained strains are relieved by the crack formation. This group of strains is called the crack-induced strain $\varepsilon_{r.cr}$ and it is essential to calculate the crack width. The average tensile stress between the cracks is

$$\sigma_r = (\varepsilon_r - \varepsilon_{r.cr}) \cdot E_{aaef} \tag{68}$$

Even if cracks do not occur at an early age, the stresses σ_r cannot be ignored, because later strains caused by drying shrinkage and other strains can increase tensile stresses which further leads to crack occurrence.

Gilbert (2017) also writes that it is essential to take into consideration internal stresses as one of the determinants of crack occurrence, which are described with the following equation for the internal part of the member:

$$\sigma_r = -\alpha_c \cdot \Delta T \cdot R_w \cdot E_{aaef} \tag{69}$$

R_w is the coefficient of internal restraint from temperature differentials, according to (Gilbert, 2017) the recommended value is 0.4.

Gilbert (2017) also emphasises the meaning of internal restraint provided by embedded reinforcement. In the case of edge restraint, Gilbert (2017) takes into account foundation and wall rigidity for determination of the distribution of restraint degree and restrained strain ε_r. He defines extreme values along the lower edge of the wall.

According to Gilbert (2017), the maximum crack width w_{max} is expressed as follows:

$$w_{\max} = s_{r,\max} \cdot \varepsilon_{r.cr} = s_{r,\max} \cdot (\varepsilon_r - \varepsilon_{r1}) \tag{70}$$

While according to CIRIA C660 (2007), the residual strain ε_{r1} is approximated by f_{ctm}/E_{cm}, and according to Gilbert (2017), it is defined as follows:

$$\varepsilon_{r1} = \varepsilon_r - \varepsilon_{r.cr} \tag{71}$$

where ε_{r1} is the sum of the elastic and creep strain caused by the average tensile concrete stress between the cracks, and $\varepsilon_{r.cr}$ is the crack-induced strain.

2.24 CIRIA C766 (2018)

In the next edition of CIRIA C766 (2018) widely commented guidelines in the subject of crack control caused by restrained deformation in concrete were discussed in detail and modified. In the case of the calculation of crack width dependent on the imposed deformations in base-restrained members in CIRIA C766 (2018), the procedure is the same as in CIRIA C660 (2007) and is based on the method defined in EN 1992-1-1 (2004). However, some descriptions were discussed in greater detail and developed. Equation (39) used to determine the restrained strain ε_r was modified as follows:

$$\begin{aligned}\varepsilon_r = & K_{c1}[\alpha_c \cdot T_1 + \varepsilon_{ca}(3)] \cdot R_1 + K_{c1} \\ & [(\varepsilon_{ca}(28) - \varepsilon_{ca}(3)) + \alpha_c \cdot T_2] \\ & \cdot R_2 + K_{c2} \cdot \varepsilon_{cd} \cdot R_3\end{aligned} \tag{72}$$

Different values of coefficient K_{c1} and K_{c2} were assigned taking into account the effect of stress relaxation as a result of concrete creep at early age strain during concrete maturing ($K_{c1}=0.65$) and during long-term situation ($K_{c2}=0.5$).

As with CIRIA C660 (2007), the size of strains which determine the crack width is described by the general Eq. (38), which is the basis for the calculation of crack width according to Eq. (41). For calculation of maximum crack spacing $s_{r,max}$, the coefficient k_1 correction was kept taking the influence of poor bond into account. In the case of the coefficient k which takes into account the self-stresses, the value 0.75 for $h=800$ mm proposed in CIRIA C660 (2007) was abandoned for the general guidelines included in EN 1992-1-1 (2004).

2.25 Zych, 2019 Model

The important assumption of this model is the opinion that the crack initiation takes place at a certain height above the place, where the wall and foundation come into contact (Anson & Rowlinson, 1988, 1990; Petersson & Thelandersson, 2001a). This height is dependent on the profile of the temperature changes in the wall. The assumption is important, because as it was presented in, among others, CIRIA C766 (2018) at a certain level above the construction connection to the foundation, the degree of restraint is smaller R_{ax}. Moreover, the height at which the crack occurrence is initiated has a significant influence on the so-called relaxation degree after crack occurrence ΔR_{ax}, which in the presented model is one of the elementary parameters used to determine the first type of crack width. It is assumed in this model that immediately after the crack occurrence, the degree of restraint R_{ax} is reduced depending on the degree of reinforcement (Zych, 2018).

The spacing of cracks of Type I [Eq. (73)] is dependent on both the size and extent of relaxation zone resulting from the first crack as well as the extent of reinforcement degree in the cross section of the zone. In general, the proposed method of calculation of crack spacing gives the results for unreinforced walls or walls that do not meet the requirements of the minimum degree of reinforcement as is proposed by Iványi (1995) and Rostásy and Henning (1989). While the increasing degree of reinforcement affects the decreasing spacing of cracks. Next, the spacing of cracks of Type II is based on the tie model of the restrained rebar on opposite ends as in EN 1992-1-1 (2004) (see Fig. 5):

$$s_{rmI} = 2H \frac{\int_{\zeta_1=0.25}^{\zeta_2=0.5} \Delta R_{ax}(\alpha_D; h_1; \zeta) \cdot d\zeta}{\int_{\zeta_1=0.25}^{\zeta_2=0.5} \Delta R_{ax}(\alpha_D = 0; h_1 = 0.4H; \zeta) \cdot d\zeta}. \quad (73)$$

where H is wall-section height, ΔR_{ax} is relaxation degree resulting from crack occurrence, α_D is the relation D_{11}/E_{cm}, D_{11} is stiffness of the cracked area cross section in the normal direction to the crack plane, and h_1 is height corresponding with the temperature profile change along the wall height from linearly variable to linearly even.

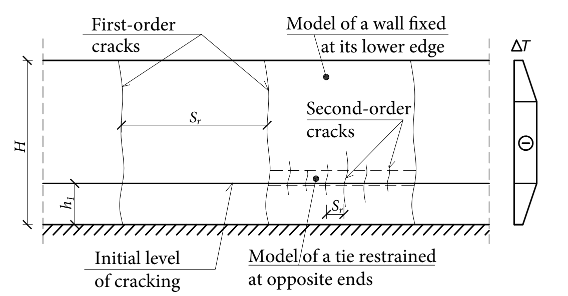

Fig. 5 Assumption of computational model concerning the type of cracks and the place of their initiation (Zych, 2019)

The model takes into account the widening of the cracks in consecutive stages of the imposed restraint occurrence and self-stresses. Measurements of strains that were performed on cracked cross sections of semi-massive structures confirm this fact (Zych & Seruga, 2019). The width increase of cracks of Type I depends on, among other factors: the size and type of load (temperature, shrinkage, external load), current extent of wall cracking, changes of mechanical properties of concrete during the whole period of maturing. The crack width which occurs first is calculated with the following expression:

$$w_{k3} = w_{k1} + \Delta w_{k2} + \Delta w_{k3}' + \Delta w_{k3S}'' + \Delta w_{k3Z}''. \quad (74)$$

where w_{k1} is initial width of the first crack, Δw_{k2} is increase in crack width during stabilized spacing of first-order cracks, $\Delta w_{k3}'$ is increase in crack width from further temperature changes, $\Delta w_{k3S}''$ is increase in first-order crack width resulting from shrinkage, and $\Delta w''_{k3Z}$ is increase in first-order crack width resulting from external load.

2.26 Schlicke et al., 2020 Model and Turner Model 2020

The change of the degree of restraint due to cracking was intensively investigated by Schlicke et al. (2020) and Turner (2020) for further improvement of the TU Graz approach. Their analyses were based on the results of experimental studies performed with the use of adjustable restraining frames (ARFs). Assuming passive conditions of restraint, changes of the partial restraint in the function of time, expected in structures, were reflected. Restraint coeffcient (a) was introduced based on the equations describing the force equilibrium and the compatibility deformations (Schlicke et al., 2020):

$$a = \frac{1}{1 + \frac{E \cdot A_m}{L} + \frac{1}{k_F}} \quad (75)$$

where $E{\cdot}A_m$ is mean axial stiffness of the specimen dependent on the actual modulus of elasticity of concrete and state of specimen's cracking, k_F is equivalent spring stiffness of the frame, and L is length of the specimen.

As indicated by Eq. (75), the degree of restraint depends on the actual stiffness of the specimen and equivalent stiffness of the frame. In general, the degree of restraint thus defined gets reduced owing to the development of the modulus of elasticity to next increase due to the progress of cracking. In the experiments (Schlicke et al., 2020; Turner, 2020) as the modulus of elasticity developed the degree of restraint was reduced from the moment of concreting from 1.0 to

0.65. In the specimen of stabilised cracking pattern and 1% degree reinforcement the degree of restraint next increased to 0.95. The change of the degree of restraint after cracking was defined on the basis of the reduction of forces due to cracking (Schlicke et al., 2020):

$$a\left(t^{II}\right) = \frac{F_{ARF}\left(t^{I}\right)}{F_{ARF}\left(t^{II}\right)} \cdot a\left(t^{I}\right) \leq 1 \tag{76}$$

where t^I is time immediately prior to cracking and t^{II} is time immediately after cracking. Next, taking account of cracking, conditions of partial restraint and the effect of time the tensile force in the specimen was described after the formula:

$$F_{ARF}(t_n) = \int_{t_o}^{t_n} \left(\Delta\varepsilon_{0,p}(t) + \frac{\sum_{i=0}^{n} \Delta w_i(t)}{L} \right) \cdot E \cdot A_m(t) \cdot a(t) dt \tag{77}$$

where t_0 is time at setting of concrete, t_n is relevant point in time, n is number of existing cracks, F_{ARF} is resultant force, and $\varepsilon_{0,p}$ is imposed deformation during hardening of concrete in the so-called passive phase, i.e., consisting of thermal strain, shrinkage strain, and viscoelastic strain.

As indicated by Eq. (77), the increase of imposed strain and change of crack width are superimposed with the currently present degree of restraint and with the stiffness of the specimen. Schlicke et al. (2020) and Turner (2020) also performed modified tests applying ARFs, whose aim was to reflect the behaviour of a part of a wall restrained along the bottom edge. On the basis of the tests they stated, *inter alia*, that in the thick members a special cracking scheme is observed consisting of a primary crack (originated by restraint from external restraints) and secondary cracks around the primary crack (formed at definitely lower tensile force which results from the active contribution of reinforcement). It was found that concrete near the surfaces is more restrained.

Turner (2020) took a similar approach to defining the changes of the degree of restraint, according to which it depends on the changes in member stiffness:

$$a(t) = \frac{1}{1 + \frac{(E \cdot A_m)(t)}{k_R \cdot L}} \tag{78}$$

where k_R is spring stiffness of the frame (frame no 1: 1137 kN/mm, frame no 2: 1053 kN/mm).

Next, dependence Eq. (78) was employed in modified equations describing the change in the stress in concrete both prior to and after cracking, which next was a basis for a modification of Eq. (53) defining the minimum area of reinforcement:

$$A_{s,\min} = \sqrt{\frac{d_s \cdot b^2 \cdot d_1^2 \cdot f_{ct,eff} \cdot (0.5 + 0.34 \cdot n)}{w_{k,\lim} \cdot E_s}} \tag{79}$$

and a modification of Eq. (52) defining the number of cracks:

$$n = \left(\frac{\sigma_c^I}{E_c} \cdot l_{cr} \cdot \frac{k_{\text{mod}}}{w_{k,\lim} \cdot a^{0.6}} - 1 \right) \cdot 1.1 \tag{80}$$

where k_{mod} is factor for effect of crack formation, 0.6…0.85 depending on requirements and stressing, and a is degree of restraint in the uncracked state.

3 Discussion

During the period of concrete maturing as well as during the period of structure exploitation, there are many factors that determine the risk of crack occurrence on the lower edge of restrained walls and also determine the final width of the cracks. On the basis of the above-mentioned computation models, some key aspects can be listed which are usually taken into account during calculations in a diversified or modified form. This proves the fact that there remains an ongoing process of searching for the most accurate models of assessing crack width. Furthermore, in this paper, aspects concerning the calculation of the following parameters are discussed: imposed deformations, restraint degree, concrete creep, the total effect of early impacts and those occurring during the exploitation period, internal deformations, crack spacing and the fundamentals of physical models. The possibility of the application of tried and tested solutions from other analytical models for further standard studies was suggested. Directions are given for further research, which more precisely specify computational models and develop them. Individual aspects can be analysed in greater detail in the subject literature. However, they have not been verified in complex models of the cracking of maturing concrete.

3.1 Aspect of the Imposed Deformations and Their Modifications

Imposed strains which result from concrete temperature changes and concrete shrinkage are elementary variables in all analytical models. Standard guidelines in different countries help to determine only the value of shrinkage strains, but the manner in which the thermal strains are determined during concrete maturing is not usually described. Of the presented standards, the only exception is the standard ACI 207.2R-07 (2007), and the guidelines in CIRIA C660 (2007) and CIRIA C766 (2018).

What is an indisputable advantage of the models developed in later years is that the impact of imposed strains is taken account of more accurately, and primarily, a more precise identification of the value of their changes at a time proper for crack monitoring (e.g., CIRIA C766, 2018). Considerable simplifications in this field or a complete lack of guidelines as to the specification of thermal strains (e.g., in EN1992-3 (2006)) may result in misassessment of cracking time, and thus the width of cracks.

In the first models, for example, Evans and Hughes (1968) and Hughes and Miller (1970), 0 imposed strains were only taken into account as mean values (i.e., without temperature gradients) resulting from different boundary conditions on the higher and lower edge of the wall. One of the first models providing such calculation possibilities was Stoffer's model Stoffer's (1978). Different deformations depending on the wall height are discussed in standard models: ACI 207.2R-07 (2007), EN1992-3 (2006), JSCE (2011) based on compensation plane method (CPM) (Al-Gubi et al., 2012).

In practice, an imposed strain, which after occurring on the section between cracks remains restraint, does not contribute to the increase of crack width. This phenomenon was already taken into account in the first computational models as the factor that reduces imposed strain by the value of $½\varepsilon_{ctu}$. In the case of BS 8007 (1987), the constant value of 100με was adopted. Other models such as Paas (1998) and Schlicke and Tue (2015, 2016) already take this effect into account in assumptions concerning deformation compatibility. In the model by Flaga and Klemczak (2016), it is arbitrarily assumed that the crack width after taking into account the stress decrease after cracking and the tensile stress in concrete is reduced by 1/3 to 2/3 of the crack width. This is described more generally by Gilbert (2017), as restrained strain reduced by crack-induced strain [see Eqs. (70) and (71)]. This phenomenon was not taken into account in the current European standard EN1992-3 (2006). However, this is taken into account in, among others, the current British guidelines CIRIA C766 (2018).

In general in the majority of models the fact that part of imposed strains remains in non-cracked sections is taken into account. There is a contradiction between this approach and that of EN1992-3 (2006) according to which all the imposed strains accumulate in the cross section of cracks, which in this particular context should overestimate their width.

3.2 Aspect of Restraint Coefficient

The next element which is strongly established in the procedure of crack width calculation is the restraint coefficient of imposed strains which results from restraint joints. The values of this coefficient given in the relevant tables or simple equations are provided to help engineers determine the restrained part of the imposed strains. However, these coefficients are available only for the simplest construction elements, while more complex constructions should be modelled in the system of the appropriate construction joints. In the first models (Evans and Hughes (1968); Hughes and Miller (1970); Stoffers (1978)) this coefficient was not applied, which can be referred to in the case of complete restraint, i.e., $R_{ax}=1$. The coefficient of the external restraint degree was openly introduced for the first time by Harrison (1981). In further studies (among others ACI 207.2R-95, 1995; CIRIA C766, 2018; EN1992-3, 2006) the coefficient was described more precisely depending on the considered case of restrained element. In some guidelines (e.g., ACI 207.2R-95, 1995) this coefficient refers to the elasticity range of strains. In other guidelines (e.g., EN1992-3, 2006) strains connected with concrete creep are also taken into account.

In most of the presented models, an assumption is made of the infinite length of the wall, additionally fixed along the bottom edge. In practice, it corresponds to cases of very long walls joined to a massive foundation slab. Analysing the cases with shorter walls joined to a flexible foundation with these models should lead to an overestimation of the necessary amount of reinforcement, especially in the upper part of the wall. In models based on the assumption that the wall is infinitely long, the coefficient was neglected, thus its value was 1.0, like in the models by Rostásy and Henning (1989) and Ivanyi (1995). The issue of the influence of the actual geometry of construction is extensive and it is the subject of many advanced analyses both in the plastic–elastic scope (Klemczak & Knoppik-Wróbel, 2015); Knoppik-Wróbel, 2015; and after cracking (Schlicke et al., 2020; Zych, 2018).

The restraint coefficient should be applied first of all to check the criterion of cracking, because in the overwhelming majority of models, it is determined for uncracked structures. However, due to the lack of more detailed models, it is also used in current standard EN1992-3 (2006) to calculate crack width. In addition, in EN1992-3 (2006) it is pointed out that this approach is poorly researched. In the author's opinion the use of the same factor in the stage prior to cracking and that after cracking is contradictory to the fact that immediately after cracking the stiffness of the system decreases. This decrease mainly depends on the reinforcement manner and the number of cracks, which may negatively affect its precise specification. What is an advantage of the approaches followed nowadays is that in the context of engineering calculations cracks widths are determined on the safe side, and in a certain group of models this factor is also used as a basis for rough assessment of cracks heights, that is a zone, where more intensive reinforcement is required.

The approach of Kheder et al. (1994a), which took into account both the degree to which the member was restrained before and after cracking, resulted in an important change in the convention of calculating crack width in members restrained along the bottom edge. The model proposed by Zych (2019) is based on this concept, where this difference is called the relaxation degree. However, this approach is more complex and thus more problematic in its application in engineering models. Next, Schlicke et al. (2020) presented an analysis of the changes of the degree of restraint in the function of the changes in the stiffness of hardening concrete and in the function of its cracking. The analysis was supported by extensive laboratory tests, and the defined factor (in the engineering approach) was used in, inter alia, specifying stresses in concrete both prior to and after each cracking episode. According to the author, further research and work on the development of analytical models should include a focus on the difference between the restraint before and after cracking.

3.3 Aspect of Concrete Creep

Concrete creep in the period of concrete maturing has a significant influence on the reduction of stresses caused by imposed strains. Despite the possibility to calculate the creep coefficient (e.g., in EN1992-1-1, 2004), the constant increase of strains during the changing properties of concrete as it matures seems to be problematic. Initially, this phenomena was not taken into account (Evans & Hughes, 1968; Hughes & Miller, 1970; Stoffers, 1978) and it was then regarded as responsible for a 50% reduction of thermal and shrinkage stresses (BS8007, 1987; Harrison, 1981). In some later models, creep also was not taken into account (Ivanyi, 1995; Paas, 1998; Rostásy & Henning, 1989), which was the result of the specific nature of the performed experimental tests upon which the models were calibrated. The originally applied coefficient was 0.5, which took into account the phenomena of creep and was subject to further modifications to values of 0.6 (Kheder et al., 1994a) and 0.65 (CIRIA C766, 2018).

In is written in EN1992-3 (2006) that if it is well founded, creep with the use of E_{ceff} should be considered in calculations of stresses in the case of uncracked cross sections. However, the time in which the creep coefficient should be determined is not indicated. In the procedure describing the calculation of crack width, there is no information about this. In EN1992-3 (2006), what raises doubts are the relatively low restraint coefficients R_{ax} for which the values of comparable restraint cases are identical to those in BS 8007 (1987). In BS 8007 (1987), lower values of coefficient R_{ax} took into account the beneficial influence of creep. There is no description given in EN1992-3 (2006), but it may be stated that the interpretation is comparable or it can be accepted that such low values of restraint coefficient also take into account the susceptibility of adjacent members and the beneficial increase of temperature in the heating period, which is neglected in calculations in engineering models. If we adopt the interpretation that the restraint coefficient R_{ax} takes into consideration creep as in BS 8007 (1987), it can be stated at the same time that creep is taken into account in the models of base-restrained walls by a restraint coefficient of 0.5.

Of models presented earlier in this paper, the most comprehensive approach toward concrete creep in the analysis of cracking of maturing concrete was presented by Gilbert (2017). As in EN1992-3 (2006), Gilbert uses concrete creep to calculate stresses with the effective modulus of concrete elasticity (E_{ceff}). In addition, Gilbert takes into consideration the ageing coefficient, but most importantly he uses creep strains directly in the calculation of crack width, Eqs. (70, 71).

There is a contradiction between the above statements and the model proposed in ACI 207.2R-95 (1995) in which it is assumed that after cracking the time progressive drying shrinkage of concrete eliminates the favourable effect of creep. Consequently, the favourable effect of creep on cracks width decrease should not be taken into account. Moreover, such an assumption is most reasonable also in the case of structures subjected to other loads which increase tension.

Moreover, it must be stated that creep acts in two ways. First of all, it reduces tensile stress in concrete between cracks, which reduces the stress in reinforcement through the crack and as a result of this, the crack width is smaller. Second, creep has a negative influence on concrete-steel bonding. Creep at the location of bonding contributes to the later dislocation of rebars in concrete and the widening of cracks. For this reason, thorough research and analyses are required of concrete–steel bonding in the area of cracks that were formed during concrete maturing and widened due to additional external loads. This phenomenon has not yet been the subject of detailed experimental tests.

3.4 Aspect of Additivity of Imposed Strains and Exploitation Loads

Models which consider the total effect of early age and long-term imposed deformations and exploitation loads should be used in future calculations of crack width. The current standards (ACI 207.2R-07, 2007; EN1992-1-1, 2004; EN1992-3, 2006) do not take such cases into consideration for either the issue of cracking criterion or the calculation of crack width. There are two scenarios to consider. In the first scenario, early age deformations do not cause any cracks and thus cracks are often neglected in further analysis. This creates the false assumption that stresses in early age deformations are not important in the period of building exploitation. This is usually justified by the argument that the values of stresses during the period of concrete maturing are small in comparison with the values of stresses during the period of building exploitation. In the second scenario, cracks occur in the period of concrete maturing. The designer checks the width of the cracks in the period of concrete maturing and also during the period of building exploitation. Then, to prevent further deterioration, s/he decides to apply reinforcement limiting the width of the cracks.

As far as the cracking criterion is concerned, the occurrence of different types of deformations and stresses was defined in CIRIA C660 (2007), and then modified in CIRIA C766 (2018). In the case of the calculation of crack width, the necessity to consider the total effect of early age stresses and long-term exploitation stresses was described in DIN EN 1992–1/NA (2011). Bamforth et al. (2009) presented a two-stage approach toward the calculation of crack width for a member under imposed deformations. The designer performs calculations immediately after the occurrence of a crack and when the crack widens in the later period. A model which considers the total effect of early and late imposed deformations and also exploitation stresses was proposed by Zych (2019). In his model, the current crack width in maturing concrete is calculated as the width increase resulting from consecutive strains and stresses.

An obvious drawback of the vast majority of the models described above is that they are not compatible with the models dedicated to strains and loads occurring at a later stage. Theoretically this limits their applicability; however, in engineering practice, when it is necessary to perform calculations, the use of different models of completely different bases, i.e., sometimes for the period of concrete maturing, for mature concrete at other times, may lead to contradictions as, e.g., in cracks spacing or the extent of strains/loads affecting the given crack width. Thus, further detailed research on additivity of early age and long-term imposed deformations and external stresses is required. Such research would enable the calibration of models.

3.5 Aspect of Self-stresses

Following the restraint coefficient of imposed strains by external restraints, models occasionally provide restraint coefficients of internal strains. This issue has a bigger significance in the case of massive constructions than in semi-massive constructions (CIRIA C766, 2018).

Standards EN1992-3 (2006), EN1992-1-1 (2004) take self-stresses into account in calculation of the width of cracks which are mainly caused by imposed strains. However, these standards apply only coefficient k which enables less reinforcement in the method of the simplified control of cracking to reduce crack width to the limit value. In addition, these guidelines enable the consideration of the total effect of self-stresses and imposed strains in a conventional manner.

Guidelines in CIRIA C766 (2018) propose models for calculating crack widths caused by self-stresses in massive constructions, in which the role of imposed strains is significantly smaller. Taking into account the durability of rebars calculating the depth of surface cracks is also an important issue, as was proposed by Flaga (2011). This approach enables the more economical determination of the effective surface of reinforcement under tension, especially in constructions of bigger mass. This solution was adapted in model proposed by Zych (2019) which takes into consideration the increase of crack width caused by imposed deformations as a result of self-stresses.

In general, in semi-massive construction, to which walls restrained along the lower edge often belong, factor k applied in the formula for $A_{s,min}$ (acc. to EN1992-1-1, 2004) should take into consideration not only the member geometry but also the relationship between the probable self-stresses and the stresses caused by external joints, because these two components determine the time of cracking occurrence. This concerns both self-stresses generated by temperature fluctuations during the period of concrete maturing and shrinkage strains caused by concrete drying in a later period. In the case of these uneven strains, there is a lack of detailed research and analyses assessing their influence on the change of the width of cracks that were formed due to early imposed strains.

3.6 Aspect of Crack Spacing

In computational models, the maximum crack spacing is often taken into account. It results from the assumption that the strain differences between steel and reinforcement which occur between cracks determine the width of these cracks. However, in a base-restraint wall there are many more factors affecting crack spacing than there are in the model of a restrained tie on the opposite edges. For this reason, the calculation of maximum crack spacing tends to be quite problematic.

In the first models considerable simplifications were applied. They resulted from the fact that the crack spacing is within a wide range of probable values, which is quite often dependent on the wall height H (e.g., Evans &

Hughes, 1968; Hughes & Miller, 1970; Ivanyi, 1995; Paas, 1998; Stoffers, 1978). At a certain stage of calculation, the spacing was also treated as an arbitrary value which equals H (Rostásy & Henning, 1989) independently of, among others, the amount of reinforcement. Currently, in most models (Model Code, 2013); (Bamforth et al., 2009; Barre et al., 2016; CIRIA C660, 2007; CIRIA C766, 2018; DIN EN1992-1-1/NA, 2011; EN1992-3, 2006; Gilbert, 2017) spacing results from the relationship between concrete-steel bond stresses and concrete tensile strength, i.e., an assumption similar to the model of a restrained tie on opposite edges. Two of the above-mentioned trends regarding crack spacing calculation determine the basic differences in the obtained results. Spacing at the level of H concerns expansion cracks which reach wall coping, the spacing of which results from joints in the wall base and is independent of the amount of wall reinforcement. By contrast, the second concept of crack width calculation focuses on the issue locally, assuming that the next crack will occur independently of the external restraints in a wall at a distance at which the tensile stress increase in concrete will be so big that the concrete tensile strength is exceeded. In this concept, reinforcement is the fundamental variable. Certain doubts are raised by the fact that the second approach is based on a tie model not on the base-restraint wall model. A different approach concerning the determination of stresses in a wall is presented in ACI 207.2R-95 (1995), in which a crack occurs in the middle of each uncracked section which in practice corresponds with the cases of poorly reinforced walls, i.e., the situation when the reinforcement in a crack cross section does not affect the next place of crack occurrence. There are also intermediate models in which crack spacing, especially with regard to the magnitude of tensile strength, is dependent on the current height of cracking (Flaga, 2011; Schlicke & Tue, 2015, 2016; Turner, 2020). Moreover, Turner (2020) and Schlicke et al. (2020) proved that in thick members cracking happens along a special scheme, namely, it consists of the so-called primary crack and symmetrically located shorter secondary cracks. The last group of models makes crack spacing dependent on both the wall height and reinforcement (Al-Rawi & Kheder, 1990; Zych, 2019). Certainly, the calculation of crack spacing is impeded by the fact that the real crack spacing mainly depends on the restraint degree of a member which changes whenever the wall is cracked. This change mainly depends on the stiffness of external joints and the amount of reinforcement.

Therefore, there is a number of contradictions in the calculation of cracks spacing, which to a large extent result from arbitrary simplifications of the given model. Apart from the differences in the assumptions described above as to what factors affect cracks spacing, a distinction can be made between models in which cracks layout is considered stabilised (e.g., CIRIA C766, 2018; EN1992-1-1, 2004; Evans & Hughes, 1968; Rostásy & Henning, 1989) and those in which the calculations take account of stage cracking, which is more corresponding to reality (e.g., ACI 207.2R-95, 1995; Schlicke & Tue, 2015; Schlicke et al., 2020; Turner, 2020). The advantage of the latter models is the possibility of an evaluation of the scale of cracking and precise determination of crack width, which is economically justified in reinforcement spacing.

3.7 Physical Basics of the Models

The milestones proposed in particular models and highlighted in the present paper indicate a significant progress in the calculation tools as concerning both their more scientific description and a larger number of aspects taken into account (e.g., cracks height or member stiffness after cracking). Some of the described models are based on: the empirical knowledge (Evans & Hughes, 1968), matching parts of models which originally were dedicated to external loads (EN1991-1, 2004; EN 1992-3, 2006), matching to experimental evidence (Rostásy & Henning, 1989) or to FEM results (JCI, 2008), and still other ones are based on deformation compatibility (Schlicke & Tue, 2015; Schlicke et al., 2020; Turner, 2020). The differences between the models result not only from the adopted method or type of adopted assumptions, but also the amount and type of data in relation to which they are calibrated. Consequently, there are fundamental differences not only in the calculation method but foremost in the possibility of taking account in the given model of for instance: aspects increasing the precision of estimated width of a crack, assessment of the scale of cracking or specification of cracks height. The most common assumption is the calculation of cracks width as a product of their maximum spacing and imposed deformations. Compared with simplified models a considerable part of the present models enables more economical design of structures. In the case of the former ones calculations should be on the safe side, but they result in solutions much more uneconomical.

Due to the existence of certain research results and various proposals for modelling the development of cracking of the base-restrained walls, in the context of EN 1992-1-1 (2004), this issue is described as being based on "British" beliefs, in which the use of the tie model is also valid for members working in other static schemes. According to EN1992-3 (2006), the model of a base-restrained wall has not been sufficiently analysed. However, in the context of cracks spacing there appears an essential contradiction in a completely different static scheme of tie model and wall restrained along the bottom edge.

Even in cases where the model takes positive foundation flexibility into account (e.g., Rostásy & Henning, 1989), the calculations usually focus on so-called dilatation cracks and disregard cracks which do not reach the upper edge of the wall and need to be analysed separately. The assumption is entirely correct but only in the case when such a crack will in fact occur in the structure. In such cases, however, a more practical solution is to use joints that at least partially compensate for imposed strains at these points. In this respect, a more versatile approach is included in ACI 207.2R-07 (2007), providing the possibility of calculating tensile stresses at a given level of the wall, and applying adequate reinforcement.

In some guidelines, there is lack of an analytical basis in the equation used for the calculation of crack width or the analytical basis considers only a part of a model. For instance in ACI 207.2R-07 (2007) in Eq. (32) there is lack of quantity describing the imposed strains. Equation (32) was adjusted to the observation of crack occurrence during the period of concrete maturing. However, in ACI 207.2R-07 (2007) the way of calculating the stresses in reinforcement steel is vital and in the case of base-restrained walls, it is much more complex than in, for example, EN1992-3 (2006). The guidelines JCI (2008) do not consider any physical basis known from other models, but they use the theory of probability, a series of calculations performed in 3D-FEM and comparisons with the state of cracking of 728 structural members.

It is also arguable whether to take account of the effect of relaxation immediately after cracking (Flaga & Klemczak, 2016), which although periodically reduces cracks width, but in less reinforced members significantly affects the reduction of stresses causing further cracking. Without any doubt it is necessary to take account of this fact in the models including the stage cracking of a member.

3.8 Guidelines for the Future

In connection with the periodic update of standard guidelines in different countries and also ongoing work over the new Eurocode version, potential corrections and follow-ups can be proposed on the basis of the review of computational models presented above. The subject is connected with both the adoption of the appropriate criterion of cracking and a model to determine the width of cracks in semi-massive structures. In the future, the most important issues will be as follows:

- Taking into consideration the combination of stresses caused by imposed deformations generating stresses in the early age stages of concrete maturing together with the later occurring stresses during exploitation in both the cracking criterion and models for crack width calculations. The most advanced standard in this subject is DIN EN 1992-1-1/NA (2011), which points to the need for such a superposition and constitutes a step forward in computational assumptions. However, there is lack of detailed computational guidelines in this scope.
- The introduction of a variable degree of restraint with regard to the imposed deformations in the future standard guidelines. The value of this restraint during, for example, the period of concrete maturing or during the period of exploitation of the structure depends on the current case of restraint. According to CIRIA C766 (2018), this has a significant influence on the risk of cracking and the crack width.
- Defining the height of the area in which it is necessary to apply reinforcement to prevent early cracking as is proposed by Schlicke and Tue (2016) or Flaga (2011). This will contribute to more economical distribution of the reinforcement.
- Taking into account the total influence of not only imposed deformations or external loads on the calculated crack width but also self-stresses which are caused by uneven deformations in the cross section of the wall. Currently, in EN1992-1-1 (2004), internal stresses are taken into account with the use of coefficient k only when the minimum reinforcement is determined.
- Considering the alternative models for the analysis of restrained members cracking along the lower edge with the use of kinematic compatibility (Schlicke & Tue, 2016).
- Distinguishing between at least two calculation stages: stage 1 immediately after the crack occurrence; stage 2 later when the crack widened (Bamforth et al., 2009; Zych, 2019).
- Giving more precise interpretations of restraint coefficient in comparison to the current version of EN1992-3 (2006) in the scope of the concrete creep phenomena.

The proposals listed above concerning the supplements or changes in the current version of EN1992-3 (2006) will only be possible when a sufficient amount of data is gathered in subject literature.

3.9 Further Research Recommendations

The most important recommended directions for further scientific research are as follows:

- Examination of the phenomena of the earlier loss of bond between concrete and steel resulting from cracks occurring during maturing of concrete. In addition, there is a need to examine the influence of this early loss of bond between concrete and steel on additional strains or stresses occurring later when the concrete is mature. This phenomenon is not included in the current approach of standards (EN 1992-1-1 2004; EN 1992-3 2006), a constant value of factor k_1, i.e., a constant mean concrete tensile strength to mean bonding stresses ratio, is stipulated.

- More detailed examination and modelling of the influence of concrete creep on the changes in imposed strains ε_r both before and after crack occurrence. Most current guidelines which take concrete creep into consideration rely on only one unchanging coefficient.
- Experimental research and computational analyses concerning changes to the width of early cracks caused by imposed strains and external loads occurring later. Current scientific research does not provide a complex study of this problem (Bamforth et al., 2009; Zych, 2019). In particular, there is a lack of appropriate scientific tests.
- An interesting trend in creating analytical models is the "merging" of results of numerical calculations performed within an adequate range of variables, to generalise the model by a specific parameter (Paas, 1998).

4 Conclusions

This research study presents the development of analytical models from the year 1968 of the calculation of the crack width in reinforced walls restrained along the lower edge. Some of these models may become an alternative solution to the design problem, assuming that they meet the basic assumptions of the presented models.

The presented analytical models were commented on in the range of the most important computational aspects, i.e., imposed deformations, self-stresses, restraint coefficient, concrete creep, long-term imposed deformations, exploitation stresses, crack spacing and the physical basics of the model. Detailed conclusions have been presented earlier in this paper in the discussion about computational aspects.

Despite partial criticism of the presented computational models, they are engineering tools that should, in most cases, guarantee solutions on the safe side. To some extent, they also form the basis and inspiration for further work on the development of these models in various computational aspects.

The current guidelines of the standard and computational capabilities of basic commercial engineering programs are at a level that does not usually allow designers to precisely predict either the size of the imposed deformation or its consequences. It is, therefore, necessary to obtain information from specialist literature. Moreover, in the case of using simplifications, which are always adopted at the design stage, knowledge of the physics of the issue is important, which from a practical point of view allows proper interpretation of the results.

Large number of computational aspects which are decisive in the complex analytical verification of cracks in maturing concrete proves that this subject is difficult. Thus, for engineering purposes, the creation of simplified methods based on detailed solutions seems to be justified.

Attention has been drawn to the current possibilities for the introduction of supplements to the standard models of calculation of crack width in walls restrained along the lower edge. In addition, possible further directions of studies in the discussed subject have been indicated.

Currently, one of the most important aspects of the development of the analytical models of cracks is examining the overall influence of early imposed deformations together with the loads occurring later during the period of exploitation as it takes place in reality in the case of most semi-massive constructions. However, this would require the creation of a database of measuring data which would gather results from a wide range of experimental tests which would then be the basis for the creation of complex analytical models.

Abbreviations
CPM: Compensation plane method; FEM: Fine element method; RC: Reinforced concrete; SLS: Serviceability limit state.

Acknowledgements
Not applicable.

Author contribution
MZ made the whole work. I hereby declare that my contribution to this Review Article is 100%. The author read and approved the final manuscript.

Authors' information
MZ (RILEM TC 254-CMS member) is an academic at Cracow University of Technology. In 2011, he was granted the academic degree of Doctor of Technical Sciences at Cracow University of Technology, and in 2018, he was granted a postdoctoral degree. In 2019 Associate Professor. His interests include the following areas: water-tightness and durability of semi-massive RC tank walls, early age cracking, as well as the design and strengthening of RC and prestressed tanks for the storage of liquids.

Funding
No funding, not applicable.

Availability of data and materials
Not applicable.

Declarations

Competing interests
I declare that I have no competing interests.

Received: 31 March 2022 Accepted: 22 August 2022
Published online: 06 January 2023

References

ACI 207. (1973) Effect of restraint volume change and reinforcement on cracking of mass concrete. ACI Committee 207, American Concrete Institute.

ACI 207.2R-95 (1995) Effect of restraint, volume change and reinforcement on cracking of mass concrete. ACI Committee 207, American Concrete Institute.

ACI 224R-01 (2001) Control of cracking in concrete structures. ACI Committee 224, American Concrete Institute.

ACI 207.2R-07 (2007) Report on thermal and volume change effects on cracking of mass concrete. ACI Committee 207, American Concrete Institute.

Al Rhawi, R. S., & Kheder, G. F. (1990). Control of cracking due to volume change in base-restrained concrete members. *ACI Structural Journal, 87*(4), 397–405.

Al-Gubi, M., Jonasson, J. E., Nilsson, M., Hedlund, H., & Hösthagen, A. (2012). Simplified methods for crack risk analyses of early age concrete. Part 1: Development of Equivalent Restraint Method. *Nordic Concr Research, 46*, 17–38.

Anson, M., & Rowlinson, P. M. (1988). Early-age strain and temperature measurements in concrete tank walls. *Magazine of Concr Research, 40*(145), 216–226.

Anson, M., & Rowlinson, P. M. (1990). Field measurements for early-age strains in concrete. *Magazine of Concr Research, 42*(153), 203–212.

Bamforth, P. B., Denton, S., & Shave, J. (2009) A new approach for the design of reinforcement to control cracking resulting from continuous edge restraint to continuous. In: Concrete: 21st Century Super. *Building a Sustainable Future*, Fib Symposium, (pp 22–24). London, Hero.

Bamforth, P. B., Shave, J., & Denton, S. (2015) Design for early age thermal cracking. In: *Bridge Design to Eurocodes* (pp 239–249). England: Thomas Telford Limited. https://www.icevirtuallibrary.com/doi/abs/10.1680/BDTE.41509.0019 AND https://www.icevirtuallibrary.com/doi/epdf/10.1680/BDTE.41509.0019

Bamforth, P. B. (2007). *Early-age thermal crack control in concrete*. CIRIA C660.

Bamforth, P. B. (2018). *Control of cracking caused by restrained deformation in concrete*. CIRIA C766.

Barre, F., Bisch, P., Chauvel, D., Cortade, J., Coste, J.-F., Dubois, J.-P., Erlicher, S., Gallitre, E., Labbé, P., Mazars, J., Rospar, C., Sellier, A., Torrenti, J.-M., & Toutlemonde, F. (2016) *Control of Cracking in Reinforced Concrete Structures*: Research Project CEOS.fr. Wiley, New Jersey.

Beeby, A. W., & Forth, J. P. (2005) Control of cracking walls restrained along their base against early thermal movements. In: *Proceedings 6th International Congress On Global construction, ultimate concrete opportunities* (pp 123–132). London: Thomas Telford.

Bödefeld, J. (2010) Rissmechanik in dicken Stahlbetonbauteilen bei abfließender Hydratationswärme. Dissertation, Universität Leipzig.

BS 5337. (1976). *Code of practice for the structural use of concrete for retaining aqueous liquids*. British Standards Institution.

BS 8007. (1987). *Design of concrete structures for retaining aqueous liquids*. British Standards Institution.

Buczkowski, W. (1993). Thermal load of rectangular tanks buried in the ground. *Inżynieria i Budownictwo, 12*, 506–508.

DBV. (2006). *Merkblatt Rissbildung, Begrenzung der Rissbildung im Stahlbeton- und Spannbetonbau*. Deutscher Beton und Bautechnik-Verein E.V.

DIN EN 1992-1/NA. (2011). *Nationaler Anhang—National festgelegte Parameter—Eurocode 2 Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken, Teil 1-1*. Allgemeine Bemessungsregeln und Regeln für den Hochbau.

DIN EN 1992–1 (2010) Eurocode 2, Bemessung und Konstruktion von Stahlbeton- und Spannbetontragwerken, Teil 1–1: Allgemeine Bemessungsregeln und Regeln für den Hochbau. Deutsche Fassung, EN 1992–1–1:2004 + AC:2010, Germany.

Emborg, M. (1989) Thermal Stresses in Concrete Structures at Early Ages, Dissertation, Luleå University of Technology.

Eurocode 2. (2003). *Design of concrete structures: General ruls and rules for buildings, Draft prEN EN 1992-1-1:2003*. Brussels: CEN.

Eurocode 2. (2004). *Design of concrete structures—Part 1–1: General rules and rules for buildings, EN 1992-1-1*. Brussels: CEN.

Eurocode 2. (2006). *Design of concrete structures—Part 3: liquid retaining and containment structures, EN 1992–3*. CEN.

Evans, E. P., & Hughes, B. P. (1968). Shrinkage and thermal cracking in a reinforced concrete retaining wall. *Proceedings of the Institution of Civil Engineers, 39*(1), 111–125.

Falkner, H. (1969). *Zur Frage der Rißbildung durch Eigen- und Zwängspannungen infolge Temperatur in Stahlbetonbauteilen, Heft 208*. Deutsche Ausschusses für Stahlbeton.

Fib. (2013). *Model code for concrete structures 2010*. Ernst & Sohn.

Flaga, K. (2004). *Shrinkage stress and reinforcement in concrete structure, [Monograph 295]*. Cracow University of Technology.

Flaga, K. (2011). *Shrinkage stress and reinforcement in concrete structure, [Monograph 391]*. Cracow University of Technology.

Flaga, K., & Klemczak, B. (2016). *Structural and technological aspects of thermal-shrinking stresses in massive and medium-thick concrete structures*. Cracow Univ. of Technology.

Gergely, P., & Lutz, L. A. (1968) Maximum crack width in reinforced concrete flexural members. *Causes, Mechanism, and Control of cracking in concrete*, American Concr Institute SP-20 87–117

Gilbert, R. I. (2017). Cracking caused by early-age deformation of concrete—Prediction and control. *Procedia Engineering, 172*, 13–22. https://doi.org/10.1016/j.proeng.2017.02.012

Harrison, T. A. (1981). *Early-age thermal crack control in concrete*. CIRIA.

Hognestad, E. (1962). High strength bars as concrete reinforcement, Part 2, Control of flexural cracking. *Journal of PCA Research and Development Laboratories, 4*(1), 46–63.

Hughes, P. B., & Miller, M. M. (1970). Thermal cracking and movement in reinforced concrete walls. *Proc of the Institution of Civil Engineers, 45*(2), 65–86.

Iványi, G. (1995). Bemerkungen zu Mindestbewehrung in Wänden, Minimum reinforcement in the walls. *Beton- Und Stahlbetonbau, 90*(11), 283–289.

JCI. (2008). *Guidlelines for Control of Cracking of Mass Concrete*. Japan Concrete Institute.

Jędrzejewska, A., Kanavaris, F., Zych, M., Schlicke, D., & Azenha, M. (2020). Experiences on early age cracking of wall-on-slab concrete structures. *Structures, 27*, 2520–2549. https://doi.org/10.1016/j.istruc.2020.06.013

JSCE. (2011). *Guidelines for concrete. No .15: Standard specifications for concrete structures*. Design Japan Society of Civil Engineers.

Kheder, G. F., Al-Rawi, R. S., & Al-Dhahi, J. K. (1994a). A study of the behavior of volume change cracking in base restrained concrete walls. *Mater and Structures, 27*(7), 383–392.

Kheder, G. F., Al-Rawi, R. S., & Al-Dhahi, J. K. (1994b). Study of the behavior of volume change cracking in base-restrained concrete walls. *ACI Materials Journal, 91*(2), 150–157.

Kheder, G. F., & Fadhil, S. (1990). Strategic reinforcement for controlling volume-change cracking in base-restrained concrete walls. *Mater and Structures, 23*(5), 358–363.

Kiernożycki, W. (2003). *Concrete massive structures*. Cracow: Polish Cement.

Klemczak, B., & Knoppik-Wróbel, A. (2015). Degree of restraint concept in analysis of early-age stresses in concrete walls. *Engineering Structures, 102*(11), 369–386.

Knauff, M. (2012). *Calculation of reinforced concrete structures according to eurocode 2*. PWN Scientific Publishers.

Knauff, M., Grzeszykowski, B., & Golubińska, A. (2018). *Examples of calculating reinforced concrete structures. Cracking*. PWN Scientific Publishers.

Knoppik-Wróbel, A. (2015) Analysis of Early-Age Thermal-Shrinkage Stresses in Reinforced Concrete Walls. Dissertation, Silesian University of Technology.

Krause, H. J., & Horstmann, M. (2018). Planung und Bemessung von WU-Konstruktionen—Entwurfsgrundsätze und deren statisch konstruktive Umsetzung. *Beton- Und Stahlbetonbau, 113*(S1), 20–35.

Leonhardt, F. (1978). *Vorlesungen über Massivbau*. Springer.

Paas, U. (1998). *Mindestbewehrung für verformungsbehinderte Betonbauteile im jungen Alter*. Beuth Verlage GmbH.

Pettersson, D., Alemo, J., & Thelandersson, S. (2002). Influence on crack development in concrete structures from imposed strains and varying boundary conditions. *Construction and Building Mater, 16*(4), 207–213.

Pettersson, D., & Thelandersson, S. (2001a). Crack development in concrete structures due to imposed strains. Part I: Modeling. *Mater and Structures, 34*(1), 7–13.

Pettersson, D., & Thelandersson, S. (2001b). Crack development in concrete structures due to imposed strains, Part II: Parametric study of a wall fully restrained at the base. *Mater and Structures, 34*(1), 14–20.

Rostásy, F. S., & Henning, W. (1989). Zwang in Stahlbetonwänden auf Fundamenten. *Beton- Und Stahlbetonbau*. https://doi.org/10.1002/best.198900300

Rostásy, F. S., & Henning, W. (1990). *Zwang und Rissbildung in Wänden auf Fundamenten, Heft 407*. Deutscher Ausschuß für Stahlbeton.

Rostásy, F. S., & Onken, P. (1994). A practical planning tool for the simulation of thermal stresses and for the prediction of early thermal cracks in massive concrete structures. In R. Springenschmid (Ed.), *Proceedings of the International Symposium RILEM: Thermal Cracking in Concrete at Early Ages*. CRC Press Taylor & Francis Group.

Rüch, H., & Jungwirth, D. (1976). *Stahlbeton-Spannbeton Bd. 2; Berücksichtigung der Einflüsse von Kriechen und Schwinden auf das Verhalten der Tragewerke*. Werner-Verlage.

Schleeh, W. (1962). Die Zwängspannungen in festgehaltenen Wandscheiben. *Beto- Und Stahlbetonbau, 57*(3), 64–72.

Schlicke, D. (2014) Mindestbewehrung zwangbeanspruchter Betonbauteile unter Berücksichtigung der erhärtungsbedingten Spannungsgeschichte und der Bauteilgeometrie, Dissertation, TU Graz.

Schlicke, D., Hofer, K., & Nguyen, V. T. (2020). Adjustable restraining frames for systematic investigation of cracking risk and crack formation in reinforced concrete under restrained conditions. *Advanced Techniques for Testing of Cement-Based Materials, 2*, 211–239.

Schlicke, D., & Tue, N. V. (2015). Minimum reinforcement for crack width control in restrained concrete members considering the deformation compatibility. *Structural Concrete, 2*, 221–232.

Schlicke, D., & Tue, N. V. (2016). Mindestbewehrung zur Begrenzung der Rissbreite unter Berücksichtigung des tatsächlichen Bauteilverhaltens Teil 2: Anwendung für Wände auf Fundamenten und Abgrenzung zum Risskraftnachweis nach EC2. *Beton- Und Stahlbetonbau, 111*(4), 210–220.

Stoffers, H. (1978). *Cracking due to shrinkage and temperature variation in walls*. Delft University of Technology and I.B.B.C.

Turner, K., Schlicke, D., & Tue, N.V. (2017) Systematic investigations with restraining frames for reinforced concrete. In: *2nd International RILEM/COST Conference on Early Age Cracking and Serviceability in Cement-based Materials and Structures* (EAC2), (pp 579–584), Brussels, ULB-VUB.

Turner, K. (2020) Ganzheitliche Betrachtung zur Ermittlung der Mindestbewehrung für fugenlose Wasserbauwerke. Doctoral Thesis, TU Graz.

Van Breugel, K. (1982) Development of temperature and properties of concrete as a function of the degree of hydration. In: *Proceedings Rilem International Conference on Concrete at Early Ages*, (pp 179–185), Avril: Presses de 1`ENPC. https://doi.org/10.1016/0262-5075(82)90045-8

Van Breugel, K. (1995). Numerical simulation of the effect of curing temperature on the maximum strength of cement-based materials. In R. Springenschmid (Ed.), *Proceedings of the International Symposium RILEM: Thermal Cracking in concrete at Early Age* (pp. 127–134). CRC Press Taylor & Francis Group.

Zych, M. (2018). Degree of external restraint of wall segments in semi-massive reinforced concrete tanks: Part II Rectangular and cylindrical segments. *Structural Concr, 19*(3), 829–838. https://doi.org/10.1002/suco.201700037

Zych, M. (2019). A new model for crack control in reinforced concrete tank walls. Part I: Analytical investigation. *ACI Structural Journal, 116*(3), 85–94.

Zych, M., & Jaromska, E. (2019). Differences between DIN EN 1992-1-1/NA2011 guidelines and EN-1992-1-1:2004 standard with respect to the design of the minimum reinforcement and crack width from imposed deformation. *Archives of Civil and Mechanical Engineering, 19*(3), 627–638.

Zych, M., & Seruga, A. (2019). A new model for crack control in reinforced concrete tank walls. Part II: Comparison with experimental results. *ACI Structural Journal, 116*(3), 95–105.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.